This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.
+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.
+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

ELEMENTS OF PHYSICS

CREW AND JONES

Educ T 219.07.305

THE INTERNATIONAL METRIC SYSTEM OF WEIGHTS AND MEASURES *

Facts about the Metric System

1791. — Reported to the French National Assembly by the Academy of Sciences.

1799. — Accepted by Holland, Denmark, Switzerland, Spain, and several minor states.

1866. — Made, by Act of Congress, "lawful throughout the United States."

1875. — Permanent organization by seventeen nations of an International Bureau of Weights and Measures, with headquarters at Sèvres, near Paris

1893. — Internationa ı made fundamental standards.

GINN AND COMPANY
Ex Libris

Ar TEM

(i) The decimal atly facilitating computation.

(ii) The extremel length, area, volume, and weight to one a

(iii) The uniform s.

The fundamental meter — the unit of length. From this of mass (gram) are derived. All other sions or multiples of these three simply l purposes *one cubic decimeter equals on weighs one kilogram.* The metric tables a ds *meter*, *gram*, and *liter* with the six nu ing tables:

Prefixes			Units
milli-	= one thousandth	$= \frac{1}{1000} = 0.001$	meter for length
centi-	= one hundredth	$= \frac{1}{100} = 0.01$	
deci-	= one tenth	$= \frac{1}{10} = 0.1$	
(unit)	= one		gram for weight or mass
deka-	= ten	$= \frac{10}{1} = 10.$	
hecto-	= one hundred	$= \frac{100}{1} = 100.$	
kilo-	= one thousand	$= \frac{1000}{1} = 1000.$	liter for capacity

* The maj a pamphlet of this title pul Department of Commerce a om the Smithsonian Physi

Harvard College Library
THE GIFT OF
GI

Linear Metric Measure

1 milli-meter (mm.)	= 0.001 meter	10 mm. = 1 cm.
1 centi-meter (cm.)	= 0.01 meter	10 cm. = 1 dm.
1 deci-meter (dm.)	= 0.1 meter	10 dm. = 1 m.
*1 meter (m.)	= 1 meter	
1 deka-meter (Dm.)	= 10 meters	10 m. = 1 Dm.
1 hecto-meter (hm.)	= 100 meters	10 Dm. = 1 hm.
1 kilo-meter (km.)	= 1000 meters	10 hm. = 1 km.

Metric Measure of Area

The table of areas is formed by squaring the measures of length, as in our common British system.

100 sq. mm. = 1 sq. cm.		1 sq. in. = about 6½ sq. cm.
100 sq. cm. = 1 sq. dm.		
100 sq. dm. = 1 sq. m.		
100 sq. m. = 1 sq. Dm. = 1 are.		
100 sq. Dm. = 1 sq. hm. = 1 hectare = about 2½ acres.		
100 sq. hm. = 1 sq. km.		1 acre = about 0.4 hectare.

Metric Measure of Volume

The cubic measures are the cubes of the linear units.

1000 cu. mm. = 1 cc.
1000 cc. = 1 cu. dm.
1000 cu. dm. = 1 cu. m.

The *cubic meter* is the unit of volume. *A cubic meter of water weighs a "metric ton"* and is equal in weight to 1 *kiloliter* of water. The thousandth part of the cubic meter, namely, 1 cubic decimeter, is called the liter. For very small volumes the cubic centimeter is used. *One thousand cubic centimeters make one liter.*

1 cu. dm. = 1000 cc. = 1 liter (l.)
1 liter = 1.05668 liquid quarts, or 0.9081 dry quarts.

Metric Measure of Weight (or Mass)

One cubic centimeter of water weighs with sufficient accuracy for all practical purposes a *gram*, which is the unit of mass. A liter of water weighs 1 kilogram, *i.e.* 1000 grams. A thousand liters of water weigh a metric ton.

* In the United States one meter equals 39.37 inches **exactly**.

3 2044 097 019 996

ELEMENTS OF PHYSICS

THE MACMILLAN COMPANY
NEW YORK · BOSTON · CHICAGO
ATLANTA · SAN FRANCISCO

MACMILLAN & CO., LIMITED
LONDON · BOMBAY · CALCUTTA
MELBOURNE

THE MACMILLAN CO. OF CANADA, LTD.
TORONTO

ELEMENTS OF PHYSICS

FOR USE IN HIGH SCHOOLS

BY

HENRY CREW

PROFESSOR OF PHYSICS, NORTHWESTERN UNIVERSITY

REVISED BY

FRANKLIN T. JONES

TEACHER OF PHYSICS, UNIVERSITY SCHOOL, CLEVELAND

New York

THE MACMILLAN COMPANY

1909

All rights reserved

HARVARD COLLEGE LIBRARY
GIFT OF
GINN & CO.
DEC 11 1930

COPYRIGHT, 1909,
BY THE MACMILLAN COMPANY.

Set up and electrotyped. Published September, 1909.

Norwood Press
J. S. Cushing Co. — Berwick & Smith Co.
Norwood, Mass., U.S.A.

FROM THE PREFACE TO FIRST EDITION

THE author's confession of faith is somewhat as follows:

1. Physics is not a series of disconnected subjects, including Mechanics, Sound, Light, Heat, and Electricity. It is, on the contrary, a body of well-organized truth, forming one great whole; illustrating, as do few other subjects, what Dickens called "the universal dove-tailedness of things." It is, in fact, *the science of matter and energy*, the most fundamental of all the physical sciences, the foundation of nearly all branches of engineering.

2. A science covering fields apparently so diverse, yet so intimately connected, demands an elementary treatment which shall be rigidly consecutive. When the student passes from Mechanics to Sound or from Heat to Electricity, he must not feel that he is taking up a new subject. If he does, something is wrong; and the chances are that his memory is being trained at the expense of his thinking power. Under such conditions he may learn many methods; he will hardly learn the one thing most to be desired — method.

Helmholtz, in his autobiographical sketch, makes one remark of especial significance to every instructor in elementary physics. Speaking of his early youth, he recalls that he met no difficulty in memorizing the poetry of the great masters because "the flow of thought was natural." Now physics, in too many of our schools, ranks as a most difficult subject. But dealing, as it does, with the familiar phenomena of daily life, and requiring, as it does, only

a small fraction of the algebraic knowledge which the average student has already acquired, the author is inclined to believe that the difficulty lies chiefly in the presentation. If the presentation be smooth and if the flow of thought be natural, the elements of physics will not approach in difficulty the first few books of Geometry.

On the other hand, the author has the utmost sympathy with the following remark of Professor Tait concerning his text-book on *Heat:*

"He who expects to find this work, elementary as it is, everywhere easy reading will be deservedly disappointed. No branch of science is free from real and great difficulties, even in its elements. Any one who thinks otherwise has either not read at all or has confined his reading to pseudo-science."

3. An elementary presentation of physics should begin by resuming what might be called the experience of the average youth of sixteen years. The number of physical facts which a student of this age has accumulated is astounding. Seldom, indeed, does the instructor appeal to him in vain for a verification of an elementary fact. The demand, therefore, is not so much for new facts, or for sheer facts of any kind, as for an orderly arrangement and an ability to use these facts.

4. The author's experience is that secondary students find the use of algebraic language in the definition of physical quantities and sometimes in the description of physical facts exceedingly helpful, especially when the notation is well chosen.

* * * * * * *

But let it not be forgotten that the algebraic definition should come last of all. First the idea must be acquired, then its description given in clear English, and, *last of all*, in algebraic shorthand.

5. The importance of limiting an elementary discussion of physics to matters that are really elementary and of avoiding mere descriptions of certain more or less ephemeral forms of apparatus cannot be overestimated.

* * * * * * *

6. Physics is an experimental science. Its foundations are experimental results. With rare exceptions, such as possibly Maxwell's "Electromagnetic Theory of Light," all important contributions to the subject have been made through experiment. Experiment is the court of highest appeal in all physical results predicted by mathematical analysis. To attempt, therefore, to teach sound physics and good scientific method without laboratory work on the part of the student is to attempt the impossible. Accordingly the present volume is intended only for use in connection with a course of laboratory instruction.

In conclusion, I wish to express great indebtedness to my friends, Professor C. A. Perkins of the University of Tennessee and Mr. J. D. Thompson of Chicago, for many valuable suggestions. To another friend, Mr. C. E. Linebarger of the Lake View High School, I am under obligations for many important excisions in the manuscript.

To my colleagues, Professor Holgate, Dr. Tatnall, Dr. Snyder, and Mr. Jakway, I owe thanks for their assistance in various directions.

I shall be greatly indebted to any reader who will be kind enough to point out sins either of omission or commission, and especially those portions of the book which need to be more clearly expressed.

HENRY CREW.

EVANSTON, ILLINOIS,
May, 1899.

REVISER'S PREFACE

In physics the pupil receives instruction from three sources — the teacher, the laboratory, and the text-book. The first is the vital agent; the last two are properly aids to the teacher. In this book it is hoped that he will find an efficient assistant.

Three elements — namely, (i) simplicity, (ii) logical sequence, and (iii) repetition — are essential to ideal teaching.

It is the aim of this book, as far as possible, to assist the teacher by providing an outline of the "fundamentals of physics," which shall be easily within the comprehension of the average pupil sixteen years of age; which shall appeal to the everyday experiences not only of boys but also of girls, and shall show them physics as a science of daily life; which shall reveal without overemphasis the historical development of physics; which shall be arranged with such logical sequence that the science may appear the orderly unit that it is, not a series of more or less disconnected chapters on Mechanics, Sound, Heat, Electricity, and Light; which shall give also an opportunity both in time and material for what is generally called "drill."

The book is written primarily to assist the pupil in explaining the natural phenomena of the world about him. While it is arranged in such a way that it should prove easy reading, it is planned for study not as a reader in physics nor as an encyclopedia. At the close of a year's work, however, it may fairly be assumed that the pupil has acquired a working knowledge of what we have called

the fundamentals of physics. The book represents no more work than can be thoroughly mastered in one year's time. Moreover, plenty of leeway is allowed for excursions into many interesting theories and experiments which cannot be properly explained in a volume of this size; — the molecular theory of matter, the kinetic theory of gases, the theory of color, the electromagnetic theory of light, the theory of electrolytic dissociation, the ionic hypothesis, the discharge of electricity through gases, aerial flight, wireless telegraphy and telephony, and radioactivity. There is also abundant time for more detailed study of such commercial processes and mechanisms — applications of fundamental principles — as the teacher may desire to introduce. Some mechanisms, such as the clock, cream separator, hydraulic press, steam engine, gas engine, dynamo, motor, camera, opera glass, etc., are referred to and even briefly described in the text. Others, just as important, such as refrigerating apparatus, steam and water turbines, magnetos, electric and hydraulic elevators, locomotive engines, cranes, steam pumps, automobile engines, etc., are left to the teacher for explanation by lantern slides, models, or diagrams, as he chooses. Such a plan saves such an immense amount of time in first going over the work that a class may easily have eight or ten weeks for review at the end of the year.

Abundant material for a many-sided review is provided in the Appendix. It is believed that such a collection of questions, selected from the examination papers of schools in all parts of the country, closely represents the consensus of opinion among teachers, not only of what pupils should *know*, but also of what they should be able to *do* at the close of a year's work in physics.

The thanks of the reviser is gratefully tendered to the many teachers and principals whose combined assistance

made this collection of examination questions possible. Each question is credited to the school from whose paper it was taken.

The reviser is under the deepest obligation to the author of the "Elements of Physics" for his extensive assistance and for his constant coöperation and advice. Full responsibility is assumed by the reviser for all parts of the book, — especially for all changes from the original editions, — and to him are to be charged all errors.

Professor Frank P. Whitman of Adelbert College, Professor Dayton C. Miller of Case School of Applied Science, Miss Mildred Day Potter of Laurel School, Mr. George B. Eisenhard of Lincoln High School, and Mr. Harold B. Reed of East High School, all of Cleveland, read parts of the manuscript and gave many valuable suggestions, for which thanks are rendered.

The last three assisted in reading the proof together with Mr. N. Henry Black of Roxbury Latin School, Mr. Irving O. Palmer of Newtonville High School, Mr. E. J. Rendtorff of Lake Forest Academy, Mr. Charles H. Smith of Hyde Park High School, and Mr. Willis E. Tower of Englewood High School, to all of whom thanks are due for corrections and assistance. Mr. H. B. Spelman of University School made many of the drawings. The Remington Arms Co., the Tiffany Electrical Clock Co., and other firms were very kind in supplying pictures from which diagrams were made. To many friends and acquaintances among science teachers, and especially to those in Cleveland and vicinity, acknowledgment is made of their suggestions, not a few of which are here incorporated.

FRANKLIN T. JONES.

CLEVELAND, OHIO,
August, 1909.

CONTENTS

METRIC SYSTEM, TABLES, USEFUL NUMBERS . . *Inside covers*

PAGES

INTRODUCTORY 1–2

CHAPTER I

MOTION

Translation — Rotation — Velocity and Speed — Acceleration — Laws of Uniformly Accelerated Motion — Solution of Problems 3–19

CHAPTER II

GENERAL PROPERTIES OF MATTER

Inertia — Newton's Laws of Motion — Gravitation — Matter a Vehicle of Energy — Work — Energy — Machines — Power — Elasticity 20–107

CHAPTER III

SPECIAL PROPERTIES OF MATTER

Liquids — Pascal's Law — Principle of Archimedes — Specific Gravity — Gases — Boyle's Law 108–145

CHAPTER IV

WAVES

Wave Motion — Water Waves — Waves in Strings — Resonance 146–163

CHAPTER V

SOUND

Production of Sound — Sound a Wave Motion — Characteristics of Musical Tones — The Musical Scale — Musical Instruments 164–194

CHAPTER VI

Heat

Heat and Temperature — Conduction — Convection — Radiation — Effects of Heat — Quantity of Heat — Nature of Heat — Heat Engines 195–243

CHAPTER VII

Magnetism

Magnets — The Compass — The Earth a Magnet — Theory of Magnetism — Magnetic Induction 244–257

CHAPTER VIII

Electrostatics

Electrification and Electric Charges — Electrostatic Induction — The Condenser 258–274

CHAPTER IX

Electric Currents

Voltaic Cells — Induced Currents — Transformer — Dynamo — Telephone — Effects of Electric Currents — Motor — Ohm's Law 275–330

CHAPTER X

Light

Rectilinear Propagation — Photometer — Diffraction — Reflection — Refraction — Total Reflection — Light a Wave Motion — Color — Lenses — Mirrors — Images — Optical Instruments 331–388

APPENDIX

Questions for Review 389–422

The Solution of Numerical Problems 422–427

Index 429–434

ELEMENTS OF PHYSICS

ELEMENTS OF PHYSICS

INTRODUCTORY

SOME students enter upon the study of physics with keen anticipation of the explanation of X-ray tubes, wireless telegraphy, dynamos, and many very complex and correspondingly interesting and wonderful machines; others are fearful of encountering strange and more or less difficult facts with which they have little or no acquaintance. Both groups are considerably surprised to find that this study is mainly an explanation and systemization of knowledge about common things, — the motions of the earth, the formation of dew, the electric light, the pump, the curved path of a baseball, the rising of cream, eclipses, echoes, shadows, rain and snow, and what not, — with which they are already more or less familiar.

The purpose of the study of physics is quite as much to furnish the student with a logical and easily remembered arrangement of his present knowledge as to put him in possession of new facts.

Every subject — whether language, science, or mathematics — has its own peculiar vocabulary; and physics is no exception. Its language, though remarkably clear and concise, is not particularly difficult; but, if progress is to be easy, then certain fundamental definitions and modes of

expression peculiar to the description of observations and experiments must be thoroughly mastered. The student will find his interest in the subject not far from proportionate to the effort he puts forth.

In physics, as in other subjects, the study of the first few pages is likely to offer the greatest difficulties, mainly because new ideas or old ideas in new form are at that time being introduced most rapidly. The student is assured, however, that in the following pages unreasonable demands will not be made upon his ability to understand. For instance, in the first chapter he will meet careful definitions of motion, velocity, and acceleration, and equally careful methods of expressing them. Practically all these ideas concerning motion and velocity are the property of the student long before he begins the study of physics. Acceleration may be a new and difficult idea; hence, considerable thought and hard work may be necessary before it is thoroughly understood.

Throughout the subject the easy and difficult are bound to appear together. Each has its part, not only in the logical presentation of the subject, but also in the growth in ability which should result from the study of physics.

CHAPTER I

MOTION

1. Most of the motions with which we are already familiar are exceedingly complicated. Perhaps the simplest are the motions of a street car, of a railroad train, or of the hands of a clock; but we shall have accomplished a great deal when we can describe even these with reasonable accuracy. In a volume of this size it will be desirable to consider only a few of the less complex motions.

For convenience when one wishes to refer to a body so small that its dimensions may be neglected in comparison with other distances involved, he calls this body a **particle.** Thus the radius of the earth, approximately 4000 miles, is so small in comparison with the distance of the earth from the sun, 93,000,000 miles, that the astronomer frequently treats our planet as a particle. A golf ball driven high into the air may perhaps be considered as a particle; but while it stands on the tee one must regard it as a body of very appreciable size.

But before one can speak definitely about the *motion* of a particle, he must be able to define its position.

Position of a Particle in a Plane

2. When geographers describe the position of a town, they assume that the reader already knows the position of some point on the earth, which may be called a **point of reference.** Such a point of reference for the surface of the

earth is the intersection of the meridian through Greenwich with the equator. This fixed, the location of a town is definitely described when two things are known about it; viz., its *latitude* and its *longitude*. Thus the island of St. Helena is in 15° 55′ south latitude and 5° 43′ west longitude.

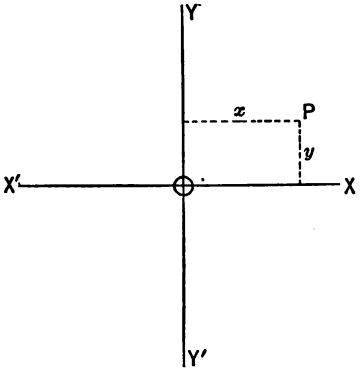


FIG. 1. Showing one method of locating a point in a plane.

A similar method is very frequently used in physics; *e.g.* the position of a point, P (Fig. 1), with reference to another point, O, *in the same plane*, is described when two mutually rectangular straight lines are drawn through the point O, and the perpendicular distances of the point P from these two lines respectively are given. The point O is generally called the **origin**. The horizontal line of reference, $\overline{OX}$, is generally called the **axis of X**; and the vertical line of reference, $\overline{OY}$, the **axis of Y**. The longitude, x, of the point P is called its **abscissa**; while the latitude, y, is known as its **ordinate**. Abscissas are positive when drawn to the right from the axis of Y; negative when drawn to the left. Ordinates are positive when drawn upward from the axis of X; negative when drawn downward.

In the laboratory, we shall have frequent occasion to draw curves; and this is generally done by locating points in the manner just described.

Navigators and surveyors generally employ a different method of locating one point with reference to another in the same plane. They give the straight-line **distance** from

the first point to the second; then they give the **direction** of the line joining the points. This method also is very frequently used in physics. Let O (Fig. 2) be the point of reference, and $\overline{OX}$ any direction which we know. There is no doubt about the exact position of a particle at P as soon as we know the distance r, between the points O and P, and the angle u, between the lines $\overline{OP}$ and $\overline{OX}$. The angular distance u we shall always consider *positive when measured in a direction contrary to that in which the hands of a clock move.* This direction is called **counter-clockwise.** In like manner, u is considered negative when measured in a *clockwise* sense. The line $\overline{OX}$ we may call the **line of reference.**

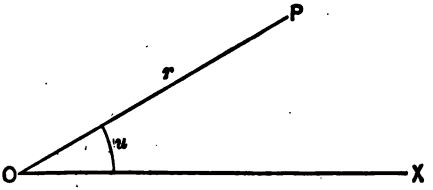


FIG. 2. A second method of locating a point in a plane.

PROBLEMS

1. Draw a pair of axes at right angles to each other. With reference to these axes, locate the points A, B, C, and D, which have abscissas and ordinates as follows:

POINT	ABSCISSA	ORDINATE
A	$+2$	$+7$
B	$+5$	-10
C	-8	-4
D	$+3$	-5

2. A city is numbered from a pair of main streets intersecting at right angles. A house is located at 1900 Fifty-fifth Street, southeast. Draw a diagram to represent its position.

3. What city is in longitude 88° W., latitude 42° N.?

4. A farm is surveyed from the northeast corner. The other corners are distant respectively 40 rods 30° east of south, 80 rods 30° west of south, and 60 rods due west of this corner. Draw a diagram of the farm.

5. The hour hand of a clock is $+45°$ from the horizontal position, the minute hand $-90°$. Draw a diagram of the clock face. What time does it show? Is more than one answer correct? Explain.

Quantities which change in Value

3. The quantities which one meets in physics are of exactly the same kind as those studied in ordinary arithmetic, except in one particular. The difference is as follows. In arithmetic we deal mainly with quantities having fixed values, such as "10 horses," "5 gallons of water," etc. These are called **constant quantities.** In physics, however, we shall very often have to deal with quantities which are all the time changing their value. For instance, a vessel leaves New York and steams due east at the rate of 8 miles an hour. What is her distance from New York at any given time? Her distance is increasing all the while, and we cannot answer the question until we are given the time at which her distance is desired. For her distance is different at each instant. Quantities which are continually changing are called **variable quantities.** The angle between the hour hand and the minute hand of a watch is a good illustration of a variable quantity, for this angle is all the while increasing or diminishing, yet, at any one instant, has a perfectly definite value.

Measurement of Quantities

4. In the determination of the position of a point we have seen that the measurement of a distance is always involved; sometimes also the measurement of an angle. *The measurement of a distance is the comparison of this distance with some standard distance which we agree to call unity.* A comparison of this kind is, indeed, the essential feature in all measurement.

Unit of Length. Meter. Centimeter

5. Almost the entire scientific world has agreed to take as the **standard length** the distance at the temperature of

melting ice between two marks on a certain platinum-iridium bar preserved in the International Bureau of Weights and Measures at Sèvres, near Paris. This distance is called a **meter**. The one hundredth part of the distance

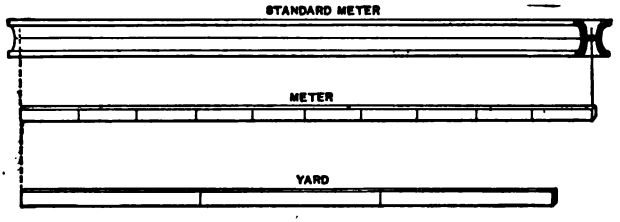


FIG. 3.

1 meter = 39.37 inches
1 yard = 36 inches

between these two marks is taken as the **unit of length** and is called a **centimeter**.

It will be observed that this definition is purely arbitrary, a mere matter of convention. The fact that the meter is so chosen as to be nearly equal to one ten-millionth of the distance from the earth's equator to its north pole is interesting; but it does not enter into the definition of the unit of length.

The student should estimate his own height, also the length and width of this page in centimeters; and then test the accuracy of his estimates by actual measurement with a meter stick.

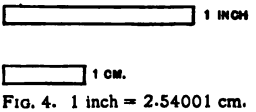


FIG. 4. 1 inch = 2.54001 cm.

English and American engineers generally employ the **yard as the standard of length** and the **foot as the unit of length**. In the United States the yard is defined by act of Congress as $\frac{36}{39.37}$ of the meter.

Measurement of Areas

6. Very seldom, indeed, are areas measured by direct comparison with a unit of area. Generally it will be found more convenient to measure the *linear* dimensions of the surface in question, and, from these, to *compute* by geometry the area of the surface.

Measurement of Volumes. Unit of Volume

7. Standards of volume, such as the bushel, the quart, the cubic foot, are already familiar. In scientific work the **cubic centimeter** is adopted as the **unit of volume.** An

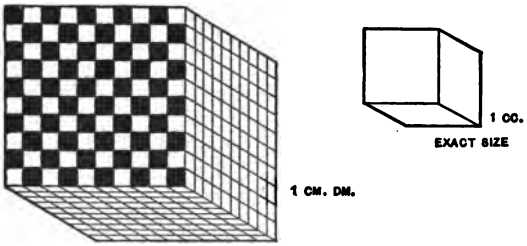


Fig. 5. 1 liter = 1 cu dm. = 1000 cc. = 1.0567 liquid qt.

acquaintance with the metric system makes the wisdom of this choice at once apparent.

In the case of volumes whose linear dimensions can be measured, *e.g.* a parallelopiped or a sphere, it is frequently more convenient to compute the volume than to measure it directly.

PROBLEMS

1. Write out the table for metric linear measure, also English linear measure.

2. Write out the table for metric areal measure, also English square measure.

3. Write out the table for metric cubic measure, also English cubic measure.

4. What is the value of the inch in centimeters, of the centimeter in inches?

5. What is the value of the meter in inches?

6. What is the value of the liter in cubic inches?

7. What is the value of the liquid quart in cubic inches?

Definition of Motion. Translation. Rotation

8. Motion is change of position. — If a particle is at P (Fig. 6) at one time and later is at Q, it has moved because its position has changed. If it moves along the straight line PQ, its motion is **rectilinear.** When a body moves so that all particles describe parallel paths, the motion is a **pure translation.** The motion of the piston rod of an engine or of a sled sliding to and fro in its own tracks is a pure translation.

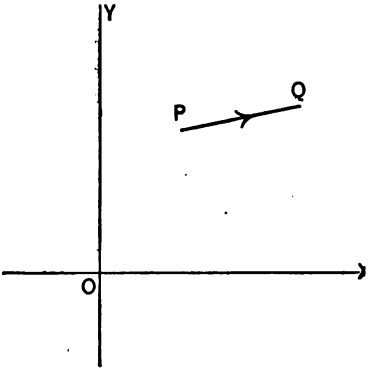


Fig. 6. Change of position.

A very common type of motion is that of a wheel rotating on a shaft (Fig. 7). Here the path of each particle is a circle and the motion is a **pure rotation.**

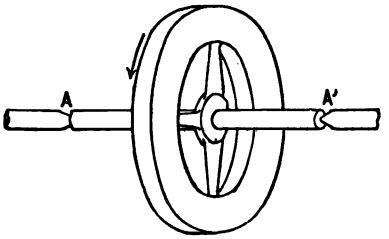


Fig. 7. A case of pure rotation.

Frequently a body is translated and rotated simultaneously, as in the wheels of a train, bicycle, or automobile. As one attempts to analyze the motions of the parts of even the simplest machine, he sees how complex these

ordinary motions are. Every one has long been familiar with the rotation of the earth on its axis and its revolution about the sun; and yet it is very difficult to conceive, much less describe, one's own path in space.

PROBLEMS

In the following motions of rigid bodies distinguish clearly which are cases of pure translation, which are cases of pure rotation, and which are cases involving both translation and rotation:

1. The motion of a door on its hinges.
2. The motion of a carriage wheel with reference to its axle.
3. The motion of a carriage wheel with reference to the road on which it travels.
4. The motion of the blade, with reference to the handle, when one opens a pocketknife.
5. The motion of the following parts of a steam engine — piston, connecting rod, flywheel. (See Fig. 194.)
6. The motions of the parts of a moving belt.
7. The motion of a screw driven into wood.
8. The various motions of the earth.

Definition of Velocity. Rate

9. In getting from one point to another one is interested not only in *how far* he goes, but also in *how long* it takes. Accordingly it is necessary to introduce two new words into our vocabulary; namely, *speed* and *velocity*, which for the present we shall use to express the same idea. Each means *how fast*, though there is a further scientific distinction which will not be explained here. If a train is scheduled to travel 300 miles in 12 hours, it must go on the average 25 miles in an hour and its average speed is said to be "25 miles per hour," written for compactness $25 \frac{\text{mi.}}{\text{hr.}}$. Evidently, this "speed" or "velocity" is obtained by divid-

ing a "distance" by the "time" required to pass over this distance, so that we may say

$$\text{velocity} = \frac{\text{distance}}{\text{time}}$$

Whenever any quantity, in this case a distance, is divided by a time, the quotient is a **rate.** Thus the rate of interest is the amount of money one pays for the use of a dollar, divided by the number of years he has used the dollar. The death rate is the total number of deaths among a thousand people during any period divided by the number of years in the period. It must be carefully noted as a fundamental fact in the language of physics that "**rate**" **always means "divided by time."**

Hence, **velocity is average distance traversed per unit of time,** or velocity is rate of change of position, or **velocity is rate of motion.**

This definition may be briefly expressed in algebraic symbols by using the initial letters for distance, velocity, and time, thus,

$$\bar{v} = \frac{d}{t}, \quad \text{or} \quad d = \bar{v}t$$

The symbol $^{-}$ is used to represent an *average velocity.*

The student is reminded that all such equations are introduced into physics to simplify and abbreviate the expression of the *relations between physical quantities.* **The thing to be understood and remembered is the relation rather than the equation.**

Time. Unit of Time

10. By universal consent the sun and stars are our timekeepers. "U. S. Observatory Time" on the jeweler's clock is a familiar sign. **The mean (*i.e.* the average) time occupied by the sun in one apparent revolution about the earth is known as the mean solar day,** and is univer-

sally employed as a standard of time. **The $\frac{1}{86400}$ part of a mean solar day is called a second, and is used by the scientific world as the unit of time.**

Intervals of time can be compared with marvelous accuracy by means of watches, or clocks, and chronographs.*

Unit of Velocity and Speed

11. Having adopted the centimeter as the unit of length and the second as the unit of time, the proper scientific **unit of velocity is a velocity of one centimeter per second. The engineer, however, generally employs the foot per second as the unit of velocity.**

PROBLEMS

1. Find the average speed, in miles per hour, of a steamer which makes the passage from New York to Liverpool, 3032 miles, in 6 days.

2. An 18-hour train from New York to Chicago, 926 miles, must make what average speed?

3. A 13-inch shell travels a distance of 14,700 feet in 7 seconds. Find its average speed.

4. A gun is fired at a distance of 1 km.; the report is heard 2.9 seconds later. From this observation, compute in meters per second the velocity of sound in air.

5. A locomotive whistles when 1 mile away. The sound is heard 4.7 seconds later. What is the velocity of sound in air in feet per second?

6. (*a*) Light travels from the sun to the earth in about 8 min. 20 sec. The distance of the sun from the earth is about 93 million miles. Find the velocity of light in miles per second.

(*b*) The velocity of light is 300 million meters per second. Find the distance of the sun in kilometers.

7. Assuming that the circumference of the earth at the equator is 25,000 miles, find the speed of a particle on the equator.

* An interesting popular account of such instruments will be found in the first chapter of Williams' *The Romance of Modern Mechanism* (Lippincott).

8. What is the average speed of a runner who goes a quarter mile in 48 seconds?

9. In a 100-meter race the winner made a record of 11⅘ sec. What was his average speed?

10. A horse trots a mile in 2:05. What is its average speed?

11. An automobile from a flying start travels a mile in 57 seconds. Find its average speed.

Velocities in Nature not Uniform

12. When a body passes over equal distances in equal times its velocity is said to be uniform. If all the motions we meet in nature had velocities which were uniform, the vocabulary of physics would be even smaller than it now is. Although the behavior of a good clock and the rotation of the earth on its axis are marvelously close approximations to constant speeds, it is an interesting fact that no instance of perfectly constant speed has yet been discovered in nature or invented by man.

Definition of Acceleration

13. The motion of a body whose velocity is changing is said to be **accelerated.** The change in velocity may be either an increase or a decrease, and may be at a uniform rate or not. If a ball is thrown vertically upwards into the air, it is a well-known fact that it moves more and more slowly, until finally it stops, and then, as it descends, travels more and more rapidly. In this case the change in velocity is very nearly at a uniform rate. All falling bodies move with very nearly a uniformly accelerated motion. **Acceleration is the change of velocity per unit of time**; or **rate of change of velocity.**

If the velocity of a train at a given instant is 88 feet per second and, after 11 seconds, the velocity is 22 feet per second, the change of velocity in 11 seconds has been

66 feet per second and the average change of velocity has been 6 feet per second for each second; this is written for compactness 6 $\frac{\text{ft.}}{\text{sec.}^2}$ and is read 6 feet-per-second per second; hence,

$$\textbf{acceleration} = \frac{\textbf{change of velocity}}{\textbf{time}}$$

Or, still more simply,

$$a = \frac{v}{t}, \quad \text{or} \quad v = at$$

The symbol v is used to represent *change of velocity*, or the *velocity at a particular instant.*

Unit of Acceleration

14. In agreement with unit of velocity as one centimeter per second $\left(\frac{\text{cm.}}{\text{sec.}}\right)$, the logical **unit of acceleration is the centimeter-per-second per second**, often written $\frac{\text{cm.}}{\text{sec.}^2}$. The corresponding engineer's unit is **one foot-per-second per second**, written $\frac{\text{ft.}}{\text{sec.}^2}$ In general,

Unit acceleration is an acceleration in which unit change of velocity is produced in unit time.

Motion of a Particle along a Straight Line

15. If a particle is moving at a uniform rate, the distance it will travel in a given time may be calculated from the definition of velocity found on page 11, distance = average velocity × time.

If, however, the motion is accelerated, some means of finding the average velocity must be devised if the distance traveled is to be calculated. When the distances traveled in each unit of time are known, they may be added together and divided by the number of units of

time to get the average. For instance a train travels 20 miles, 30 miles, 28 miles, 36 miles, and 25 miles, respectively, in five successive hours. The average rate of travel is

$$\frac{20 + 30 + 28 + 36 + 25}{5}, \text{ or } 27.8 \ \frac{\text{mi.}}{\text{hr.}}$$

Here the change of velocity is not at a uniform rate, and these data tell us nothing about the velocity of the train at any given instant within the five hours.

Where the velocity *does* change uniformly, the change of velocity, v, at the end of any time, t, may be calculated from the definition of acceleration found on page 14, namely, change of velocity = acceleration × time, or $v = at$.

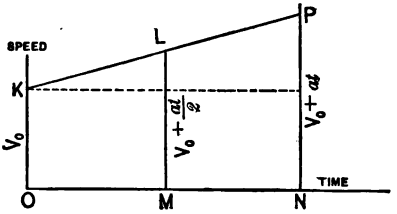


FIG. 8. Galileo's method of averaging.

Now, knowing the initial velocity, V_0, and the change of velocity, v, the final velocity is their sum, $V_0 + at$, and the average velocity, $\bar{v}$, may be calculated by the following method devised by Galileo:

If $\overline{KO}$ (Fig. 8) represents the initial velocity (V_0) and $\overline{PN}$ the final velocity ($V_0 + at$), the average velocity is evidently represented by the median, LM, of the trapezoid $OKPN$, since

$$\overline{LM} = \frac{\overline{KO} + \overline{PN}}{2}$$

which means

$$\textbf{average velocity} = \frac{\textbf{initial velocity} + \textbf{final velocity}}{2}$$

or

$$\bar{v} = \frac{V_0 + (V_0 + at)}{2} = V_0 + \tfrac{1}{2}at$$

Hence the distance (d) traveled in time t is

$$d = \bar{v}t = (V_0 + \tfrac{1}{2}at)t = V_0t + \tfrac{1}{2}at^2$$

If the particle starts from rest, the average velocity is $\frac{1}{2}at$, and

$$d = \tfrac{1}{2}at^2$$

A very useful relation between velocity at a particular instant, acceleration, and distance may be expressed by eliminating t between the equations $v = at$, and $d = \frac{1}{2}at^2$, thus obtaining

$$v^2 = 2\,ad$$

which means that the square of the velocity at any instant may be calculated from the product $2 \times$ acceleration $\times$ distance.

Laws of Uniformly Accelerated Motion

16. The relations thus derived are the so-called **laws of uniformly accelerated motion,** which may also be expressed in words as follows:

When a body starts from rest and is uniformly accelerated,

(i) **The velocity attained at the end of any interval of time is proportional to the time elapsed, and is computed by multiplying the acceleration by the time, or $v = at$.**

(ii) **The distance traversed during any interval of time is proportional to the square of the time elapsed, and is computed by multiplying the average velocity ($\frac{1}{2}at$) by the time, or $d = \frac{1}{2}at^2$.**

(iii) **The speed at any distance from the point of rest may be computed by taking the square root of twice the product of the acceleration by the distance traversed, or $v^2 = 2\,ad$.**

When the particle does not start from rest, if the initial velocity of the particle be represented by V_0, these three expressions assume the following forms:

$$V = V_0 + at$$
$$d = V_0 t + \tfrac{1}{2} at^2$$
$$V^2 = V_0^2 + 2\,ad$$

These laws include, as a special case, the "laws of falling bodies," since it has been shown by experiment that the acceleration due to the attraction of the earth is a constant whose value is about $980 \frac{\text{cm.}}{\text{sec.}^2}$ or $32 \frac{\text{ft.}}{\text{sec.}^2}$.

The Solution of Problems

17. The above relations may be thus expressed:

(i) Final velocity
$$= \text{initial velocity} + \text{change of velocity.}$$

(ii) Average velocity
$$= \frac{\text{initial velocity} + \text{final velocity}}{2}.$$

(iii) Distance = average velocity × time.

(iv) (Final velocity)^2
$$= (\text{initial velocity})^2 + 2 \times \text{acceleration} \times \text{distance.}$$

Many problems may be most simply solved by ordinary arithmetical analysis without the use of a formula. Illustrations of the application of this method are given in the appendix. If it is preferred to make use of the relations stated above among velocity, acceleration, distance, and time, we have merely to decide:

(i) for which of the quantities the numerical values are given;

(ii) for which quantity the numerical value is required; and

(iii) what relation connects the known and unknown quantities.

(iv) Then by a simple substitution find the value of the unknown.

The student is cautioned to keep clearly in mind the physical relations existing among these quantities.

In determining the denomination of answers remember that, if distances are expressed in centimeters and times in seconds, speeds are in centimeters per second and accelerations in centimeters-per-second per second, $\left(\frac{\text{cm.}}{\text{sec.}^2}\right)$, *or*, if distances are expressed in feet and times in seconds, speeds are in feet per second, and accelerations in feet-per-second per second, $\left(\frac{\text{ft.}}{\text{sec.}^2}\right)$.

PROBLEMS

1. A body starts from rest and is accelerated at the rate of 10 cm.-per-sec. per second.

(1) How fast is it moving at the end of 3 sec.? 8 sec.? 15 sec.?

(2) How far has it moved at the end of 3 sec.? 8 sec.? 15 sec.?

(3) After it has traveled 500 cm. how fast is it moving?

(4) How long has it taken to move 1000 cm.?

2. Express 50 km. per hour in meters per minute; in centimeters per second.

3. Express 50 mi. per hour in feet per minute; in feet per second.

4. A train running at 72 km. per hour is stopped by a sudden application of the brake. What average acceleration must be produced by the brakes in order to stop the train in 8 sec.?

5. When steam is turned on again, a constant acceleration of 50 cm per second in every second is produced. How long will it be before the train has again acquired its original speed of 72 km. per hour?

6. How far will the train travel while again acquiring its original speed in the fifth problem?

7. A particle has an initial speed of 25 cm. per second and receives an acceleration of 10 cm.-per-sec. per second. How fast will it be traveling at the end of 15 sec.? What distance will it travel in 20 sec.?

8. The speed of a street car decreases from 20 mi. per hour to 5 mi. per hour in 10 sec. What average retardation (negative acceleration) is produced by the brake?

9. The intercollegiate record for the mile run (1909) is 4 min. 17⅘ sec. What is the average speed of the runner in feet per second?

10. How many feet of belt will travel in 8 hr. over the face of a pulley 10 in. in diameter, when the pulley makes 120 revolutions per minute?

11. A bicycle wheel is 70 cm. in diameter. How many revolutions does it make per minute when the rider is traveling 24 km. per hour?

12. At the International Balloon Races at Berlin in 1908 a balloon burst at a height of about 2900 ft. The two passengers reached the earth in safety in about 2 min. What was their average acceleration?

References

LODGE — *Elementary Mechanics* (Van Nostrand, New York), pp. 308. Price, $1.00. The first forty pages of this little volume are recommended as the best thing the beginner can take up at this stage.

MACH — *Science of Mechanics*, trans. by McCormack (Open Court Pub. Co.), pp. 534. Price, $2.50. Excellent chapter on Galileo's achievements, pp. 128–155.

WATSON — *Text-Book of Physics* (Longmans & Co., 1899), pp. 896. First six chapters contain an excellent discussion of motion.

CLIFFORD — *Dynamics*(Macmillan), pp. 221. Part I. Price, $1.90.

MAXWELL — *Matter and Motion* (Soc. Promotion Christian Knowledge), pp. 128. Price, 40 cents.

CHAPTER II

SOME GENERAL PROPERTIES OF MATTER

18. As one casts about in this world he very early discovers that certain forms of matter are endowed with certain remarkable properties, including the ability to assimilate food, the power of reproduction, sensibility, etc. Such matter is called *living matter*, and forms the subject of study for the **biological sciences.** All other matter is said to be inanimate; and it is with this, especially, that the **physical sciences** have to do.

But physics does not deal with all the phenomena of inanimate matter. There are certain pieces of matter which are so distant from us that they can be studied only by especially designed instruments, such as the telescope; accordingly the study of the sun, moon, planets, etc., is carried on under a special department of physical science called **Astronomy.**

In like manner, those phenomena of matter which involve a change in its composition are considered under the head of **Chemistry.**

Those properties of inanimate matter which are *not* connected with a change in its chemical composition are then the special subjects of study pursued under the head of **Physics.**

Up to this point we have been considering some of the motions of particles or bodies, and have confined our attention entirely to the motion, not at all to the mov-

ing object. We next proceed to study some of these properties which all pieces of matter have in common; properties possessed alike by bodies at rest and by bodies in motion, and by bodies of every possible chemical composition. Properties of this kind are called **general properties.**

We shall consider four of these:

I. **Inertia,**
II. **Gravitation,**
III. **Capacity for Energy,** and
IV. **Elasticity.**

Afterwards we shall study some of the **special properties** of inanimate matter, such as capillarity, specific gravity, conductivity, magnetic quality, transparency, and color, and we shall then see how these special properties are employed to classify the different kinds of substances.

Matter is something with which we are familiar *in a general way* from our earliest years. We have learned to recognize it from its properties. Always associated with it is something which is not matter and which is called **energy. The study of physics is the study of matter and energy.***

FIRST GENERAL PROPERTY OF MATTER

I. Inertia

19. One of the commonest experiences of life is that of "sizing up" a body by pushing, pulling, or "hefting" it. When one sees a barrel rolled along the street, he can always tell whether it is empty or is filled with something

* At this point both teacher and pupil may be interested in reading a collection of definitions of matter given on pages 14–15 of Tait's *Properties of Matter* (A. & C. Black, London, 1894). On pages 808–811 of the same book will be found Professor Flint's very interesting collection of "Hypotheses as to the Constitution of Matter."

that has considerable weight. The empty barrel rebounds to a greater extent as it passes over a small stone, and is more easily turned aside from a straight course.

Even a wagon drawn by a team behaves so differently when empty and when loaded that an observer does not need to look into the box of the wagon to see whether it contains a load or not.

Imagine three balls, each six inches in diameter, each painted black, one a rubber football, one a wooden ball from a bowling alley, one an iron cannon ball. Imagine them all started rolling with the same speed over a moderately smooth sidewalk. Any one can tell instantly, by observing the behavior of the three balls, which one is filled with air, which one is made of wood, and which one of iron.

Everybody knows that the empty barrel is more easily set in motion than the full one, and that it is more easily stopped than the full one. Any one can tell, by kicking the barrel, whether it is full or empty. In like manner, any one can go up to the three black balls, each of the same size and color, each at rest on the ground, and can distinguish by a push with the foot, the iron ball from the wooden ball, the wooden ball from the football. But just how is one able to arrive so quickly at the correct conclusion? The answer is that every one knows from his previous experience with matter these facts:

(1) **That it requires an effort to put matter into motion when the matter is already at rest; that it requires an effort to stop matter after it has once been set in motion** (*Definition of Inertia*).

(2) **That, with any given body, the amount of effort is greater in proportion as the change of velocity is greater.**

(3) **That, for any given change of velocity, the amount of effort required is in each case greater when the amount of matter is greater; and less when the amount of matter is less.**

Inertia a Property of All Matter

20. The first of these general facts is described by saying that **all matter possesses inertia.** Now, inertia is simply the Latin word for laziness; but the *laziness* of matter is peculiar in this respect, namely, in that stopping matter when once in motion (Fig. 9) is as difficult as starting it when once at rest.

What experience of your own can you cite to show that liquids, as well as solids, possess inertia?

Describe a simple experiment to show that air has inertia.

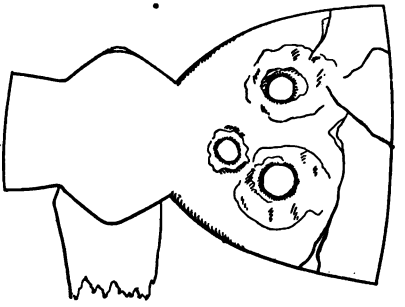


Fig. 9. The moving bullet does not easily stop.

In all the experience of the human race no exception has been found to the statement that all matter possesses inertia. How, then, shall we explain the fact that a marble started rolling on a level blanket so soon stops? Here the blanket forms a slight hill in front of the marble, and hence exerts an effort, so to speak, against the marble. But suppose the marble be put into motion on a smooth, level table; here the marble rolls much farther before it stops; but the very slight roughness of the table is sufficient shortly to bring the marble to rest. The effort which a sheet of level plate glass can oppose to the motion of the marble is slighter still, yet sufficient to stop the marble by and by. The smoother the road and the freer the bearings are from friction, the longer a bicycle will run when coasting on level ground.

Suppose now that a bicycle wheel is entirely free from friction, not only the friction of the bearings, but the fric-

tion of the air. How long will it continue to spin when the frame is held above the ground and the wheel set in motion? All our experience goes to show that, under these ideal conditions, the wheel would never stop. The earth in its rotation on its axis comes very nearly * being such a body; the earth is nearly free from any effort to stop it, and we find that it keeps on moving, year after year, with essentially the same speed.

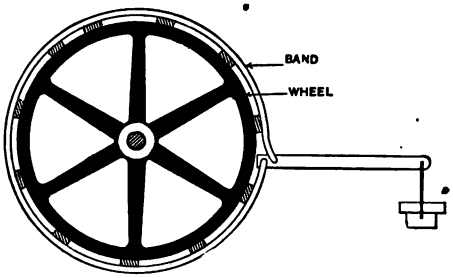


FIG. 10. Friction brake band.

In like manner, if a body has a motion of pure translation (see p. 9), the natural and logical inference is that the body will keep on moving indefinitely *if no effort is exerted against it.*

But, as a matter of fact, **in all cases of actual motion, we find that there is some outside influence at work which interferes with the uniformity of the motion.** Thus in the case of a bullet fired vertically upward, we find that the bullet does not keep upward forever, but soon returns to the earth. In the case of a kite, on the other hand, the combined effort of the string and the weight of the kite are not sufficient to bring the kite down. An equally peculiar case is that of an empty bottle, corked and then immersed under water. The bottle at once rises to the surface.

* The student who pursues physics farther will find that the ocean tides are equivalent to two ridges of water which are held in a fixed position while the earth rotates between them; these are equivalent to a friction brake, and the earth is thus slightly retarded, perhaps 0.44 sec. in a century. — *Ency. Brit.*, Art. "Tides," Sec. 48.

In order to explain these various phenomena in a really simple and comprehensive way, it will be necessary first to define two new terms, viz. *Mass* and *Momentum*.

Definition of Mass

21. In the second and third of the general statements on page 22, it is pointed out that we all know something about estimating the amount of matter in a body by the difficulty of starting or stopping it.

It is customary in the physical sciences to employ the word "mass" to denote the amount of matter (i.e. the amount of inertia) in a body. And hereafter we shall use the word "mass" in this sense, namely, as a synonym for the amount of matter in a body, or its inertia.

Unit of Mass

22. There is a piece of matter preserved in the International Bureau of Weights and Measures at Sèvres, near Paris, a piece of platinum-iridium, which the scientific world has agreed to call the standard kilogram, *i.e.* 1000 grams (Fig. 11). **The gram, which is therefore one one-thousandth part of this mass of platinum, is taken as the unit of mass.**

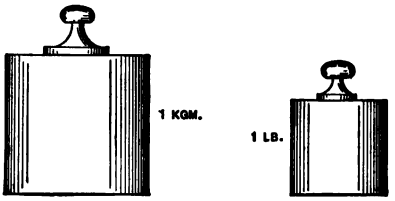


Fig. 11. 1 kgm. = 2.2046 lb.

When hereafter we shall speak of the mass of a body as "25 grams," we shall mean that the body has 25 times as much matter in it as there is in one gram; that the effort required to start it moving with a given speed is 25 times that required to start one gram moving with the same speed; or, what happens to be the same thing, that it weighs 25 times as much as one gram.

Similarly, in the Standards Office at Westminster, London, is a standard mass of metal known as the *pound.* 1 kgm. = 2.2046 lb.

On Fundamental and Derived Units

23. The standard of mass, and also the standards of length and time, have been defined in a purely arbitrary way. They do not depend upon any other standards for their value. *A unit which is chosen in this arbitrary manner, without reference to other units, is called a* **fundamental unit.** In modern physics there are practically only three fundamental units, viz. the unit of mass, the unit of length, and the unit of time.

All other units are on a different basis, as will be seen by considering one of them, say the unit of speed. The whole scientific world uses as unit speed, that speed with which a particle will traverse unit distance in unit time. In the metric system, we say unit speed is one centimeter a second. The unit of speed thus depends upon the units of time and length. It is, therefore, said to be a **derived unit.**

Derived units are defined as those which depend for their value upon the fundamental units. In like manner, the unit of acceleration, the unit of area, and the unit of volume are all derived units. So are all the units employed in this text-book, except the three fundamental ones just mentioned.

Units which are based upon the centimeter, gram, and second are called "C.G.S." units. Units based upon the foot, pound, and second are called "F.P.S." units.

Density

24. From the earliest times it must have been observed that there are certain bodies which float on water, while others sink more or less rapidly. Wood nearly always floats, no matter how large a piece is taken, and lead always sinks provided the piece is solid, no matter how small. It is a well-known fact that lead, volume for volume, is a heavier substance than wood. To describe the manner in which mass and volume are related the term "density" has been introduced and is defined as follows: **The density of any substance is the mass per unit of volume of that substance,**

or $$\textbf{density} = \frac{\textbf{mass}}{\textbf{volume}}$$

The mass of one cubic centimeter of water at a temperature of 4° (centigrade about 39° Fahrenheit) is almost precisely one gram. Hence, **the density of water is one gram per cubic centimeter,** $\left(1\,\frac{\text{gm.}}{\text{c.c.}}\right)$. The mass in grams of any amount of water, then, is numerically equal to its volume in cubic centimeters, and 10 c.c. of water weigh 10 gm., 40 c.c. weigh 40 gm., and so on.

The mass of one cubic foot of water, however, is about 62.5 lb. Hence, **the density of water is 62.5 lb. per cubic foot,** $\left(62.5\,\frac{\text{lb.}}{\text{cu. ft.}}\right)$, and 10 cu. ft. of water weigh 625 lb., 50 cu. ft. weigh 3125 lb., and so on.

Definition of Linear Momentum

25. We have already seen that the mass of a body is one of the factors which measure the difficulty of starting or stopping the body. But, the difficulty of starting or stopping a body in translation depends also upon how fast

you start it going, or upon how fast it is moving when you attempt to stop it. It depends, as Newton said, upon the "quantity of motion" in it; or, as we now say, upon its linear **momentum.**

Since, then, the difficulty of stopping a body in translation depends both upon its mass and upon its velocity, we say,

$$\textbf{linear momentum} = \textbf{mass} \times \textbf{velocity} = m \times v$$

where m is the mass and v the velocity of translation of the body. Thus the momentum of a ball whose mass is 80 grams and whose velocity is 225 $\frac{\text{cm.}}{\text{sec.}}$ is $80 \times 225 = 18{,}000$ units; and the momentum of a boat whose mass is 30,000,000 pounds, moving 15 $\frac{\text{ft.}}{\text{sec.}}$, is $30{,}000{,}000 \times 15 = 450{,}000{,}000$ units of momentum.

Comparison of Masses

26. If two bodies of equal mass be moving toward one another in the same straight line with the same velocity, their momenta will be equal in magnitude but opposite in direction. If, therefore, the two bodies strike and stick together (as two balls of putty might do, Fig. 12) so as to form one body, the combination has zero momentum; that is, the combination is at rest. This gives us the means of finding when two bodies have equal masses. For in the case just mentioned, if the masses had been unequal, the combination would not have come to rest, but would have moved in the direction in which the larger mass was moving before collision.

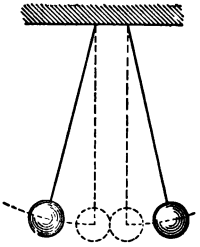


Fig. 12. The masses are equal when, on moving equal distances in the same time, with equal velocities, they have equal momenta.

A balance for comparing masses on this principle has been devised by the English physicist, Hicks.

Having once obtained two equal masses, it becomes possible to add them together and thus to produce a series of masses, the values of which are 2, 5, 10, etc., times the original mass. We may thus manufacture a complete set of standard masses. These are generally called, not a "set of masses," but a "set of weights." The reason for this we shall see later, when we come to study under the head of *Gravitation* a much more accurate and convenient method of comparing masses.

Rotational Inertia

27. Two children who are seesawing, one at each end of a board, know very well that the seesawing goes on more smoothly and with greater uniformity when they are out near the ends of the board than when they are in near the middle. But when they are near the middle, the board is much more quickly started or stopped by a third party taking hold of the end. The mass of the board and the children is the same in each case; only the distribution of the mass is different.

The children and the board constitute a typical rotating system. The axis of rotation is the line of contact of the board and the log over which the board is balanced. The point to which attention is here directed is that **the resistance which a body offers to being set in rotation,** *i.e.* its **rotational inertia,** is not measured by its mass merely, but depends also upon the distribution of the mass.

Imagine two fly wheels to have equal masses (Fig. 13). In the first case, we shall suppose the mass to be distributed mostly near the axis of rotation; in the second case, the mass is placed far out, mostly near the rim of the wheel. Which wheel should be more easily set in rotation?

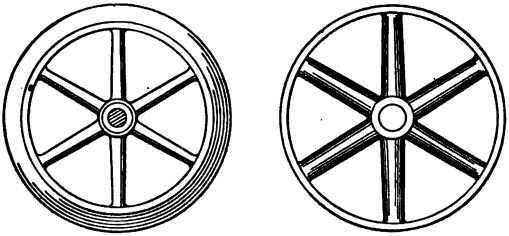


FIG. 13. Which fly wheel is more easily set in rotation?

From an inspection of Fig. 14 more than one reason can be given why it is more difficult to "swing a sledge" than to use an ordinary carpenter's hammer.

Sewing machines, corn shellers, and engines generally are provided with fly wheels in which the mass is placed as far as possible from the axis of rotation, for the ability of a particle to keep a fly wheel in rotation is measured by the distance of the particle from the shaft, as well as by its momentum.

Those who pursue this subject farther will find that the **rotational inertia of a particle depends not merely upon the**

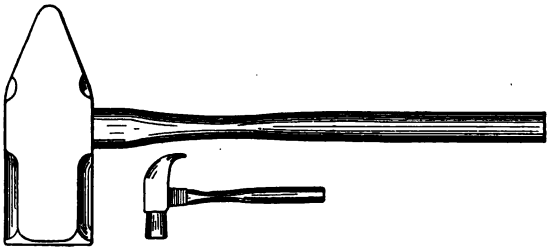


FIG 14. A sledge hammer. A carpenter's hammer.

mass of the particle, but upon the product of the mass and the square of its distance from the axis of rotation. This quantity which measures the rotational inertia of any particle is generally called its moment of inertia.

We shall find later that this quantity which we have called **moment of inertia plays the same part in rotation that mass does in translation.** We have thus been led to substitute, for our general notion of rotational inertia, a definite quantitative idea, which in more advanced work will be of the greatest use to us.

PROBLEMS

1. How is it that, in a fly wheel weighing one ton, the effect of the wheel in steadying the motion of the engine is very much increased by placing most of the mass out near the rim of the wheel?

2. If an extra pound is to be added to a bicycle wheel, where will it prove to be the greater hindrance to starting, when placed in the tire or in the hub? Why?

3. Why is it that in the balance wheel of a watch (Fig. 15) most of the material is placed in the rim of the wheel?

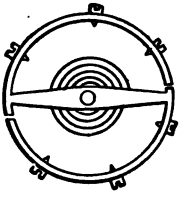


FIG. 15. The balance wheel of a watch.

4. Two quoits are made to weigh 3 lb. each. One of them is a disk 6 in. in diameter; the other is a ring whose outside diameter is 6 in. and whose width is about 1 in.

In pitching these quoits, which one could you control more easily? Why?

CASE I. NO EXTERNAL INFLUENCE AT WORK

Momentum a Constant

28. As we all know, it is not an easy matter "to leave a body to itself"; but Galileo and Newton have correctly inferred the behavior of a body *when left to itself*, and have described this behavior in the following simple way:

Every body continues in its state of rest, or of uniform motion in a straight line, except in so far as it is compelled by force to change that state. *Principia*, Book I. This fact is generally called **Newton's First Law of Motion,** and means that a body at rest remains at rest unless acted on

by an external influence, and that, for a body in motion, its linear momentum remains constant.

This statement is true, so far as is known, without any exception whatever: for bodies at rest as well as for bodies in motion; for a body of any size, shape, or composition. But it is true only under the condition stated, viz. that the body, or system of bodies, is affected by no outside action.

Before one can say whether or not a body is acted upon by outside influences, he must be very careful to define just what the body is which he is studying. This done, the distinction between *internal* and *external* influences is easy.

If, therefore, a heavy fly wheel, set in rotation and "left to itself," gradually diminishes in its rate of rotation, we may fairly expect to find that the wheel is *not* really left to itself and is *not* entirely free from external influences. In such an expectation no one has yet been disappointed.

If a skater coasting on level ice finds himself moving more and more slowly, he suspects resistance to his motion in the air, and frictional resistance between the ice and the skates.

If a baseball, thrown vertically into the air, changes its velocity (as it does, first diminishing, then stopping, and then increasing in speed), we may rest satisfied that an outside influence is at work. In this case it is popularly called *gravity*.

Since, then, bodies left to themselves move at uniform velocity, we may say that **motion is a state of matter quite as natural as rest.** For bodies in motion and left alone keep in motion: while bodies at rest and left alone remain at rest.

Case II. An External Influence at Work

Force and Moment of Force

29. In the preceding pages we have frequently been compelled to speak of the motion of a body as influenced by "resistance," by "external action," by "some outside influence," or by "some external effort." These terms have all been used to express the same idea; but such loose, vague, and varying names are not in keeping with the spirit of modern physics. We accordingly take the first opportunity to define exactly **what is meant by these various external influences which change the momentum of a body.** For the common feature of all these actions which affect a body from without is that they do not permit the body to move with unchanged momentum: **they continually alter the velocity of the body,** and hence change its momentum.

In physical science two definite names are used to include all these influences. They are called **Forces** if they **change the linear momentum of the body,** and **Moments of Force** if they **change the rotation of the body.** In other words **forces produce acceleration; moments of force produce change of rotation.**

Newton, looking for the shortest and clearest description of what had been observed to happen where masses, either in motion or at rest, are acted upon by forces, arrived at the following, which is generally known as **Newton's Second Law of Motion:**

The change in the linear momentum of a body is proportional to the force acting upon the body; and the direction of the change is the same as the direction of the force.

Definition of Force

30. Force is defined as that which changes the state of rest or motion of a body, or which changes its linear momentum; and it is measured by the rate at which the linear momentum is changed.

This idea we owe to Galileo (1564–1642). In popular usage force has an endless variety of meanings; in physics it is no longer used except in this one sense.

The student should observe carefully that in this definition nothing whatever is said as to the origin or source of the change of momentum. The source of the external influence may be electric, magnetic, or gravitational; it matters not. If the momentum is changed, some external force is said to be acting on the body; and the measure of this force is the change of momentum per unit of time.

This fact may be expressed as follows:

$$\text{Average force} = \frac{\text{change of linear momentum}}{\text{time}}$$

That is, $F = \frac{mv}{t} = m \times \frac{v}{t} = m \times$ rate of change of velocity, or, since acceleration is rate of change of velocity,

$$\text{force} = \text{mass} \times \text{acceleration}$$

Hence we may write $F = \frac{mv}{t} = ma$

and define **force** as the **product of mass and acceleration**, a definition which is exactly equivalent to that given above.

Unit of Force. The Dyne

31. A certain force acts on a mass of 10 grams and gives it an acceleration of 1 $\frac{\text{cm.}}{\text{sec.}}$ per second. Evidently the force would have to be 5 times as great to give the same

mass an acceleration of $5 \frac{\text{cm.}}{\text{sec.}}$ per second. Likewise a force only $\frac{1}{10}$ as great would be necessary to give a mass of 1 gram an acceleration of $1 \frac{\text{cm.}}{\text{sec.}}$ per second. This last force is called a *dyne*. Hence, **the dyne is defined as that force that can produce an acceleration of $1 \frac{\text{cm.}}{\text{sec.}^2}$ when acting on a mass of 1 gram.**

When a mass of one gram is allowed to fall freely towards the earth, one observes that it gains velocity at the rate of $980 \frac{\text{cm.}}{\text{sec.}}$ per second. Therefore, if one keeps this mass from falling, he must exert upon it a force of 980 dynes. Since a mass of one pound is equal to a mass of about 453 grams, the hand has to exert a force of $980 \times 453 = 443{,}940$ dynes in order to support a mass of one pound. The force required to support one kilogram is $980 \times 1000 = 980{,}000$ dynes.

In many instances it will be convenient to speak of a **force of one gram**, meaning 980 dynes, similarly for a **force of one pound.**

The dyne is called the absolute unit of force. The student will get a reasonably definite idea of its very small magnitude by holding in the hand a mass of one milligram. The hand will then exert a force of nearly one dyne.

An Interesting Special Case. Centrifugal and Centripetal Force.

32. Occasionally one reads of the bursting of a fly wheel or emery wheel with attendant damage and loss of life. The reason usually assigned is that "the engine raced"; namely, ran too fast. Apparently, whenever a body is rotated, a force is exerted whose tendency is to tear the body in pieces. This force is called the **centrifugal force.** The force just necessary to counteract

this force is called **centripetal force.** But the particles of which bodies are made have the property of sticking together to a degree characteristic of the material of the body. This property, called **cohesion,** is strongest in solids, less in liquids, and almost lacking in gases. Evidently the *cohesion* of the particles of a body resists the centrifugal tendency and holds the particles together unless the centrifugal tendency becomes too great.

If we think of a particle in the rim of a rotating wheel, its direction of motion at any instant is along the tangent to the circle in which the particle moves (Fig. 16). The direction of the tangent is continually changing, hence the

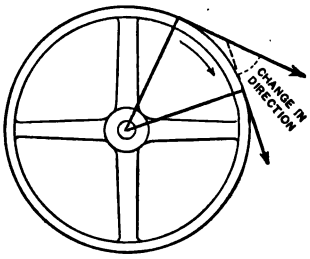


FIG. 16. Change in direction of a rotating particle.

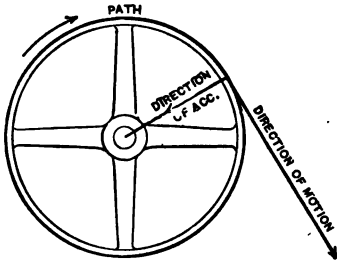


FIG. 17. Acceleration is toward center.

direction of the motion is changing and, according to Newton's *First Law of Motion,* some force must be acting to produce this change of direction. And, according to Newton's *Second Law,* the direction of this force is in the direction of the change.

It may be proved, though we shall not attempt it here, that **any particle in such a rotating wheel** (Fig. 17) **is accelerated toward the center of the wheel, and the numerical value of its acceleration is the square of its linear velocity divided by the radius of its path,**

or, acceleration toward center $= \dfrac{(\text{linear velocity})^2}{\text{radius}}$

That is, $$a = \frac{v^2}{r}$$

Since force is defined as mass × acceleration, the force which produces the acceleration, $\frac{v^2}{r}$, in a mass, m, is, of course, $\frac{mv^2}{r}$.

The centrifugal force is just counteracted by this centripetal force, so we know them to be equal in amount and opposite in sense.

Hence, centrifugal force $= \dfrac{\text{mass} \times (\text{linear velocity})^2}{\text{radius}}$

or $$F = \frac{mv^2}{r}$$

One can now see why a fly wheel bursts if rotated too rapidly. The mass is concentrated in the rim so as to get the greatest effect in steadying the motion of the machinery. As the velocity increases, the centrifugal force increases as its square ($F \propto v^2$), and the force may easily become so great as to overcome the cohesion of the particles in the wheel.

Applications of Centrifugal Force

33. When sugar is freed from molasses in the sugar factory, clothes dried in a modern laundry, or cream is separated from milk, a centrifugal machine is used.

Some of the best and most rapid modern pumps are centrifugal pumps. Both spring and ball governors on steam engines operate according to centrifugal principles, as will be seen by inspecting Figs. 19 and 20. In Fig. 19 as the speed increases, centrifugal force drives the masses 1 and 2 away from the axis. Each mass is attached to a rod, R, which rotates the eccentric, E, thus controlling the admission of steam into the cylinder of the engine.

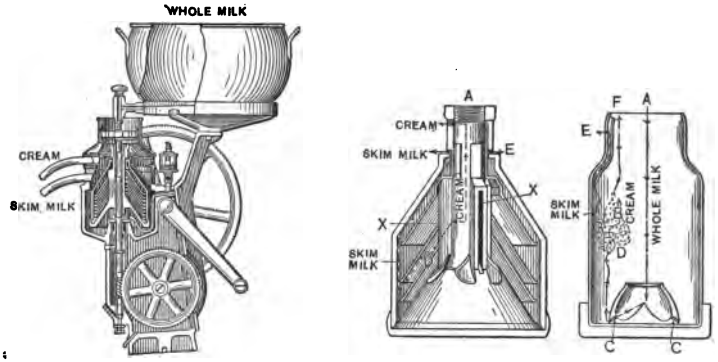


FIG. 18. De Laval cream separator.
(Separation takes place at *D*.)

In Fig. 20 the balls are carried away from the center with increase in speed, thus adjusting the passage of steam into the engine cylinder.

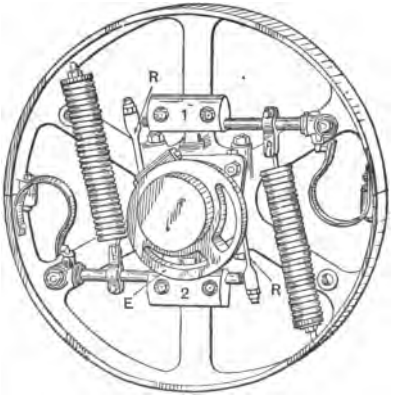


FIG. 19. A Buckeye spring governor.

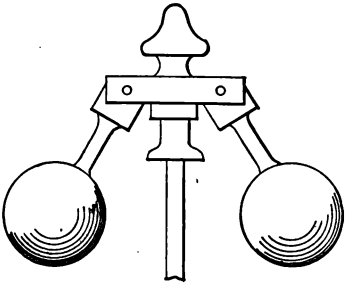


FIG. 20. A ball governor.

The curves of a race track are "banked" and the outer rail of a railroad curve is raised to overcome the centrifugal force.

PROBLEMS

1. A mass of 1500 gm. is acted upon by a force of 6000 dynes. Find the acceleration produced in the mass.

2. A force of 100 dynes is able to produce an acceleration of 2 in a certain body. What is the mass of the body?

3. What force would be required to change the momentum of a body from 250 to 1500 in 5 sec.?

4. A boy swings about his head, in a circle of 1 m. radius, a bullet whose mass is 120 gm. What is the pull on the string when the bullet is making 1 revolution per second?

5. A locomotive whose mass is 100,000 kgm. is rounding a curve of 400 m. radius at a speed of 10 m. per second. What is the force which the locomotive exerts upon the outer rail? What holds the engine to a circular path?

6. Why does a bicycle rider "lean in" as he rounds a corner?

7. A force of 2,000,000 dynes is applied to a mass of 10 kgm. for 5 sec. Compute the change in velocity.

8. When a bicycle is being ridden through a wet or slushy street, how is it that the water collects in a little ridge at the middle of the tire?

9. A skater who weighs 6000 gm. describes a circle of 30 m. radius with a speed of 6 m. per second. Find the horizontal force tending to throw him over. Why does he incline himself toward the center of the circle?

10. How many times a minute must a mass of 3 gm. revolve in a horizontal plane at the end of a string 50 cm. long in order that the force on the string may equal 1000 dynes?

The Resultant of Two Forces

34. The three characteristic properties of a limited straight line are length (*i.e.* magnitude), direction, and the sense in which it is drawn. If P (Fig. 21) is 5 units of distance from O, it might lie anywhere on the sphere whose radius is 5. To locate P precisely, both its distance and direction from O must be given. Likewise with a force, we must know the magni-

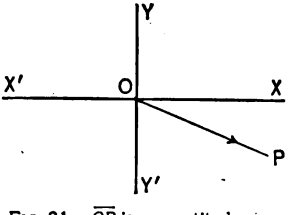


Fig. 21. $\overline{OP}$ is a quantity having direction.

tude of the force, its direction, its sense, and where it is applied — *i.e.* its point of application — before we can prophesy its effect in producing motion. In these respects force is seen to resemble a straight line. The magnitude and direction of the force correspond to the length and direction of the straight line $\overline{OP}$; the *sense* in which it acts is indicated by the arrow on the straight line. **A force, then, may be represented in direction and magnitude by a straight line.** This gives us a very convenient method for finding the combined effect of several forces all acting on the same body.

It is a well-known fact that a raindrop usually falls in an oblique path. Let us suppose a raindrop (Fig. 22) to form at O and reach the earth at E. The forces acting upon it are the attraction of the earth in the direction $\overline{OG}$ and a wind acting in the direction $\overline{OW}$.

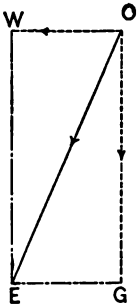


Fig. 22. The path of a raindrop.

Newton's *Second Law of Motion* means, for one thing, that each force produces its own effect regardless of the action of other forces. Applying this principle to the forces acting on the raindrop, the wind would carry the drop from O to W in the same time the drop was falling from O to G; or if the two acted simultaneously, the actual path of the drop would have been OE. Note that $\overline{OE}$ is the diagonal of the parallelogram of which $\overline{OW}$ and $\overline{OG}$ are sides.

A single force which will produce the same effect as two or more forces is called their resultant. The separate forces are called the components.

Parallelogram of Forces

35. This simple application of Newton's *Second Law* is known as the **parallelogram of forces.** It is really an experimental law which may be stated as follows:

If the adjacent sides of a parallelogram are constructed so as to represent two concurrent forces, the concurrent diagonal represents their resultant (Fig. 23).

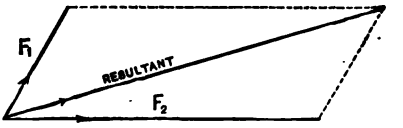


FIG. 23. The concurrent diagonal represents the resultant.

The same principle may be extended to more than two concurrent forces by regarding the resultant of any two as a single force, finding its resultant with the third force, and so on.

If the two concurrent forces are at right angles, their resultant may be computed by the Pythagorean theorem, as illustrated in Fig. 24.

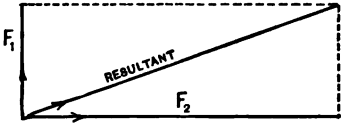


FIG. 24. $(\text{Resultant})^2 = F_1^2 + F_2^2$.

If the components are not at right angles, an approximate value for the resultant may be obtained by measuring the diagonal of a parallelogram constructed to scale (Fig. 23). The value represented by the length of the diagonal will be the resultant.

Resolution of Forces

36. The principle of the parallelogram of forces may also be applied to the resolution of a single force into components. If, as we have seen, the resultant and its components are related in the same manner as the diagonal and concurrent sides of a parallelogram, it is at once evident that for any given resultant ($\overline{OP}$) there may be as many pairs of components as there are possible parallelo-

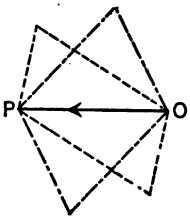


FIG. 25. As many components as we choose.

grams on $\overline{OP}$ (Fig. 25), which is as many as we choose. Ordinarily, the most convenient pair of components to select is at right angles.

The application of the principle to specific cases affords few difficulties and the student is left to apply the parallelogram relation as occasion requires.

Equilibrium of Non-parallel Forces

37. Any three forces so related that they may be represented in direction and magnitude by adjacent sides and the concurrent diagonal of a parallelogram reversed in sense (Fig. 26) are said to be in equilibrium. Evidently their resultant is zero.

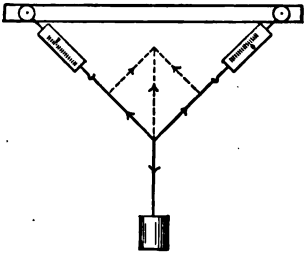


FIG. 26. Three forces in equilibrium.

Any system of forces acting upon a particle and so related that their resultant is zero is said to be in equilibrium.

In general, if a body is not acted upon by any forces, or only by a system of forces whose resultant is zero, the body will remain at rest or move uniformly in a straight line (Newton's First Law of Motion).

PROBLEMS

1. Two forces acting at right angles have magnitudes 3 and 4. Find their resultant.

2. Why is the mud on your shoe apt to fly off when you kick a doorstep?

3. A force of 50 units acts upon a mass 30. Find the acceleration produced.

4. Why are brakes necessary on a train? Why are air brakes better than hand brakes? Why is it not necessary to put brakes on a carriage when an automobile of equal weight must be equipped with them?

5. Show from Newton's First Law of Motion that some force must be acting on a body describing uniform circular motion. What is its direction?

6. Resolve a force of 60 units into two equal components at right angles.

7. What property of air makes it possible for a bird to fly?

8. A pair of *equal* forces acts upon a body urging it to move in two directions inclined to each other at 120°. What is the magnitude and direction of their resultant? [Draw diagram.]

9. Why does a boy in crossing a piece of thin ice have to skate more rapidly than he would on thick ice?

10. What is the principle of the aëroplane?

Directed Quantities

38. On page 40 it was shown how a force may be represented in direction and in magnitude and in the sense in which it is acting by a straight line with an arrow drawn upon it. Precisely the same reasoning shows that **motions, velocities, accelerations, and momenta may also be represented in direction and magnitude by straight lines, and hence, the resultant of any two motions, velocities, accelerations, or momenta may be found by the parallelogram method.**

The application of this principle to the following problems will be readily seen without further explanation.

PROBLEMS

1. Starting from a certain point, a man walks 3 miles north and then 4 miles east.

Draw a triangle which will represent his final position with reference to his starting point.

In this case, which side of the triangle do you call the resultant?

2. A stream which is 300 feet wide flows from north to south. A man attempts to row a boat directly across it, that is, he heads the boat directly toward the east. But while he is crossing, the current carries him downstream. The result is that he lands on the opposite bank at a

point which is 500 feet distant from his starting point. Draw a triangle which will represent his change of position. Which sides of the triangle do you call the components, and which the resultant?

3. A wheelman rides 2 miles east, then 10 miles north, and then 2 miles west. Draw a figure which will represent these three changes of position, and their resultant.

4. A steamer travels forward 400 feet while its flag is being hoisted to the top of a staff 60 feet high. What distance did the flag actually travel while being raised?

5. After walking 3 miles along a road running northeast, how far have I traveled east, how far north, from my starting point?

6. A particle is accelerated vertically upward with an acceleration of 3, and horizontally eastward with an acceleration of 4. Find the total acceleration of the particle.

7. Represent by a diagram the velocity of a boat which is being rowed east at the rate of 4 miles an hour, while the current is carrying it south at the rate of 1 mile an hour.

8. A wheelman is riding north at the rate of 8 miles an hour against a head wind which is retarding him at the rate of 2 miles an hour. What speed would he make if there were no wind?

9. What speed would this same wheelman make if he turned around and rode with the wind?

10. A canal boat is towed at the rate of 3 miles an hour. How fast must a man walk its deck in order that he may remain at rest with respect to the towpath?

11. A wheel rider travels east at the rate of 6 miles per hour while the wind is blowing from the north with a speed of 6 miles per hour. From what direction will the wind appear to strike the rider?

12. A boat is rowed on the river so that its speed in still water would be 6 miles an hour. Suppose the river flows at the rate of 4 miles an hour. Make a diagram showing the direction in which the boat must head in order that its motion may be at right angles to the current.

13. An automobile is traveling 30 miles per hour due west. A north wind is blowing 15 miles per hour. What is the apparent wind velocity to the driver of the machine?

14. A steamship is traveling 22 knots per hour southwest. How fast is it traveling west? how fast south?

Moment of Force

39. In the earlier pages of this chapter we have frequently spoken of "external influences" changing the rotation of a body.

A spinning top rotates more and more slowly: here friction is the *external influence.* The top is the body or *system* under consideration; the floor on which it spins and the air are *external* to the system.

We have seen, page 33, that in physical science one definite name has been given to all these influences which affect the rotation of a body. They are called **Moments of Force.**

Definition of Moment of Force

40. Moment of Force is defined as **that which changes the rate of rotation of a body; and it is measured** (Fig. 27) **by the product of force and the perpendicular distance from the axis to the direction of the force; *i.e.* force by lever arm.**

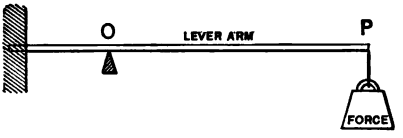


FIG. 27. Moment of force = force × lever arm.

If the rotation of a body is changing (whether growing faster or growing slower, it matters not), there is a moment of force at work somewhere. Whether the moment of force has some magnetic, electric, or gravitational origin does not here concern us: the moment of force acting is, in each instance, measured in the same way, viz. as the product of force by lever arm.

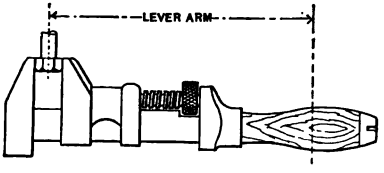


FIG. 28. Moment of force in a monkey wrench.

Moment of Force acting on a Rigid Body

41. When one straight line is fixed in a rigid body, such as the hinge line in a door, the only motion which that body can have is one of rotation.

The effect of a force acting on any such body is, in general, to change its motion of rotation. The amount of this change will obviously depend upon two things:

(1) the force applied, and

(2) the distance of the line of action of the force from the axis of rotation.

Thus, if one wishes to shut a door and can exert only a limited force, he knows that this force will be much more effective when applied near the edge of the door than when applied near the hinge line.

In like manner, when one wishes to turn a heavy wheel, he always takes hold near the rim, and not near the hub. He does this in order to make the perpendicular distance between the force and the axis as great as possible.

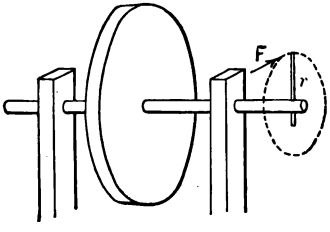


FIG. 29. Moment of force on a particle, at the end of an arm, r.

Hence, the effect of a force in producing rotation about any axis can be measured by the following product, viz. the force acting × the perpendicular distance from the axis to the direction of the force, namely, force × the lever arm. See Fig. 29.

Equilibrium of Parallel Forces

42. Equilibrium of non-parallel forces is explained by the "parallelogram of forces." To explain equilibrium of parallel forces, we use the **principle of moments.**

Let us suppose two men carrying a load suspended from a bar represented by AB (Fig. 30).

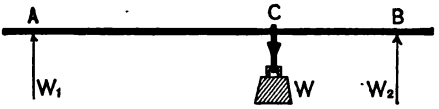


FIG. 30. Resultant of parallel forces.

Between them they must support the whole load, so the part A carries, W_1, plus the part B carries, W_2, = the total load, W, or

$$W_1 + W_2 = W$$

Evidently the resultant of two parallel forces is equal to the sum of the forces. If the men keep the bar horizontal, there is no rotation, and the resultant moment of force must be zero. Take *any* point C as the center of moments about which any rotation would be observed,

the moment at A is $W_1 \times \overline{AC}$ in a clockwise sense;
and
the moment at B is $W_2 \times \overline{BC}$ in a counter-clockwise sense.

If the resultant is zero, these moments must be equal in amount and opposite in sense; or

$$W_1 \times \overline{AC} = W_2 \times \overline{BC}$$

or, if we take the ratio between W_1 and W_2,

$$\frac{W_1}{W_2} = \frac{\overline{BC}}{\overline{AC}}$$

which means that the loads carried by the men are inversely proportional to their respective lever arms.

These facts are not peculiar to this problem, but apply as well to any instance where parallel forces are in equilibrium, and may be stated as follows:

The resultant of two parallel forces is equal to their sum and is applied at a point which divides the line joining their points of application inversely as the forces.

Evidently, the same principle could be extended to any number of parallel forces (Prob. 6, p. 48) and the condition for equilibrium be stated thus:

Any number of parallel forces are in equilibrium when

(1) **the algebraic sum of the forces is zero;**

(2) **the algebraic sum of the moments about all axes is zero.**

PROBLEMS

1. An angler, fishing with rod and line, hooks a fish. Will the fish appear to pull harder if the rod is a long one or a short one? Why?

2. A bicycle rider exerts a force of 10 million dynes on a pedal. Suppose the force to be at right angles to the pedal crank, and imagine the crank to be 18 cm. long, what moment of force does the rider exert?

3. In opening my pocketknife I catch the blade at a point 4 cm. from the pin about which the blade turns; and, at this point, I exert a force of 500,000 dynes. What moment of force is required to open the knife?

4. A mass of 20 gm. is revolving uniformly once in 5 sec. in a circle of 30 cm. radius. Find the centrifugal force.

5. At one end of a board which is 20 ft. long is placed a stone weighing 40 lb. At the other end of the board is placed a stone weighing 90 lb. Neglecting the weight of the board, find the point at which it must be supported in order that these two stones may just balance each other.

6. Forces 6, 8, and 10 acting on a pivoted body at perpendicular distances 3, 4, and 5 respectively from the pivot each tend to produce rotation in the same direction. Where must a force of 20 be applied to preserve equilibrium?

7. Taking the period of the moon's revolution around the earth as 28 days, and its distance from the center of the earth as 240,000 miles, calculate its acceleration toward the earth.

8. The load on a beam 18 ft. long and supported at each end is 4000 lb. If this load is 6 ft. from one end, what is the load on each support?

9. A loaded wagon weighing 2000 kgm. is crossing a bridge 40 m. long. When it is 10 m. from one side, how is the weight divided between the abutments of the bridge?

ON SYSTEMS OF BODIES. NEWTON'S THIRD LAW OF MOTION

43. If we take acceleration as the test for the presence of force, questions such as the following are likely to arise: A railroad train is running on a level track at the uniform rate of 24 mi. an hour. There is no change in the speed or in the direction of the velocity. Accordingly, the acceleration of the train is zero. This conclusion is plain, simple, unavoidable. But how can this be, when the engine is exerting a force of millions of dynes on the train?

The answer is simply this: The pull of the engine is only one among many forces acting upon the train. In these forces we must include the friction of the rails, the friction of the bearings, and the friction of the air. These forces are all opposing the pull of the engine. If now the train be running *uniformly* at the rate of 24 mi. an hour, we conclude that these opposing forces are exactly equal and opposite to the pull of the engine; and hence that the total force acting upon the train is zero. If steam be cut off the engine, the opposing forces accelerate the motion of the train in such a way as to diminish its speed. When the train comes to rest, the opposing forces become zero; and hence the total forces are again zero and the acceleration is again zero. When the engineer again lets on steam, the forward pull of the engine is greater than the backward pull of the opposing frictional forces. Hence the motion of the train is accelerated, this time in such a way that its speed is increased. This acceleration will continue until the frictional forces are exactly equal and opposite to the pull of the engine. At this point the acceleration is again zero.

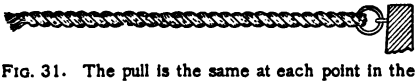


FIG. 31. The pull is the same at each point in the rope.

From this illustration the student will observe that great care is necessary to include in any system under discussion all the various parts of that system. The track and the air, retarding the train, are essentially parts of the system.

When the forces which aid any uniform motion are added to those which oppose the motion, the sum is always zero.

The fact is that forces always occur in pairs.

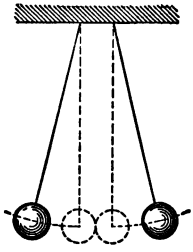


Fig. 32.
Action = reaction.

This is an observed fact of nature which Newton described by saying that **To every action there is always an equal and opposite reaction; or, the mutual actions of two bodies are equal and oppositely directed.** *Principia.* Book I.

This statement is known as **Newton's Third Law of Motion.**

When a mass is accelerated, it exhibits a reaction. And this force of reaction is one of the forces which must be taken into account in the *general* statement of Newton's *Third Law* just given. This force of reaction is well illustrated by the following experiment:

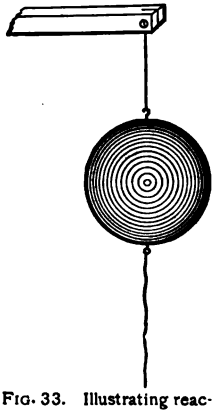


Fig. 33. Illustrating reaction which is measured by mass × acceleration.

With a piece of ordinary thread support a bag of sand, from two to four inches in diameter, as indicated in Fig. 33. To the lower side of the bag attach a piece of the same kind of thread.

Taking hold of this lower thread, one may by pulling break either thread he likes. If he pulls slowly, the lower thread will be affected only by the pull of the hand, while the upper thread will be stretched both by the pull of the hand *and* by the weight of the sand bag. In this case, of course, the upper string will break first.

If, on the other hand, one pulls the lower string quickly, it will as before feel the pull of the hand alone. But the upper string will be stretched practically by the weight of the bag only; for in this case the bag will be so rapidly accelerated that the reaction of the bag (mass × acceleration) will be sufficient to break the lower string before the upper one.

The student will find it an excellent habit to look at every force from these two points of view.

When a body, or system of bodies, is not accelerated, we may consider it as acted upon by two or more forces whose sum is always zero, or by two or more moments of forces whose sum is always zero.

The study of these particular cases where no acceleration is involved forms the subject of **Statics**.

PROBLEMS

1. A gun whose mass is 120 tons fires a shell whose mass is 800 lb. with a velocity of 2000 feet per second. What is the velocity of recoil of the gun?

2. A bullet weighing 10 gm. and moving with a velocity of 55 cm. per second lodges in a mass of putty weighing 100 gm., previously at rest, and the two then move on together against no resistance. How great is the velocity of the whole after the collision?

3. A 30-gm. bullet is fired from a gun weighing 5 kgm. with a velocity of ½ km. per second. What is the velocity of the recoiling gun?

Summary Concerning Newton's Laws of Motion

44. Inertia represents the tendency of matter to maintain its state of rest or of uniform motion in a straight line.

Mass is the amount of matter in a body.

The unit of mass in the C.G.S. system is the gram; in the F.P.S. system, the pound, written "lb."

Density is the mass per unit volume.

Linear momentum is the product of mass by velocity, that is, momentum $= mv$.

Masses may be compared by comparing their momenta, when their velocities are known.

Rotational inertia depends not only upon the amount, but also upon the distribution, of the mass, namely, its distance from the axis of rotation.

When no external influence is at work, the linear momentum of any body remains constant, or $mv =$ a constant (Law I).

When an external influence is at work, the change of linear momentum is proportional to the force and takes place in the direction in which the force acts (Law II).

Force produces change of linear momentum.

Force is measured by rate of change of linear momentum, hence $F = \frac{mv}{t} = ma$.

Concurrent forces may be compounded according to the principle of the "parallelogram of forces."

Concurrent forces are in equilibrium when their resultant is zero.

Centrifugal force is the force required to keep a body moving uniformly in a circle; its value is $m \frac{v^2}{r}$.

Moment of force is the product of force by lever arm, or $L = Fr$.

Parallel forces are in equilibrium when the algebraic sum of the forces is zero and also the algebraic sum of their moments about all axes is zero.

To every action there is always an equal and contrary reaction: or the mutual actions of two bodies are always equal and oppositely directed (Law III).

FRICTION

45. In the preceding pages attention has repeatedly been directed to the fact that in the world about us none of the motions are uniform, or even uniformly accelerated. This is in large part due to the fact that any two surfaces in contact require the application of force to cause one surface to move over the other. A boat requires impulse from oars, screw, or paddles to drive it through the water; a descending parachute depends upon the retarding action

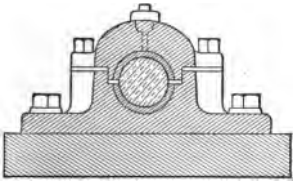


FIG. 34. Engine bearing, steel on Babbitt metal.

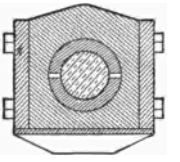


FIG. 35. Car axles, steel on brass.

of the air; any piece of machinery will gradually come to a stop after the power has been turned off. **The force called into play to resist the motion when one surface slides or rolls over another is called the force of friction.** Between smooth surfaces it is, other things being the same, less than between rough; between unlike substances usually less than between like. Hence, machine bearings are usually steel on Babbitt metal (Fig. 34) and freight car axles run in brass journals (Fig. 35).

FIG. 36. Friction is due to the interlocking of small elevations and depressions.

The origin of friction (Fig. 36) doubtless lies in the interlocking of small hills and valleys, slight elevations and depressions that remain even on the most highly polished surfaces.

Sliding Friction

46. The principal **experimental** facts regarding the behavior of one solid body sliding over another may be summarized as follows:

The frictional resistance between solid bodies

(i) **is proportional to the force with which the bodies are pressed together;**

(ii) **is independent of the area of the surfaces in contact; and**

(iii) **is, within wide limits, independent of the relative speed of the bodies.**

The first of these facts can be thus illustrated. A block (Fig. 37) weighing 500 gm. can be just kept in uniform

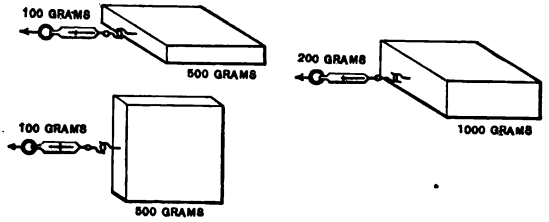


Fig. 37. Force of friction is proportional to the load, independent of surface in contact and of the speed.

motion horizontally by a pull of 100 gm. If the block is loaded an additional 500 gm., the pull increases proportionally to 200 gm., and so on as the load is further increased. **The ratio** ($\frac{100}{500} = 0.2 = \frac{200}{1000} =$ constant) **between the force of friction and the force pressing the bodies together is called the coefficient of friction.**

$$\text{coefficient of friction} = \frac{\text{force of friction}}{\text{force of pressure}}$$

It is also found that it makes no difference whether the block is placed on its broad face or an edge. The friction depends only on the pressure.

Rolling Friction

47. Why are roller bearings (Fig. 38) used in automobiles, and ball bearings (Fig. 39) in bicycles, instead

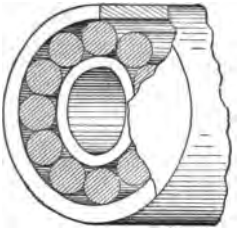


Fig. 38. Roller bearing.

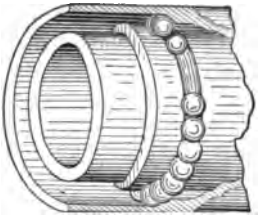


Fig 39. Ball bearing.

of ordinary bearings? Simply because experiment shows that friction between surfaces is less if they roll than if they slide over one another.

But the question now arises, Why should there be *any* friction in, say, a smooth iron car wheel rolling over a level, smooth iron track? The reply is, that the iron track (Fig. 40) is slightly and temporarily indented so that the wheel has really to run uphill all the time. Any one who rides a bicycle over a level sandy road soon becomes familiar with this phenomenon in an exaggerated form and keenly realizes the uphill features of the problem.

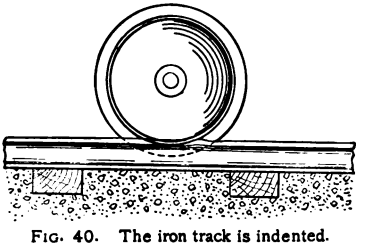


Fig. 40. The iron track is indented.

Uses of Friction

48. At no time is the usefulness of friction more plainly evident than when one attempts to cross an icy sidewalk or a waxed floor. The ordinary operations of walking or grasping an object in the hand would be impossible if

friction were absent. Nails, nuts, bolts, screws, and belts are devices dependent for their action on its presence.

Summary concerning Friction

49. Friction is the force called into play to resist the motion when one surface slides or rolls over another.

Friction is probably due to the roughness of the surfaces in contact, hence, other things being the same, —

Friction is less between smooth surfaces than between rough; between unlike than between like;

Friction is proportional to the force with which the surfaces are pressed together;

Friction is independent of the area of the surfaces in contact;

Friction is, within wide limits, independent of their relative speed.

Rolling friction is usually less than sliding friction.

PROBLEMS

1. A piece of steel weighs 1000 kgm. and a horizontal pull of 17 kgm. just keeps it in motion. What is the coefficient of friction?

2. The coefficient of friction between two pieces of wood is 0.25. The upper block weighs 200 lb. What force will just move it along a horizontal plane?

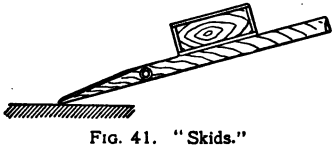


Fig. 41. "Skids."

3. A ladder leans against the wall of a house. What keeps it in position?

4. A box of dry goods rests on a pair of "skids." (Fig. 41.) It does not slip down. A barrel of sugar of the same weight rolls readily down the same slope. Explain.

5. Maud S. made a trotting record for one mile of 2 : 08¾, drawing an old-fashioned, high-wheeled sulky. John A. McKerron rounded the same track a few years later in 2 : 04¼, drawing a sulky with pneumatic tires and ball bearings. Why should there be any doubt as to which was the better horse?

6. Why is oil used on bearings?

SECOND GENERAL PROPERTY OF MATTER

II. Gravitation

50. Before the time of Galileo (1564–1642), the ideas of men concerning gravitation were simply chaotic. They had observed that some bodies, such as stones, fall very rapidly; that others, such as feathers and thin paper, fall very slowly; that heavy bodies fall with a constantly increasing speed; that lighter bodies, such as small raindrops, or feathers, fall with a practically uniform speed; that in some media certain bodies do not fall at all, as, for instance, wood in water, or iron in mercury. Galileo was the man who first harmonized these apparently contradictory facts, and described them all in a very simple way.

Up to this period nothing was known as to how far a body would fall in a given time, or as to what speed it would acquire in a given time, or as to what speed it would attain in falling a given distance.

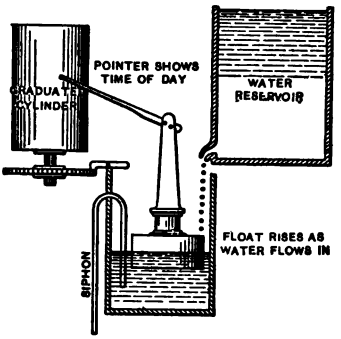


Fig. 42. An Italian water clock.

The difficulty in getting at these facts, *i.e.* the difficulty in discovering *how* a body falls, lay partly in the circumstance that the ancients had no accurate method of measuring small intervals of time, partly in the circumstance that the speed of falling bodies, after a minute interval of time, is liable to be very rapid, and partly in the fact that the predecessors of Galileo had little or no idea of obtaining knowledge by experiment.

For measuring small intervals of time, Galileo employed

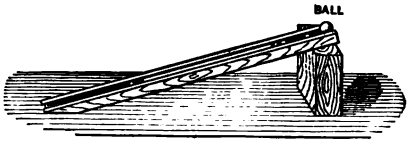


FIG. 43. Galileo's inclined plane.

a very ingenious water clock.* To prevent the attraction of the earth from giving the falling body too high a speed, he allowed the body to roll down a smooth inclined plane of small slope (Fig. 43).

GALILEO'S EXPERIMENT ON FALLING BODIES

51. But first he satisfied himself that **all kinds of matter would fall the same distance in the same time.** One ball might be made of copper, another of iron; the time of descent was the same in each case.† The resistance of the air was practically eliminated when bodies of the same size and nearly the same density were let fall. But, in many cases, large bodies and small bodies occupy *practically* the same time in falling from the same height. On the other hand, as every one knows, a bunch of cotton does *not* fall as rapidly, in air, as does a bullet of lead.

But Galileo's conclusions were that, in all such cases, the difference in speed is due to the resistance of the air. He regretted that he had no means of producing a vacuum, so that he might appeal to a direct experiment: accordingly he employed a series of fluids of different densities, such as mercury, water, air; and found that as the density of the fluid diminishes the bodies descending in these fluids fall more nearly at the same rate, *i.e.* the same distance in the same time. He argued, therefore, that, **in a vacuum,** where the density of the fluid is zero, **all bodies when starting from**

* Galileo's clock and also the clock shown in Fig. 42 are described in Cunynghame's "Time and Clocks," pp. 39–41. (E. P. Dutton & Co., New York, 1906.)

† The account of Galileo's experiments at Pisa, given in Cajori's "History of Physics" (pp. 32–33), should be read by every student.

rest, whatever their size, shape, or composition, fall through equal distances in equal times.

How thoroughly Galileo was justified in this conclusion was proved later, about the year 1650, when the air pump was invented by Otto von Guericke. One of the first experiments to be tried in the newly discovered vacuum was to place a feather and a coin in the same tube (Fig. 44) and see whether they would fall at the same rate in a region devoid of air. The result was such as to show that Galileo was perfectly correct in thinking that the earth impresses the same velocity upon all falling bodies, if allowance is made for the resistance of the air.

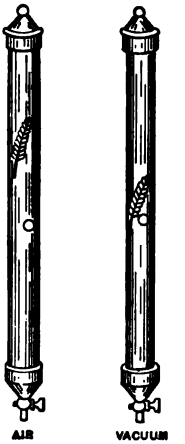


Fig 44.
Coin and feather

Galileo's Experiment on the Inclined Plane

52. Galileo was now ready to begin his experiment on the inclined plane, to which we have referred above. This is very conveniently repeated in the following form. (See Fig. 45.)

From one side of a room to another stretch a brass or steel wire of one or two millimeters diameter. The wire

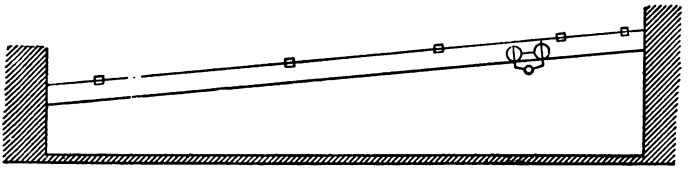


Fig. 45. Galileo's experiment.

should have a slope of about one foot in eight or nine, and should be stretched very taut.

Parallel to the stretched wire, and about six inches above it, should be stretched a string, on which some small indices of cardboard are strung, as shown in Fig. 45.

A small carriage to run on this wire track is shown in Fig. 46. It should have light wooden wheels. A pendulum, a clock, or a telegraph sounder should beat seconds in the room.

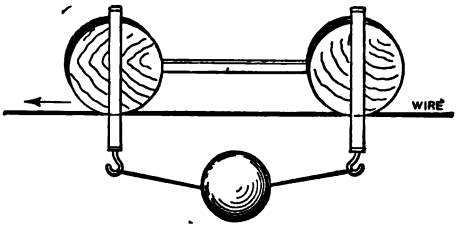


FIG. 46. Carriage for Galileo's experiment.

On the beat of the seconds' pendulum the carriage is released at the upper end of the inclined track. The position of the carriage at the end of one second is marked by sliding a cardboard index along the string to the proper place. Another trial enables one to mark the position of the carriage at the end of the second second; a third observation gives the position of the carriage at the end of the third second; and so on. Having thus marked off on the inclined plane the positions of the falling body at ends of successive seconds, one may tabulate his results as follows:

Relation between Time and Distance of Descent

Let t = time since carriage was released.
d = total distance passed over by carriage, in time t.
Ratio of length to height of inclined plane = 12.7.

TIME t	DISTANCE d	$\frac{\text{DISTANCE}}{(\text{TIME})^2} = \frac{d}{t^2}$	ACCELERATION ALONG THE INCLINED PLANE, $\frac{2d}{t^2} (= a)$	ACCELERATION IN A VERTICAL DIRECTION, $12.7\ a\ (= g)$
1 sec.	30 cm.	30.	60	762
2 sec.	118 cm.	29.5	59	749
3 sec.	273 cm.	30.3	61	775
4 sec.	478 cm.	29.9	60	762
5 sec.	726 cm.	29.0	58	737

But we have already found that, in the case of a constant acceleration, a body starting from rest will traverse a distance which varies directly as the square of the time, $d = \frac{1}{2} at^2$, or $d \propto t^2$

In this experiment, also, from the above observations, it is evident that, **in the case of earth, the distance varies directly as the square of the time**: for in the third column the figures are as nearly constant as one could expect with a rough experiment of this kind.

Acceleration of Gravity a Constant

53. In this manner, Galileo inferred that this particular acceleration, namely, **the acceleration produced, upon a freely falling body, by the earth, is also a constant.** This is a discovery with whose consequences we are so familiar that we are perhaps a trifle blind as to its importance.

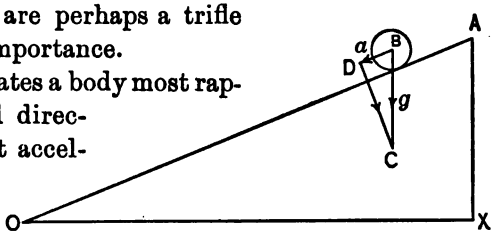


Fig. 47. Showing how Galileo "diluted gravity" in order to study it.

Gravity accelerates a body most rapidly in a vertical direction, and does not accelerate a body at all in a horizontal direction. Indeed, all that we mean by a "horizontal" direction is one in which the acceleration of gravity has no component.

In Fig. 47 let us represent by the line $\overline{BC}$ the total acceleration of gravity, which we shall denote by g. If $\overline{BC}$ is resolved into two components, one parallel to the plane, $\overline{DB}$, and one perpendicular to it, $\overline{DC}$, the component which tends to accelerate the motion of B along the plane is $\overline{DB}$. From the evident similarity of the triangles BDC and AXO, $\frac{\overline{BC}}{\overline{BD}} = \frac{\overline{AO}}{\overline{AX}}$, or $\overline{BC} = \overline{BD}\,\frac{\overline{AO}}{\overline{AX}}$

which means that the acceleration along the plane $\overline{BD}$, which we actually obtain in Galileo's experiment, must be multiplied by the ratio $\frac{\text{length of plane}}{\text{height of plane}}$ in order to obtain the value of the acceleration due to gravity, $\overline{BC}$.

This device is, therefore, competent not only to show that the acceleration of gravity is a constant, but also to give us the numerical value of this constant. The values of this constant derived from Galileo's experiment are given in the last column of the above table, but since no allowance is made for friction they are about 25 per cent too small. Using centimeters and seconds, we find that $g = 980$ approximately.

Galileo's Inferences

54. Having found that **the acceleration of gravity is constant,** Galileo at once made the following inferences, namely, (1) **that the speed at the end of any interval of time is proportional to the first power of the time.** $v \propto t$, or $v = gt$. This consequence, we have already seen (p. 16), follows in *all* cases of constant acceleration. The inference may be tabulated as follows:

Relation between Time of Fall and Speed Acquired

Time	Speed
1	g
2	$2g$
3	$3g$
...	...
...	...
t	gt

By definition, g is the number by which the speed of a freely falling body is changed in each second. Consequently, in 2 sec., the total *change* in speed will be $2g$; in 3 sec., $3g$; and in t sec., gt; or, if the body starts from rest, the results in the preceding table are expressed algebraically as follows:

$$V = gt$$

If the falling body has, at time $t = 0$, a speed of V_0, our expression for the

actual speed at any instant, t, becomes

$$V = V_0 + gt$$

(2) **Distance passed over at the end of any interval of time is proportional to the second power of the time,**

$$d \propto t^2, \quad \text{or} \quad d = \tfrac{1}{2} gt^2$$

Summary of Laws of Freely Falling Bodies

55. These are exactly the laws of a uniformly accelerated particle which we have studied in Chapter I, only here the numerical value of the acceleration ($a = g$) is practically always 980, using the centimeter and second as units.

Relation between Speed and Time, $V = V_0 + gt$.
Relation between Distance and Time, $d = V_0 t + \frac{1}{2} gt^2$.
Relation between Distance and Speed, $V = V_0^2 + 2gd$.

PROBLEMS

In considering the following questions, the resistance of the air is to be neglected; the acceleration of gravity is to be assumed as 980 $\frac{\text{cm.}}{\text{sec.}^2}$, or 32.2 $\frac{\text{ft.}}{\text{sec.}^2}$.

1. If a rifle bullet were shot directly upward with a velocity of 3300 cm. per second, how far would it rise? With what velocity would it return? How long would it remain in the air?

2. A stone is dropped into a stream from a bridge 150 ft. high. How soon will it strike the water?

3. A boy throws a baseball to a height of 30 m. With what speed does the ball leave his hands?

4. A body falls freely for 10 sec. How far does it travel? What is its final velocity? How far did it go in the first second? How far in the tenth second?

5. *a.* A gun is held horizontal at a distance of 490 cm. above the ground. How long after the gun is fired until the bullet reaches the ground?

b. During this interval, how far will it travel in a horizontal direction if it leaves the muzzle of the gun with a speed of 500 m. a second?

c. Construct a diagram describing the path of the bullet.

6. A cannon ball is projected horizontally from a wall 100 ft. above the sea. How soon will it strike the water? How far will it travel if its initial speed is 2000 ft. per second?

7. What is the "range" of a gun? Why is the muzzle of the gun elevated to increase its range?

8. Through what distance would a body have to fall under gravity to acquire a speed of 60 km. an hour?

9. A baseball is thrown over a tree 35 ft. high. How long is the ball in the air? If the thrower stands 40 ft. from the foot of the tree, where should the ball strike the ground? What is the horizontal velocity of the ball?

10. A package is thrown horizontally with a velocity of 6 m. per second from the door of a train. If it starts from a point 2 m. above the station platform, how long before it strikes the platform and how far from the train does it strike?

Periodic Motion

56. *Linear Periodic Motion.*—There is another motion with which all students, even beginners, are already familiar in a general way. This motion is typified by that of a particle, *P* (Fig. 48), supported by a vertical spiral spring, and vibrating freely in a vertical straight line. The motion of any point on a guitar string which has just been plucked and let go furnishes another illustration of this motion. If a body be suspended by a rubber band, then slightly displaced up or down and suddenly released, its motion also will be that which we are about to study.

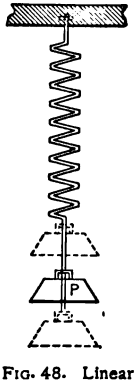


Fig. 48. Linear periodic motion.

In each of these cases observe that, when the body vibrates freely,

(*a*) The motion is one of translation; because the particles of the body describe equal and parallel paths.

(*b*) The path is a straight line of limited length.

(*c*) The particle moves most rapidly at the middle of its path, and comes to rest at each end of its path.

(*d*) The motion gradually dies down, unless kept up by some "outside" means.

(*e*) The motion is periodic; namely, repeats itself at equal intervals of time.

Such would be a popular and qualitative description of the motion.

57. *Angular Periodic Motion.*—In the case of solid bodies having a fixed axis of rotation, we frequently meet a very similar motion, and one with which, again, all students are familiar in a general way.

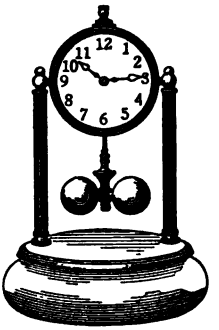


FIG. 49. Clock with a torsion pendulum.

The pendulum of a clock is a body whose angular position is constantly changing. It typifies the motion we are about to describe. The motion of the balance wheel in a watch movement also illustrates the motion under consideration. If a slender lath be clamped at one end, while the free end is pulled aside and suddenly let go, this free end will vibrate with practically the same motion as that of a pendulum left to itself.

Observe that, in each case, when the body vibrates freely,

(*a*) Its motion is an angular one.

(*b*) The amount of swing never exceeds, in any one case, a certain limited angle.

(*c*) The speed is most rapid at the middle, and ceases altogether at the end of the swing.

(*d*) The vibration gradually dies down, unless kept up by some means external to the system we are studying,

as, for instance, by the weight of the clock or the coiled spring of the watch.

(*e*) The motion is periodic.

58. *Definitions.* Angular periodic motions are very common, every rotating wheel describing this variety of motion. The following definitions will be perhaps more easily remembered when referred to a rotating wheel than when referred to the pendulum, though applicable to both.

Definitions. **The period of the motion is the time it takes to make one round trip.**

The amplitude of the motion is the radius of the circle in which the particle moves; or **it is the greatest distance the particle reaches from the middle point of its path.**

The phase at any time is the fraction of the round trip then completed, measuring time and rotation from the same starting point.

A complete vibration means one round trip.

THE PENDULUM

59. A small mass, *B* (Fig. 50), **suspended by a very light thread from any point, *O*, so that it is free to vibrate is called a simple pendulum.** When such a pendulum is drawn aside from *D*, its position of rest, and let go, the pendulum bob, *B*, is urged back toward *D*, passes on to *E*, comes to rest, and begins a similar return trip. The round trip, from *B* to *E* and back again, is a complete vibration, and the time for this round trip is the period of the pendulum. The motion is in all particulars like that described in Sec. 58.

The accelerating force is the attraction of the earth, called the attraction of gravity, g, and is represented by the straight line $\overline{BC}$. When the bob is in the position *B*, the component of gravity producing acceleration toward *D* is represented by the straight line, $\overline{BA}$.

The length of such a pendulum is usually taken as the distance from O to the middle point of the bob, B. In the ordinary clock pendulum (Fig. 51) the length cannot

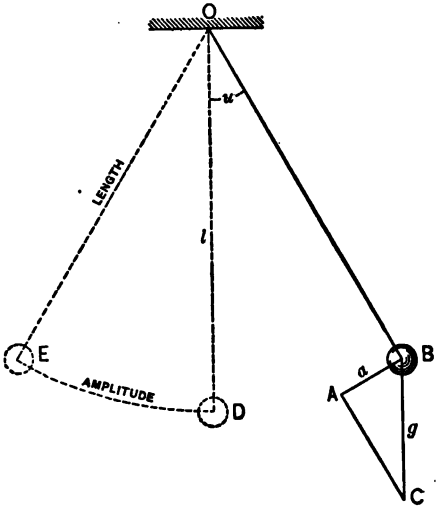


FIG. 50. A simple pendulum.

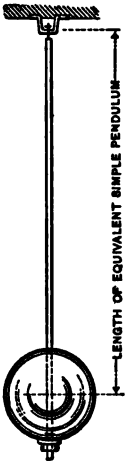


FIG. 51. The clock pendulum is a compound pendulum.

be found in this simple manner, and hence the pendulum is called *a compound pendulum*. In many cases this *length* may be easily calculated, so that we may often employ the compound pendulum as the equivalent of a simple one.

LAWS OF THE PENDULUM

60. Experiments on the simple pendulum — some of which were first performed by Galileo and some by Huygens (1629–1695), and which may easily be repeated in the laboratory — have established the following facts which may be called the "**laws of the pendulum**:"

(i) **The period is practically independent of the amplitude when the amplitude is small.**

(ii) **The period is independent of the mass of the pendulum, or the material of which it is made.**

(iii) **The period is directly proportional to the square root of the length.**

(iv) **The period is inversely proportional to the square root of the acceleration due to gravity.**

These experimental facts may be summarized as follows:

The period (T) of a pendulum is proportional to $\sqrt{\frac{\text{length}}{\text{gravity}}}$, or $T = 2\pi\sqrt{\frac{l}{g}}$.

The Clock Pendulum as a Timekeeper

61. It is a well-known fact that any pendulum, vibrating independently, "dies down" and eventually stops.

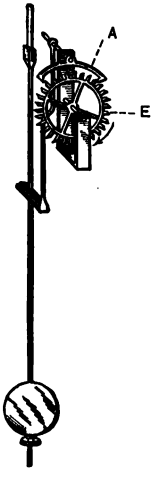


Fig. 52. Clock escapement.

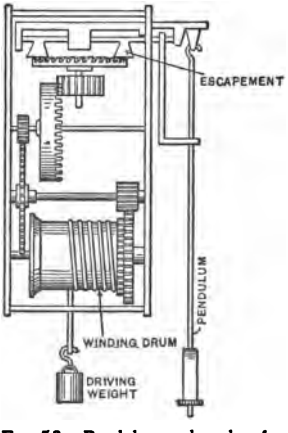


Fig. 53. Pendulum and works of a grandfather's clock.

Another fact not so well known is that successive swings are made in equal times, or **the vibrations of a pendulum**

are isochronous. This isochronism makes the pendulum useful for regulating the motion of the "works" of a clock which in turn supply the push needed to keep the pendulum in vibration.

The mechanical device called an escapement — one form is shown in Fig. 52 and another in Fig. 53 — consists of an escape wheel (E) connected by a train of toothed wheels (Fig. 53) with the driving weight or spring of the clock. As the pendulum swings to the right, the anchor, A (Fig. 52), allows one tooth only of the escape wheel to press against the projecting notch on the anchor, thus pushing the pendulum just enough to keep it vibrating. An inspection of Figs. 52 and 53 will show how the works of the clock are compelled to move at a rate dependent on the period of its pendulum. How is such a clock "regulated"?

PROBLEMS

1. The length of a pendulum is 1000 cm. at a place where the value of the acceleration of gravity is 980 $\frac{\text{cm.}}{\text{sec.}^2}$. Find the period.

2. The period of a pendulum is 2 sec. when the length is 1 m. What is the acceleration of gravity at that place?

3. It is well known that metals expand on heating. What will be the effects of the cold weather of winter and the hot weather of summer upon the rate of a clock having a brass pendulum? Upon a clock having a shellaced wooden pendulum rod?

4. Compare the lengths of pendulums whose periods are 1, 3, and 5 units respectively.

5. A clock whose pendulum is 100 cm. long keeps correct time at a place where the value of the acceleration of gravity is 980 $\frac{\text{cm.}}{\text{sec.}^2}$. How many vibrations does it make in one day?

6. A pendulum makes 86,000 vibrations per day. What is its length?

UNIVERSAL GRAVITATION

62. Before the time of Newton (1642–1727) Copernicus had demonstrated that the sun was the center of the solar system; but neither he nor any one else had shown what it was that kept the planets in motion about the sun.

Newton pointed out, as we have already seen (Sec. 28), that bodies do not require any force to keep them in motion. When no forces are acting, the body moves at a uniform pace in a straight line. Motion is just as natural to a body as rest. But when either the amount or the direction of the velocity is altered, then we know some force is at work.

In the case of the planets moving in almost circular paths about the sun (Fig. 54), the *direction* of the velocity of each planet is constantly changing. What is the force which is ever changing the velocity of the planets? We have already seen (p. 36) that when a particle moves in a circle, from whatsoever cause, there is a force attracting the particle toward the center.

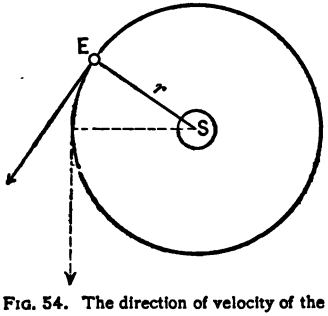


FIG. 54. The direction of velocity of the earth is continually changing.

In like manner, the moon revolves about the earth in a path which is almost circular. What is the source of *its* attraction? The eight satellites of Jupiter also move in nearly circular paths. Why do they not move off along straight lines, for the straight line is the path of a body when left to itself? The answer is given in

Newton's Law of Universal Gravitation

63. Before considering Newton's answer to this question, let us recall the fact that the dimensions of each of these planets are so small compared with their distances from the sun that they may be treated as particles. Newton showed by mathematical computation that if we suppose the sun to attract each of the planets with a force which varies directly as the mass of the planet, and inversely as the square of the distance between the planet and the sun, this supposition will explain, in the most satisfactory manner, all the motions of the planets. Going one step farther, he supposes that **any two particles, or masses which may be considered as particles, m and m', attract one another with a force proportional to the product of the masses, mm', and inversely proportional to the square of the distance, r, between them.** Expressing this in the language of algebra, $F = \frac{mm'}{r^2}$ dynes. This statement is known as **Newton's Law of Universal Gravitation.**

Newton next proved that this supposition would explain the circular path of the moon. And by demonstration, which, however, must be reserved for the more advanced student, he showed that the reason ordinary bodies at the surface of the earth are heavy is that the earth attracts them just as it does the moon. The whole earth acts as if its mass were concentrated at its center, so that bodies at the surface of the earth act as if attracted by a particle whose mass is that of the earth, and whose distance is about 4000 miles.

How bodies attract one another Newton discovered and described in a most convincing manner. Why bodies attract one another is as much of a mystery as ever.

Mass and Weight

64. Mass has been defined as the amount of matter in a body, and the unit of mass is the "gram." In accordance with the Law of Gravitation the earth attracts any mass with a force which will change its velocity at the rate of about 980 cm.-per-second in a second. That force which changes the velocity of 1 gm. by 1 cm.-per-second in a second is defined as 1 dyne. Hence, **the earth's attraction for 1 gm. is 980 dynes.** This attraction is called 1 gram-weight. Evidently, in dynes, 2 grams-weight would be 2×980, and so on. In general

grams-weight

$$= \text{grams-mass} \times \text{the acceleration due to gravity}$$

or $$W = mg.$$

In engineers' units the weight of a pound, which is the commercial unit of force in America, takes the place of the dyne.

The Value of Gravity

65. The earth is a spherical body but not an exact sphere. Its equatorial diameter is 7926.592 miles or 12,756,380 meters, while its polar diameter is only 7899.58 miles, or 12,712,912 meters. The highest mountain is about 6 miles high and the deepest mine about $1\frac{1}{3}$ miles deep.

Evidently, according to Newton's Law of Gravitation, a body should receive different accelerations due to gravity depending upon its distance from the center of the earth. Experiment confirms this supposition. The following table gives the value of this acceleration in C.G.S. units:

Absolute Value of Gravity

Atlanta	979.523	New Orleans	979.323
Austin	979.282	New York	980.266
Boston	980.395	Philadelphia	980.195
Cambridge	980.397	Princeton	980.177
Charleston	979.545	Salt Lake City	979.802
Charlottesville	979.937	San Francisco	979.964
Chicago	980.277	St. Louis	980.000
Cleveland	980.240	Terre Haute	980.071
Cincinnati	980.000	Washington	980.111
Colorado Springs	979.489	Worcester	980.323
Denver	979.608	London	981.201
Ithaca	980.299	Paris	980.941
Kansas City	979.989	Potsdam	981.274
Little Rock	979.720	Para	978.03
Montreal	980.742	Spitzbergen	983.08

Mass (amount of matter) is unaltered by change of location, but since weight is proportional to gravity ($W = mg$), its value will change with the value of gravity. **The weight of any mass above the earth's surface is,** then, in accordance with Newton's Law, **inversely proportional to the square of the distance from the center of the earth.**

PROBLEMS

1. Shortly after the invention of the pendulum clock, one was sent from Paris, where it kept correct time, to Cayenne, in South America. In the latter place the clock lost $2\frac{1}{2}$ min. daily. The length of the pendulum was adjusted so that the clock kept correct time in Cayenne; and in this condition the instrument was sent back to Paris, where it now *gained* $2\frac{1}{2}$ min. daily. Explain this phenomenon.

2. There are 454 gm. in 1 lb. What is the force, in dynes, with which the earth attracts a mass of 1 lb., at a place where $g = 980$ C.G.S. units?

3. Find the weight of 7 kgm. in dynes. $g = 960$.

4. The period of a given pendulum at Denver is exactly 1 second. The period of the same pendulum at Chicago is 0.9995 second. The acceleration of gravity at Denver is 979 C.G.S. units. What is its value at Chicago?

5. What will be the weight 5 miles above the earth's surface of a mass weighing 10 kgm. at the surface of the earth?

6. What will be the weight of a 100-lb. ball if taken to the moon, 240,000 miles away from the earth?

7. A spring balance is made and graduated at a town where $g = 983$. At another town where $g = 979$, the spring balance is used in a meat market. Who gains by this error, the man who buys the meat or the man who sells?

8. From the basket of a balloon hangs a spring balance. This balance carries a mass of 100 gm. The balloon ascends with acceleration of 220 C.G.S. units. What will be the apparent weight of the 100 gm. during the ascent?

9. A ball thrown vertically upwards is caught by the thrower 5 sec. later. Neglect the resistance of the air, and compute the height to which the ball was thrown.

10. The radius of the earth is 636,000,000 cm. At what linear speed must a body move along the surface of the earth to lose all apparent weight?

11. Why is the observed value of gravity less at the equator than at the poles?

12. What is the length of a seconds pendulum at Spitzbergen? at Cincinnati? at Para?

Center of Gravity

66. Since the earth acts essentially as if it were a particle 4000 miles away, it is evident that any one part of a body at the surface of the earth is attracted in a direction which is in general slightly different from that of any other part; but this difference of direction is so exceedingly minute that we generally speak of the weights of the various particles in a body as parallel forces. And the total weight of a body, we know, results from the individual weights of its individual particles.

Let us, for instance, string two heavy masses, M_1 and M_2 (Fig. 55), upon a light rod. Let W_1 be the weight of one mass and W_2 the weight of the other. We then know, by the rule for addition of parallel forces, that the total

weight of the system, W, is $W_1 + W_2$. But through what point of the system this total weight, W, acts we have yet to discover.

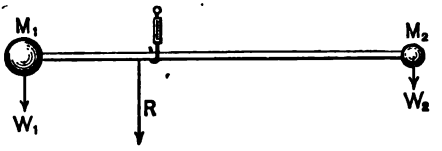


FIG. 55. The total weight acts through the center of gravity.

From whatever point we may hang the system, the weight of it, as read off on the spring balance, will be the same. But there is only one point on the bar at which we can place the balance and have the system remain horizontal. That is, there is only one point of support about which the weights will offer no moment of force tending to rotate the bar. When once we have placed the hook of the balance at this point of zero moment, we have discovered the point through which the resultant W acts. The weight is not zero; but the arm of the resultant weight is then zero; or, what is more strictly true, the sum of all the moments tending to rotate the system about this point of support is zero. This point is known as the **center of gravity** of the system.

DEFINITION OF CENTER OF GRAVITY

67. The center of gravity is accordingly defined as that point through which the total weight of the body — considered as a single vertical force — acts. The center of gravity is located practically at the center of mass. From this definition it is evident that if a body be supported through this point it will be in equilibrium, *i.e.* it will not rotate, because there is no resultant moment of force acting upon it, and it will not be translated because the point at which it is supported is fixed. This leads to an easy method for determining, *by experiment*, the position of the center of mass in a body.

Determination of Center of Mass

68. If a body is straight and slender, as in the case of a wire or a walking stick, we may locate the center of mass at once by balancing the body over a knife-edge (Fig. 56), thus discovering the point about which the moment of force is zero.

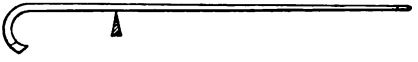


Fig. 56. The cane balances about its center of gravity.

But if the body be a lamina, *i.e.* a thin flat plate, we may infer from symmetry that the center of mass lies somewhere in the plane of the body. And by balancing the body about a knife-edge we may determine in this plane a straight line $\overline{AB}$ (Fig. 57), which contains the center of mass. If, now, we choose another position for the knife-edge, say $\overline{A'B'}$, and balance the body on it again, we shall locate the center of mass on another straight line.

Since, now, we know that the centre of mass lies on each of these two straight lines, we know that it must lie at their intersection, C. We shall hereafter frequently denote center of mass by the letters *c.m.*

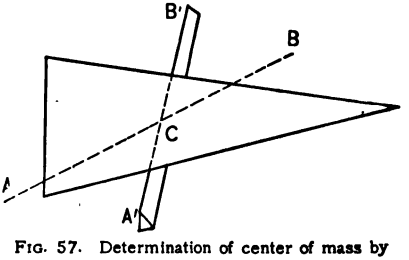


Fig. 57. Determination of center of mass by experiment.

In the case of an extended body, *i.e.* one all three of whose dimensions are considerable, we should need to balance the body about three different lines; for, in this case, each balance would determine one vertical *plane* in which the center of mass lies; and three such planes would be required to locate, by their intersection, the center of mass of the extended body.

In many cases of this kind the center of mass can be located by symmetry, as in the case of a sphere or a parallelopipedon or a cylinder; but only where the density of the body is uniform or symmetrical. In the case of "loaded" dice, for instance, the center of mass would *not* coincide with the center of figure.

It is a general principle of dynamics, first formulated by Galileo, **that the center of mass of a body starting from rest always seeks the lowest possible position in space.**

But however this principle is stated, it suggests still another slightly different method for getting the center of mass of a body extended in all three dimensions, viz. instead of balancing on a knife-edge, suspend the body from some one *point* by a thread (Fig. 58); the center of mass will seek its lowest position, and will, therefore, lie in the straight line determined by the thread.

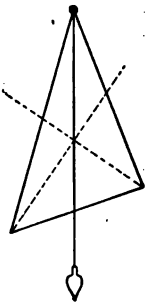


FIG. 58. The c.m. of a triangle.

If, now, the body be suspended from another point, a second line will be determined in which also the c.m. lies. The intersection of two lines must occur, else the body would have *two* centers of mass. This intersection must, therefore, be the c.m. In this way two observations, instead of three, suffice to locate the c.m. of a three-dimensional body.

EQUILIBRIUM OF HEAVY BODIES

69. We have already seen that **when the center of mass of a heavy body lies in the same vertical line with the point of support, there is no moment of force tending to rotate the body.** Such a body is said to be **in equilibrium, or, in other words, the vertical line from the c.m. falls within the base on which the body rests.** We all know from daily

experience that bodies may be in equilibrium in a variety of ways. A lead pencil may, with a little care, be made to stand upon an end which has not been sharpened, and it is said to be in equilibrium; but a breath of wind, or the slightest blow, is sufficient to "start" it, and, when once started, it falls over, "of itself," we say. If the pencil be a round one, it will rest on its side in *any* position we may give it, so long as it lies in a horizontal plane. Here, again, it is said to be in equilibrium; but if it be displaced slightly, it will still be in equilibrium in its new position; for there is no force or moment of force acting upon it in any position so long as it lies on its side.

Let us, again, consider such a body as a brick lying on any one of its sides. We all know that if the brick be slightly lifted at one end, *i.e.* if the brick be slightly rotated about one edge, it will not go on and "of itself" rotate still farther; but, if released, will return to its original position of equilibrium.

These three cases typify *all* the cases of equilibrium met with in nature. Their essential features are the following:

1. **Unstable Equilibrium.** When a body is supported in such a way (Fig. 59) that its center of mass is lowered by a slight displacement, its equilibrium is said to be **unstable**; for the slightest displacement will introduce a moment of force tending to rotate the body still farther. For illustration, imagine a cone inverted and balanced on its apex. The center of mass when the cone is ever so little inclined moves *downward* and the cone topples over.

2. **Neutral Equilibrium.** If a body is supported in such a way that its center of mass remains at the same height above the point or line of support, its stability is said to be **neutral**; for when disturbed no moment of force will be introduced to displace it farther or to restore it to its original position. Such a case is represented by a right

cone (Fig. 59) lying on its side in a level plane. The center of mass lies on a level, and has, therefore, no tendency to move in any direction.

3. **Stable Equilibrium.** When a body is supported in such a way that the slightest displacement will raise its c.m., the body is said to be in **stable** equilibrium; for the slightest displacement will introduce a moment of

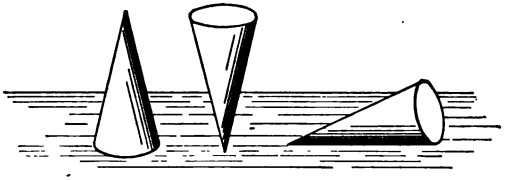


FIG. 59. Stable, unstable, and neutral equilibrium.

force tending to restore the body to its original position. Such a case is found in a right cone standing on its base (Fig. 59). Any small motion of such a cone starts the c.m. uphill, so to speak; and when left to itself, the center of mass tends to roll back, downhill.

It should be carefully observed that in each of the three different cases of equilibrium the c.m. of the body seeks its lowest position so that the vertical line from the c.m. shall always fall within the base.

PROBLEMS

1. In which of the following cases does the center of mass lie inside, and in which outside, the body: a teacup, a straight copper wire, the same wire bent into the arc of a circle, a hoop, a silver half-dollar, a base-ball, a football?

2. Locate, by geometry, the c.m. of a triangular sheet of brass. If a corner of this triangle be clipped off along a line parallel to the opposite side, the c.m. will be displaced along what line?

3. A meter scale (the mass of which is negligible) is divided into decimeters, and at the end of the first and eighth decimeters are attached masses of 1 and 9 kgm. respectively. Where is the c.m. of the system?

4. An iron bar weighs 4 lb. per foot of length. It balances about a point which is 2 ft. from one end when a weight of 5 lb. is suspended from that end. How long is the bar?

5. Why is one's stability less when coasting on a wheel than when resting his weight on the pedals?

6. Illustrate the fact that the equilibrium of a body may be stable even while the center of mass lies *above* the base of support.

7. The leaning tower of Pisa is 183 ft. high, has walls 13 ft. thick at the base, and the extreme edge of its cornice is 13 ft. 8 in. out of the perpendicular. Why does it not fall?

8. Find the center of gravity of three heavy particles whose masses are 3, 5, and 9, and which lie in straight line, at the following distances from the left-hand end of the line, 2 ft., 4 ft., and 6 ft.

THE BALANCE

70. The balance is known to most people as an instrument for comparing the weights of two different bodies; *i.e.* for comparing two different forces.

The grocer when he sells us a pound of coffee deals out an amount such as the earth will attract with a force which English-speaking people have agreed to call "the weight of one pound." *

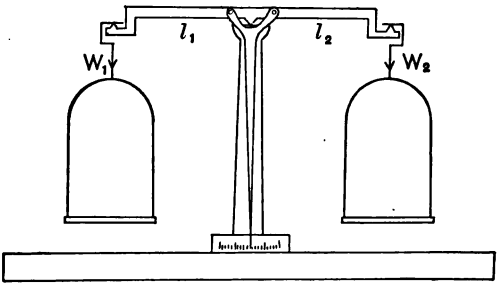


FIG. 60. Ordinary beam balance.

* Strictly speaking, "the weight of a pound" is the pull of the earth on the platinum mass diminished by the centrifugal force due to the earth's rotation. The weight of a body at the equator is, therefore, about one third of one per cent less than the pull of the earth upon it.

The essential features of the ordinary balance (Fig. 60) are:

1. A rigid beam carrying three knife-edges, one near each end, and a third, as nearly as possible, midway between these two.

2. Two scale pans supported, one on each knife-edge at the end.

3. The beam when carrying the knife-edges and the pans must be in stable equilibrium. This is accomplished by so placing the knife-edges and so distributing the mass in the beam that the c.m. of the whole system lies just *below* the central knife-edge. Why would not neutral equilibrium answer quite as well as stable?

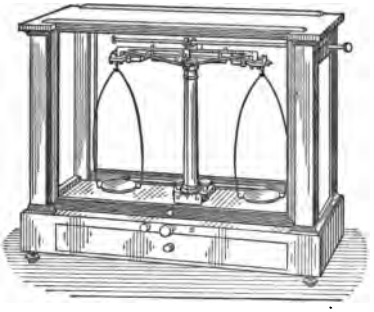


Fig. 61. Chemical balance.

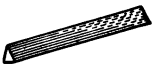


Fig. 62. A knife-edge.

4. The whole system must be sensitive to a small moment of force. This is accomplished by making the distance from the central knife-edge to the c.m. very small.

5. A pointer, or index of some kind, generally fastened to the beam, for the purpose of indicating the position of equilibrium.

Principle of the Balance

71. In physics the balance is employed to compare two masses by means of comparing their weights. This is done as follows: The position of equilibrium is observed before any mass has been added to either pan, that is, when the pans are empty. The body whose mass is required, M_1, having been placed in one pan, standard masses, amounting to M_2, are then placed in the other pan until the

moments of force which their weights exert on the beam are just equal to the moment of force of the body whose mass is required.

Moment of unknown mass = moment of known mass.

$$W_1 l_1 = W_2 l_2$$

Since these two masses tend to rotate the beam in opposite directions with equal moments, the total moment, L, will be zero.

$$L = W_1 l_1 - W_2 l_2 = 0$$

This equality and opposition of moments is indicated by the pointer coming back to the same position of equilibrium which it had when the pans were empty. **Thus, the immediate purpose of the balance is simply to tell us when we have equal moments of force.**

Now if, in addition, the arms of the balance, l_1 and l_2 (that is, the respective distances between central and end knife-edges), are equal, $l_1 = l_2$, we know that the *weights* of the two bodies, in their respective pans, are equal; and, since mass is proportional to weight, the masses in the two pans must be equal, so that **the equality of masses is an inference from the equality of moments**, and this inference would not be allowable if the earth acted as a magnet, attracting different bodies with a force depending upon the kind of material as well as the mass of material in them.

Summary concerning Gravitation

72. Newton's Law of Gravitation is summarized in the equation, $F = k\frac{mm'}{r^2}$.

Gravitational forces depend upon the masses of bodies, not upon the kind of matter of which the bodies are composed.

The earth acts as a particle with its mass concentrated at its center.

The attraction of gravity at any place on the earth's surface depends upon its distance from the center of the earth.

The weight of a body is the earth's attraction for it; hence,

The weight of a body above the earth's surface is inversely proportional to the square of the distance from the center of the earth.

The total weight of a body acts as though concentrated at its center of mass (c.m.).

A body is in equilibrium as long as the *vertical* line from the c.m. falls within the base on which the body rests.

Masses may be compared by the balance.

THIRD GENERAL PROPERTY OF MATTER

III. Matter as a Vehicle of Energy

73. Work is something of which every one has, or ought to have, a fairly definite idea. The farmer knows that it requires twice as much work to plow a two-acre field as to plow a one-acre field. For while the horses exert no more force on the plow in the first case than in the second, the total length of furrow is twice as great in the first case as in the second. In like manner, we all know that it requires twice as much work to carry 2000 bricks to the top of a house as to carry 1000 bricks to the same height. For, while the distance is the same in each case, the weight of 2000 bricks is twice that of 1000 bricks.

In general, the quantity of work done depends upon two factors, and upon two factors only: (1) the force exerted, and (2) the distance through which this force is exerted.

Definition of Work

74. Consequently the term **work** is employed in physics to denote one definite quantity, viz. **the product of a force multiplied by the distance through which it is exerted, both distance and force being measured in the same direction.**

Problem. — A mass, B (Fig. 63), is attracted by the earth with a force of 1000 dynes. What work would be required to lift it vertically 24 cm.? Evidently the answer will be the product, $1000 \times 24 = 24{,}000$ units of work, since both force and work are measured vertically.

What work will be required to roll this mass up an inclined plane whose length, $\overline{OA}$, is 24 cm.? If work were

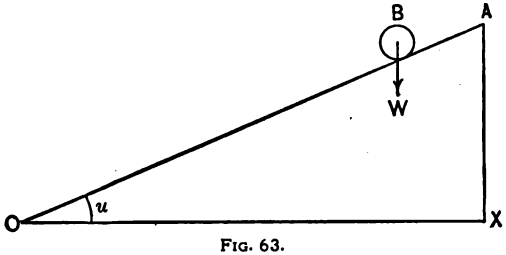


Fig. 63.

defined simply as "the product of force by distance," the answer to the problem would be, as before, $1000 \times 24 = 24{,}000$ units of work. But using the proper definition, we observe that since here the force, *i.e.* the weight of the body, B, is vertical, we must measure the distance also in a vertical direction.

Evidently, therefore, we must know the height $\overline{AX}$ before we can solve the problem at all. In other words, we must know how steep the inclined plane is before we can estimate the work required to carry the body B through the distance $\overline{OA}$.

Suppose the height $\overline{AX}$ to be 12 cm. The work in this particular case will then amount to 1000×12 units of

work. When the mass is moved along a horizontal plane, no work is done against the force of gravity because the component of the weight of the body along the horizontal plane is zero. In like manner when a load is pulled along a horizontal track, no work is done against the weight of the load. The work done is all against the forces of friction.

So, in general, **to obtain the work done we have merely to multiply the force exerted by the distance through which it is exerted,**

or $$\text{work} = \text{force} \times \text{distance}$$

and $$W = Fd$$

where F is the average value of the force throughout the distance d, and where, of course, *d and F are to be measured in the same direction.* This algebraic definition is identical with the one given above in words.

Units of Work

75. Since now the unit of force is the dyne, and the unit of distance the centimeter, the **unit of work is that amount of work which is done when a force of one dyne is exerted through a distance of one centimeter.**

This unit is of such frequent occurrence in physics that it has been given a special name. It is called an **Erg**, which is only the Greek form of our English word "work."

Since the **erg** is very small, a large multiple, **the joule**, is frequently used.

$$1 \text{ joule} = 10{,}000{,}000 \text{ ergs} = 10^7 \text{ ergs}$$

The joule is named in honor of the English physicist, James Prescott Joule (1818–1889).

Where work is measured against gravity, *i.e.* vertically, the units are named by combination of units of force and distance, thus

gram-centimeter — the work done in raising 1 gm. 1 cm.
kilogram-meter — the work done in raising 1 kgm. 1 m.
foot-pound — the work done in raising 1 lb. 1 ft.

PROBLEMS

1. How many ergs of work will be done in exerting a force of 6 dynes through a distance of 18 centimeters?

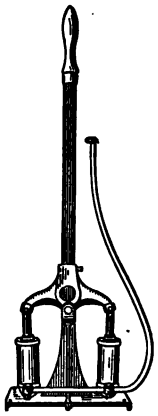


FIG. 64. Double action bicycle pump.

2. What work will be required to hoist 10 kgm. of water against an acceleration of 980 units from a well whose depth is 10 meters?

3. How many foot-pounds of work are required to carry 100 pounds of brick to the top of a building 42 ft. high?

4. If 3500 foot-pounds of work are required to propel a bicycle for a distance of one mile, what is the average force required to propel the wheel?

5. A magnet attracts a piece of iron with a force of 8000 dynes. How much work will be performed by the magnet in moving the iron through a distance of 0.15 mm.?

6. A force of one million dynes is required to operate a "double action" bicycle pump (Fig. 64). If the length of stroke is 12 cm. how much work will be required to make 50 complete strokes of the pump?

7. How much work is done in pumping out a circular well 40 ft. deep and 4 ft. in diameter, when there are 20 feet of water on the bottom? (1 cu. ft. of water weighs 62.5 lb.)

8. The mass of a pile driver ram (Fig. 65) is 1200 kgm. When it is allowed to fall through a distance of 10 m., it drives the pile 40 cm. What is the average force exerted on the pile?

ENERGY

76. We shall see later that a body may be able to do work in consequence of

1. Its position.
2. Its speed (angular or linear).
3. Its temperature.
4. Its electrical condition.
5. Its chemical composition.

Etc.

The **capacity for doing work** is called **energy**; and a system is said to possess energy when it can do work. Since the work which a system can do is the measure of its energy, **the same units are employed for energy as for work,** viz. the erg, joule, gram-centimeter, kilogram-meter, foot-pound.

Forms of Energy. Kinetic. Potential

77. If we were to attempt to study all the forms of energy which a system may possess, we should find ourselves at once in the midst of the entire subject of physics. Accordingly we shall here consider only the energy which a mechanical system may have by virtue of

1. The relative position of its parts; and
2. The relative speed of its parts.

Every one is familiar with many instances of these two forms of energy.

The boy who starts at the top of the hill on a bicycle knows that his energy of position is such that, without pedaling, it will carry him to the bottom of the hill and some distance beyond. If, coming in the opposite direction, he approaches the foot of the hill with sufficient speed, the energy due to his motion may be ample to carry him to the top without any pedaling. In this case the system consists of the bicyclist and the earth.

An uncoiled spring will not run a watch; but if, using a key, we change the relative position of its parts, by coiling it up, it will run the watch for the next twenty-four hours. When one drives a nail, he gives the hammer a high speed. The energy due to this speed does the work of forcing the nail into the wood.

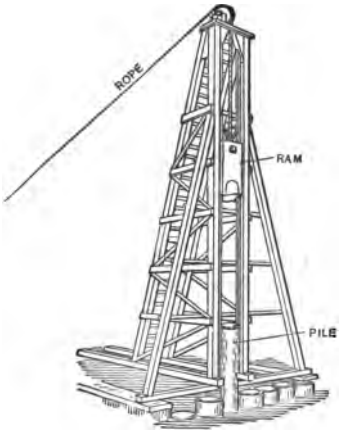


Fig. 65. Pile driver.

In the language of physics, **the energy which a body possesses in virtue of its position is called Potential Energy; while the energy which a body possesses in virtue of its speed is called Kinetic Energy.**

The energy of the iron block used in a pile driver (Fig. 65) is all potential at the top of the stroke, but all kinetic just before striking the pile at the bottom of the stroke.

The question now arises as to how much work a body can do in virtue of its position, and how much work a body can do in virtue of its speed. In other words, what are the proper measures for potential and kinetic energy respectively? Before answering these two questions, we must digress for a moment to consider the

Dissipation of Energy

78. It is well known to every one that a ball, or a bicycle, which is allowed to run down one hill from a certain height, h (Fig. 66), will not, with the speed thus acquired, ascend an adjoining hill to the same height. The

ball, or the wheel, always stops short of its original elevation. The explanation of this is something which does not now concern us. The important fact is that the kinetic energy of the wheel at the bottom of the hill is not quite sufficient to do the work of carrying the wheel to its original height.

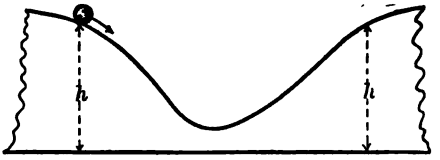


FIG. 66.

The same is observed to be true of a pendulum as it swings from its position of rest at *A* to its position of rest at *C* (Fig. 67). The point *C* is always a little lower than the point *A*, *i.e.* the kinetic energy of the pendulum at *B* is not quite sufficient to do the work of carrying the bob to its original height.

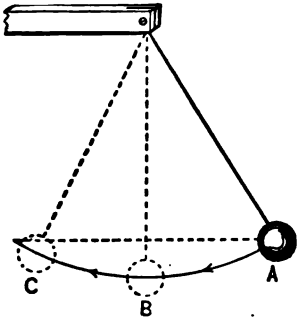


FIG. 67.

In every case of this kind it has been observed that we never get out of a machine all the work which we put into it. Some of the mechanical energy of a system is always wasted when we attempt to use that energy by transforming it, or transferring it.

This unavoidable waste of energy is brought about by friction: a portion of the mechanical energy is transformed into heat energy, where it is of less use to us than before. This tendency of energy to assume a more and more useless form is known as the principle of the Dissipation of Energy.

Conservation of Energy

79. But the most important discovery along this line, perhaps the most important discovery of the nineteenth century, is the following, viz. that **when allowance has been made for unavoidable wastes, the sum of the kinetic and potential energies of the body or system of bodies never changes, unless through some external influence.**

"Waste of energy" is a very common expression; but, as we shall learn by and by, this never means destruction of energy. There is no evidence that the slightest bit of energy has ever been annihilated. Energy is, in this respect, like matter. No human being has ever succeeded in creating or destroying any amount of either one. When, therefore, we speak of allowing for "unavoidable wastes," we refer to that energy which has been unavoidably changed into heat or some other form of energy which, for our purpose, is useless.

The law of the **Conservation of Energy** expresses the fact that **the sum total of the energy in the universe remains the same.** This energy can change from one form to another, it tends constantly to become less and less available; but, so far as is known, it has never changed in quantity.

Whenever energy makes its appearance in any particular form, such as chemical or electrical, it always does so at the expense of an equivalent amount of energy of some other form.

Digression on Perpetual Motion

80. A perpetual motion machine means either one of two things: (1) that a machine can continue to run forever, once started, without the addition of energy; (2) that a machine can give out more energy than is put into it. Every machine dissipates energy in the form of heat due to the friction of its parts. Hence a **perpetuum mobile**

demands a creation of energy. The law of the conservation of energy—than which no law is better established—means that **energy cannot be created.**

The conclusion is obvious.*

Measure of Kinetic Energy of Translation

81. By the application of the principle of the *Conservation of Energy* to many special cases, the proper measure of the kinetic energy of a body in translation has been found as follows:

If a force F acts upon a mass m initially at rest, for a time t, it will produce in it a speed of translation v, such that (Art. 30)

$$F = ma = m\frac{v}{t}$$

If the force F remain constant, then the acceleration will remain constant, and the mass m will travel, during this time, a distance d such that (Art. 15) $d = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{v}{t}\right)t^2 = \frac{1}{2}vt.$

But the work done by a constant force F exerted through a distance d is Fd, and $W = Fd = m \cdot \frac{v}{t} \cdot \frac{1}{2}vt = \frac{1}{2}mv^2.$

Now this work has all gone toward putting the mass m in motion, since, by supposition, there has been no waste from friction or other causes. We see, therefore, that the **expression, half the product of the mass by the square of the speed, is the proper measure of the work put into the body.** Relying now upon the Conservation of Energy, it is seen that this product also measures the amount of work which we shall be able to get out of the body, *i.e.* measures

* In the "Encyclopædia Britannica" and also in Phin's "Seven Follies of Science" (Van Nostrand, 1906), the student will find an amusing and instructive chapter on this piece of human folly.

its kinetic energy of translation. Let us denote this energy by $K.E.$, then

$$\text{Work} = \text{Kinetic Energy} = \tfrac{1}{2}mv^2$$

or

$$K.E. = \tfrac{1}{2}mv^2$$

Measure of Potential Energy

82. We turn next to a system whose parts have a definite relative position, and ask how much work can be gotten out of it. For instance, a coiled watch spring, a stretched rubber band, a pond of water at a definite height

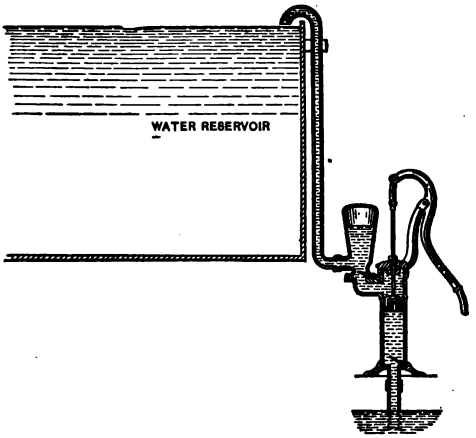


Fig. 68. Potential energy of the water = work done upon it.

above a water wheel, a stone raised to a certain height above the ground. How shall we measure the potential energy of each of these systems?

Experiment has answered this question in a clear and simple manner. *The amount of work done in bringing the system into its present position is, barring frictional losses, exactly equal to the potential energy now in the system.*

The weight of the water in the reservoir, F, multiplied by the average height, d, through which it was pumped, is

at once the work required, Fd, to give the water its present potential energy and is also the measure of its present potential energy. It must not be forgotten that here allowance is to be made for the energy wasted in friction in the pump. The student who pursues the subject farther will find that in like manner the energy of the coiled watch spring is simply the average moment of force which one exerts upon the key, multiplied by the arc through which he turns the key.

Let us call the potential energy of a system $P.E.$ Then, if this energy is transferred to the system by an average force, F, exerted through a distance, d,

$$P.E. = Fd$$

This equation is a simple application of the law of the Conservation of Energy.

Summary of the Principal Discoveries concerning Energy

83. Energy is known only in connection with matter. We know of no matter devoid of energy, and know of no form of energy which is not associated with matter.

If no energy be given to or taken away from a system, the sum of its kinetic and potential energies remains constant.

When a system at rest is left to itself, its potential energy tends to assume the form of kinetic energy. This great law of nature may be described in another way by saying that **there is a universal tendency for the potential energy of a system at rest to decrease.**

Kinetic energy is measured by half the product of the mass by the square of the velocity, $K.E. = \frac{1}{2} mv^2$.

Potential energy, like work, is measured by the product of force by distance, $P.E. = Fd$.

APPLICATIONS OF THE PRINCIPLES OF ENERGY TO MECHANICS

I. The Lever

84. The lever is a machine which, in various forms, is familiar to every lad (Fig. 69). It has a variety of uses. Sometimes it is employed to increase the force which one is able to apply at some particular point, as in raising a heavy stone or cracking a nut. Sometimes it is employed to multiply the motion of a point, as in the index

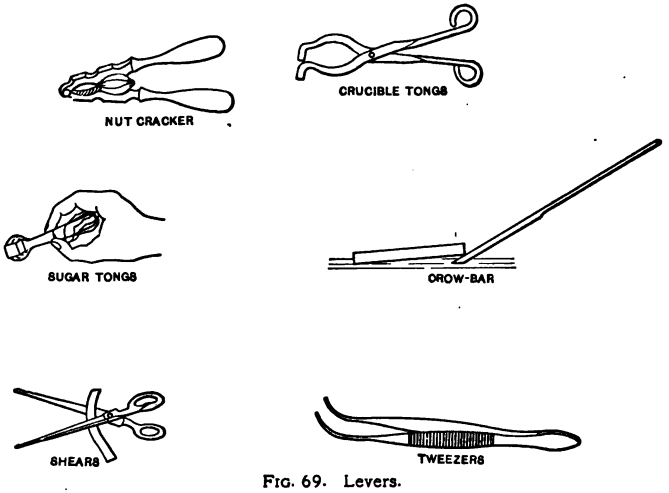


Fig. 69. Levers.

on the dial of a steam gauge; sometimes, as in sugar tongs, pliers, and tweezers, to reach otherwise inaccessible places.

The principle of the lever was first correctly enunciated by the Greek scholar Archimedes (287–212 B.C.).

The essential feature of all levers is a more or less rigid bar rotating about the edge of some body as an axis. This edge is called the **fulcrum** of the lever.

Since hardly any energy is here wasted in friction, one obtains by use of a lever practically as much work as he puts into it.

Mechanical Advantage of the Lever

85. *AB* (Figs. 70 and 71) is a lever pivoted over a fulcrum. The forces, F_1 and F_2, represent the **effort** applied and the **resistance** (resisting force), respectively, r_1 and r_2 are the lever arms, or the perpendicular distances from the fulcrum to the line along which the force acts.

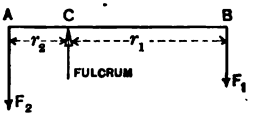


FIG. 70. Lever arms *AC* and *BC*.

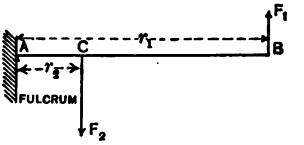


FIG. 71. Lever arms *BA* and *CA*.

The effect of each force would be to produce rotation about the fulcrum. Hence, the principle of moments may be applied to the lever, and, since the lever may be regarded as at rest, the moments F_1 and F_2 about the fulcrum are equal, or

$$F_1 r_1 = F_2 r_2$$

which may be written in the form

$$\frac{F_2}{F_1} = \frac{r_1}{r_2}$$

The physical meaning of this expression is that **the force (resistance) which one can obtain by use of a lever is to the force which he applies to the lever (effort) inversely as the length of the arms.**

The ratio, $\frac{\text{resistance}}{\text{effort}}$, is sometimes called the mechanical advantage of a machine.

Great care must always be observed in estimating the length of the arms of a lever. The fulcrum is the axis of rotation. The perpendicular distance from the axis to

the line of direction of the applied force is one arm, r_1; the perpendicular distance from the axis to the line of direction of the opposing force is the other arm, r_2. These definitions are illustrated in each of Figs. 70 and 71.

II. The Wheel and Axle

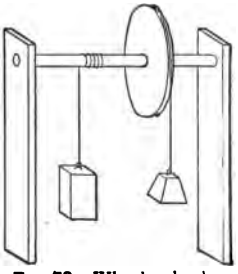


Fig. 72. Wheel and axle.

86. An inspection of Figs. 72, 73 shows at once that the principle just derived for the lever applies also to the wheel and axle and may be thus stated:

The force (resistance) which one can obtain by use of the wheel and axle is to the force which he applies (effort) inversely as the radii of the wheel and of the axle. Since radii (r_1, r_2), diameters (d_1, d_2), and circumferences ($2\pi r_1$, $2\pi r_2$) are in the same ratio, **diameters or circumferences may be used instead of radii.**

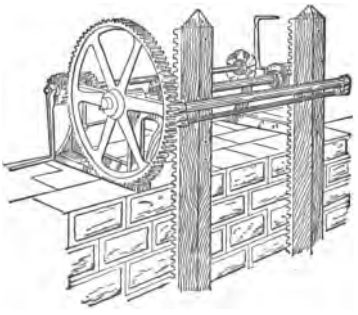
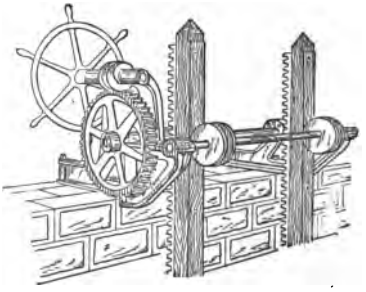


Fig. 73. Applications of the wheel and axle.



Dayton head gate hoists.

III. The Pulley

87. The pulley is simply a lever used for a particular purpose, viz. to change the direction in which a force is applied and to gain mechanical advantage.

Fixed Pulley

The bar constituting the lever is the diameter of the pulley $\overline{AB}$. (See Fig. 74.) O is the axle (axis) about which the bar is continually rotating. And hence O is the fulcrum.

Since here the arms $\overline{AO}$ and $\overline{BO}$ of the lever are equal, the forces are also equal and $F_1 = F_2$. So that **the only**

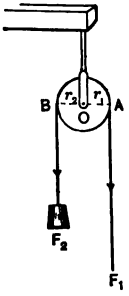


FIG. 74. The pulley a special case of the lever.

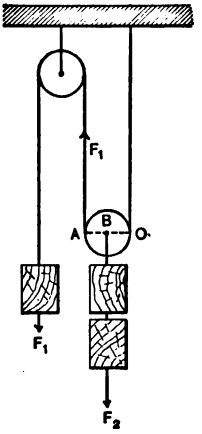


FIG. 75. The load is divided between the cords.

advantage offered by the fixed pulley is a change of direction; a pull downward can be transformed into a pull upward.

Movable Pulley

When a pulley is arranged as in Fig. 75, it is evident that, when F_1 supports F_2, the load F_2, is divided between the two parts of the cords that support it. Now it is found by experiment that, when F_2 is lifted through any distance, d_2, F_1 acts through twice that distance, $2d_2$. Since, by the law of Conservation of Energy, the work

done by the effort, F_1, is equal to the work done against the resistance, F_2,

$$F_2 \times d_2 = F_1 \times 2\, d_2$$

or

$$\frac{F_2}{F_1} = 2$$

Block and Tackle

In the case of a system of pulleys (Fig. 76), or of block and tackle (Fig. 77), the force applied, F_1, is again

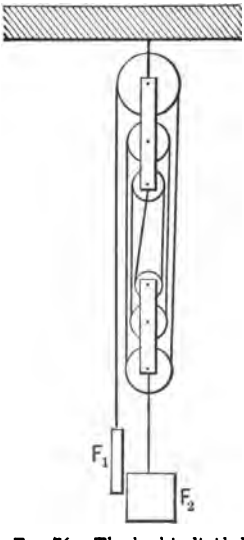


FIG. 76. The load is divided among six cords.

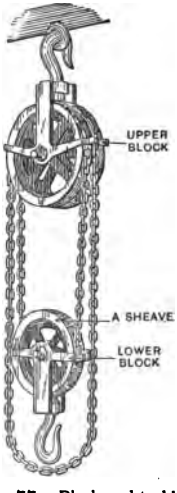


FIG. 77. Block and tackle. Differential pulley.

exerted in each part of the cord supporting F_2, and the load F_2 is shared by all the cords supporting it.

In this case (Fig. 76),

$$\frac{F_2}{F_1} = 6$$

In general, the ratio $\frac{F_2}{F_1}$ will always be equal to the

number of cords supporting F_2, or

$$\frac{F_2}{F_1} = n$$

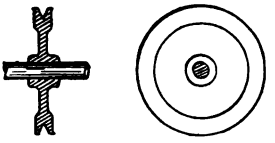


FIG. 78. A sheave.

Hence, the mechanical advantage of a system of pulleys, provided no energy is dissipated inside the pulleys, is equal to the number of parts of the cord supporting the load.

IV. THE INCLINED PLANE

88. The inclined plane is a device frequently employed to avoid the direct lifting of heavy weights. Barrels are frequently loaded into wagons by use of *skids* (Fig. 41), which are simply inclined planes. Goods are often taken aboard steamers by inclined gangways. What advantage does such a device offer?

Figure 79 represents a body resting on an inclined plane, OA. Let F_2 be the weight of the body and F_1 the force parallel to the plane required to push along the body. Let L be the length of the slope and H its vertical height.

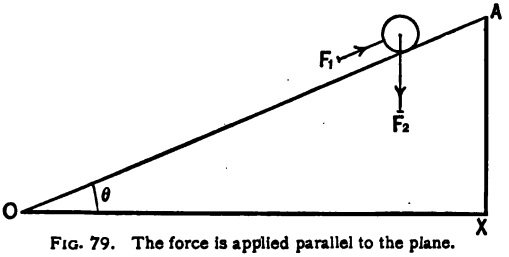


FIG. 79. The force is applied parallel to the plane.

Then the work spent upon the mass by the force F_1 is F_1L (work = force × distance) and the potential energy of the mass at the top of the plane is F_2H. But, by the law of the Conservation of Energy, these two are equal, provided friction is negligible, and

$$F_1L = F_2H, \text{ or } \frac{F_2}{F_1} = \frac{L}{H}$$

Then, **for the inclined plane the mechanical advantage, where the force is applied parallel to the plane, is the ratio of the length to the height of the plane.**

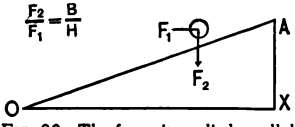


FIG. 80. The force is applied parallel to the base of the plane.

Where the force is applied parallel to the base of the plane, the mechanical advantage is the ratio of the base to the height of the plane.

The demonstration is left to the student.

V. THE SCREW

89. The screw (Fig. 81) is essentially an inclined plane wrapped as a ridge around a cylinder and the whole rotated by means of a lever.

The ridge is called the *thread* of the screw. The distance between consecutive threads measured along the

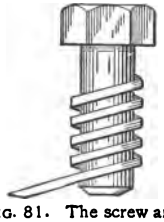


FIG. 81. The screw an inclined plane.

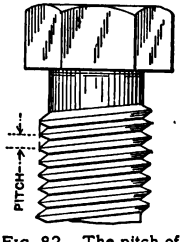


FIG. 82. The pitch of a screw.

axis is called the *pitch* of the screw. As the lever makes one complete revolution (F_1 acts through a distance $2\pi r$), the screw travels a distance equal to its pitch (F_2 acts through the distance p). By the principle of the Conservation of Energy (in an ideal frictionless screw),

$$F_1 \times 2\pi r = F_2 \times \text{pitch of the screw}$$

or

$$\frac{F_2}{F_1} = \frac{2\pi r}{\text{pitch of the screw}}$$

This ratio is the mechanical advantage for the screw.

PROBLEMS

1. A meter stick is suspended at a point 40 cm. from one end. A kilogram placed at the end of the long arm will be balanced by what weight placed at the end of the short arm?

2. A man weighing 150 lb. is lowered into a well by means of a windlass, the arm of which is 30 in. long and the axle of which is 8 in. in diameter. Find the force required to let him down with uniform velocity.

3. A screw, the pitch of which is 5 mm., is turned by means of a lever which is 120 cm. long. What force will be required to raise 2000 kgm.?

4. A weight of 300 lb. is raised 3 ft. by means of a block and tackle, the lower block of which has three sheaves. Determine the force required and the space through which it has acted.

5. The pedal crank of a bicycle is 6½ in. long. The rider exerts a force of 100 lb. on the pedal. Find the force exerted on the chain when a 6-in. sprocket is used.

6. A driving wheel has 64 teeth, and the driven wheel 33 teeth. How many revolutions per second will the latter make while the former makes 10 revolutions per second?

7. A lever is 14 cm. long. Where must the fulcrum be placed in order that a weight of 8 gm. at one end may just balance a weight of 16 gm. at the other end?

8. A toboggan slide has a vertical height of 10 m. At the bottom is a long level stretch. Three men starting at the bottom climb to the top of the slide, carrying the toboggan on which they later descend. The three men with the toboggan weigh 200 kgm. The slide itself is assumed to be frictionless, while the level stretch is not. Describe the transformation of energy, (*a*) in the ascent, (*b*) in the descent, (*c*) in the run along the level stretch. Compute the amount of energy involved in each transformation. (College Entrance Examination Board.)

Efficiency of Machines

90. In describing the simple machines the principle of Conservation of Energy has been used always with the proviso that they be frictionless. This supposition is justifiable only in the case of very simple levers, and some forms of wheel and axle and inclined plane. The pulley and screw are valuable in many cases simply because their friction is so great.

The **general law of machines** may be thus stated: **the work done by the force applied** (force × distance, F_1d_1) = **the work done by the machine** (force × distance, F_2d_2) + **work done against friction.** Evidently, F_2d_2 must always be less than F_1d_1 by the amount of the work done against friction.

The net work thus done by the machine, F_2d_2, is the *available* or *useful* work.

The ratio, $\frac{\text{useful work done by the machine}}{\text{work done by the force applied}}$, is the efficiency of the machine.

The table below shows the efficiency of some machines under certain definite conditions which it is unnecessary to describe here.

Efficiency of Some Machines

Lever	95–96 %	Pneumatic Hoist	45–95 %
Windlass	60–70 %	Mine Hoist	60–85 %
Block and Tackle	42–92 %	Centrifugal Pump	46–73 %
Worm and Wheel	50 % or less	Reciprocating Pump	30–85 %
Rope over Sheave (elevator)	80–94 %	Water Turbine	80–85 %
Differential Pulley Block		Steam Engine	90–95 %
hoisting	36–69 %	Dynamo	90–98 %
lowering	67–58 %	Motor	80–98 %
Screw Gearing	22–46 %	Transformer	95–98 %

POWER

91. One important feature of any mechanical system is the rate at which it can do work. Two steam engines may be similar in design and yet so different in dimensions that one can do in any given time twice as much work as the other, both using steam from the same boiler.

In everyday life, **power** is a word which is employed with considerable variety of meaning; but in physics it is always used to denote **the rate of doing work.** Hence, it may be thus expressed,

$$\text{power} = \frac{\text{work done}}{\text{time occupied in doing it}}$$

The fundamental idea involved in power is rapidity of working. The reason it is more fatiguing to ride a wheel uphill than to ride it along a level stretch at the same rate is that, in the former case, the rate of working is higher. The main difference between a 5 and 10 horse power motor is not that the one has done or is doing twice as much work as the other, but that it *can* do work twice as rapidly.

Units of Power

92. Three units of power are in common use:

(1) The **erg per second,** which has no special name and is inconveniently small;

(2) The **joule per second,** which, in honor of James Watt (1736–1819), the inventor of the steam engine, is called a *watt;* and

(3) The **horse power,** introduced by James Watt; defined as **33,000 foot-pounds per minute,** and generally indicated by the symbol HP.

The relation between the last two units is

$$1 \text{ HP} = 746 \text{ watts}$$

Engines and boilers are commonly rated in horse power.

Dynamos are commonly rated in kilowatts.

A kilowatt is 1000 watts and is frequently called an *electrical horse power.*

PROBLEMS

1. What must be the horse power of a pump that can lift 500 cu. ft. of water 100 ft. high in 1 min.? 1 cu. ft. of water weighs 62.5 lb.

2. What must be the horse power of an engine that lifts the hammer of a pile driver weighing 4000 lb. 40 ft. high in 3 sec.?

3. An elevator carriage and unbalanced load is 2000 lb. What horse power engine would be required to raise it 140 ft. in 50 sec.?

4. A mine hoist handles 224,000 lb. of coal in 8 hr., lifting the load 250 ft. What is the lowest power engine possible?

5. At the handle of a windlass a man may exert a force of 15 lb., moving it through 220 ft. per minute. What is the horse power of a man?

6. James Watt estimated that the average cart horse of London could travel $2\frac{1}{2}$ mi. per hour, at the same time raising, by a rope led over a pulley, a weight of 150 lb. On this basis what is the value of 1 horse power?

FOURTH GENERAL PROPERTY OF MATTER

IV. Elasticity

93. Few words in our scientific vocabulary are more confusing than the word "elasticity." It is popularly supposed that a rubber band is a very elastic substance, while steel and glass are not. Few give a thought to the elasticity of water or air. In only one particular does the popular notion of elasticity agree with the truth; namely, that a body whose size or shape has been altered by the application of force may of itself return to its previous condition when the force has been removed. Rubber is considered very elastic because it returns to its original size and shape even after a very large change.

Three States of Matter

94. Water, as we all know, exists in three conditions — **solid**, **liquid**, and **gas** — ice, water, and steam — and we also know that ice may be turned into water and then into steam, or the reverse.

(i) **Solids are those substances that have a shape of their own and need no support or containing vessel to maintain that shape.**

(ii) **Liquids are those substances that always take the shape of the containing vessel, but remain in a collected mass at the bottom.**

(iii) **Gases are those substances that always expand so as to fill any vessel in which they are placed.**

(iv) **The term "fluid" includes both gas and liquid.**

The student will note that these definitions express in a simple manner our everyday experience with any solid, liquid, or gas.

Definition of Elasticity

95. When a bicycle tire is pumped up, we all know that means putting more air into smaller volume — the gas is compressed. If the valve is opened, the air escapes, occupying the same volume as at first. When water is forced by a pump, it also is very slightly compressed, but recovers its original volume when the pressure is removed. If, however, iron is put between the jaws of a vise, not only the volume but also the shape of the piece changes; still, unless pressed too strongly, it also recovers its original size and shape on being removed from the vise. Careful examination shows that there are only two kinds of changes that may occur inside the body under the action of force.

(i). Change of volume; (ii) change of shape.

These are called **strains.**

Every body whose volume is strained, provided the strain is not too great, will recover its original volume. This is a property possessed by all bodies and is called **elasticity.**

Gases and liquids are perfectly elastic, but possess only elasticity of volume. Solids possess elasticity of both volume and shape.

Hooke's Law

96. Between the change in length of any solid and the force required to produce the change, there is an interesting relation which may be illustrated as follows:

A steel wire AB (Fig. 83) is securely suspended from any support. Weights are attached as shown. At B a light pointer is clamped firmly to the wire and pivoted over the fulcrum F, so that the long arm C is just oppo-

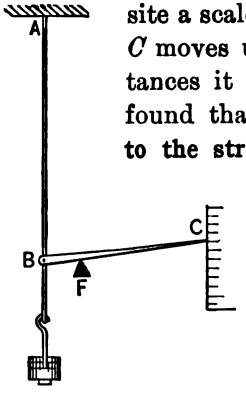


FIG. 83. Elongation is proportional to the stretching force.

site a scale. As weights are added, the pointer C moves up the scale. By comparing the distances it moves with the weights added, it is found that **the stretch produced is proportional to the stretching force.** This observation was first published in 1676 by Robert Hooke and is accordingly known as **Hooke's Law.**

Limit of Elasticity

97. In the experiment which we have just employed to demonstrate Hooke's Law, if we continue to add heavier and heavier weights, there comes a point beyond which the stretch produced is no longer proportional to the stretching force and, when the weights are removed, the wire (if it has not broken) remains permanently lengthened. The **limit of elasticity** has been exceeded; hence, we say that **the limit of elasticity of any substance is the point beyond which its elongation, or bending, or change of volume becomes permanent.**

The strength of a rope means the pull (force) it can withstand without breaking. The strength of a wire, rod, or beam is the force it can withstand without exceeding its limit of elasticity.

Gases and liquids have no limit of elasticity as described above for solids, but always return to their original volume when the pressure is removed; hence, they are said to be *perfectly elastic.*

The Coefficient of Elasticity

98. As we have seen, the application of force to a body may change its size or its shape, and any such change is called a **strain.** We have also seen that, up to the elastic limit, a force is brought into play which will restore the

original size or shape. The amount of such a restoring force per unit of area is called a **stress. The elasticity of a substance is measured by the ratio** $\frac{\text{stress}}{\text{strain}}$.

We can now see why the popular idea concerning the elasticity of a rubber band is erroneous. By a simple pull between the fingers the length of the band may be more than doubled without breaking. In other words, the effect produced — strain — is large in comparison with the restoring force — stress. Hence the elasticity, $\frac{\text{stress}}{\text{strain}}$, is small. In the case of a metallic wire of the same cross section as the rubber, the stretch is scarcely visible and, even when carefully measured, the strain is very small in comparison with the stress. Hence the ratio, $\frac{\text{stress}}{\text{strain}}$, is large. In other words, iron, steel, brass, even copper, are much more elastic than rubber.

Other Coefficients of Elasticity

99. In the preceding section the **longitudinal coefficient** of elasticity, sometimes called Young's Modulus, is described in a very elementary way. In this book no attempt will be made to explain the **coefficient of elasticity of volume** or the **coefficient of elasticity of shape.** These are called, respectively, **bulk modulus** and **rigidity modulus.**

References

Cajori — *History of Physics* (Macmillan). Contains excellent chapters on the discoveries of Galileo and Newton.

Duff and others — *A Text-book of Physics* (Blakiston). An excellent reference book on general physics.

Tait — *Properties of Matter* (A. & C. Black, London). Though rather difficult for the average student, this book should be in every school library.

Cunynghame — *Time and Clocks* (E. P. Dutton & Co., New York). Gives very interesting accounts of various forms of clocks — Galileo's, water clock, grandfather's clock, etc.

CHAPTER III

SOME SPECIAL PROPERTIES OF MATTER

100. In the preceding pages we have considered four properties of matter which are perfectly general. It is an experimental fact that matter possessing any one of these possesses all the others. This by no means exhausts the list of general properties, but it probably includes those which are most important in explaining everyday experiences.

PROPERTIES OF LIQUIDS

101. We shall first consider some of the special properties of liquids, and from the three following points of view:

I. Behavior of Liquids at Rest and under Gravity; Hydrostatics.

II. Behavior of Liquids at Rest and freed from Gravity; Surface Tension.

III. The Behavior of Liquids in Motion will be considered briefly in the following chapter on Wave Motion.

I. Hydrostatics

102. The water in a teapot (Fig. 84) stands at the same height in the spout as in the pot. The danger of a leak in the hull of a boat arises from the fact that the boat will continue to "fill" until the water stands at the same level inside and out. Mercury in a U-tube stands at the same level in each arm, provided the diameter of

the tube is uniform or large. If, in any of these cases, the level is disturbed for an instant, it is quickly restored when the force producing the disturbance is removed.

In other words, **water, or any other liquid, seeks its level.**

FIG. 84. Water seeks its level.

DEFINITION OF PRESSURE

103. In physics the term **pressure is always employed to denote the force applied to unit area of that surface,** or

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

For example, a steam pressure of 150 pounds means 150 pounds per square inch, a gas pressure of 4 ounces means 4 ounces per square inch, while normal atmospheric pressure means about 1033 grams weight per square centimeter.

PRESSURE IN A LIQUID

104. Sometimes it is difficult to tell at a glance whether a body is solid or liquid. Paving tar frequently keeps the shape of the barrel several days after the barrel has been chopped away. Before long, however, it will begin to flow and spread out into a mass with a level surface. Such bodies are called "viscous." Oil and molasses on a cold day show the same property of viscosity, and all liquids possess it to some degree. But, regardless of the viscosity of the liquid, the final result is always the same: **The surface of the liquid at rest is horizontal.** If we think of any layer of the liquid at a particular level, as *AB* (Fig. 85), the liquid particles are apparently at rest; or, in other words, there is no force driving the particles from one point to another in this layer of liquid. This fact

may be thus stated: **The pressure in a liquid at rest is the same at all points in a horizontal plane.**

Because a liquid *does* come to rest, one more fact may be inferred: **Pressure in a liquid is normal (at right angles)**

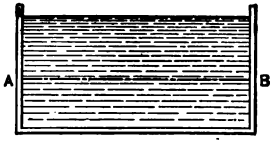


FIG. 85. The free surface of a liquid is horizontal.

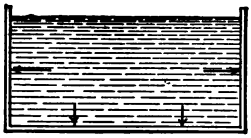


FIG. 86. Pressure is normal to the sides and base.

to the surface pressed upon. For if the pressure were not normal, there would be a component of force producing motion of the liquid in some direction. Absence of the motion means absence of such a component.

How Liquids Transmit Pressure. Pascal's Law

105. Any vessel (Fig. 87) in which a number of holes have been bored is fitted with a plunger and filled with water. When the plunger is thrust in the water pours out of all the holes with equal force. Evidently force exerted on one part of a liquid is transmitted to all other parts. The particles of liquid seem to press upon one another like shot in a box (Fig. 88), each transmitting pressure to its neighbors. Thus the pressure applied to any particle is transmitted to all, producing the pressure which exists on bottom and sides of the containing vessel.

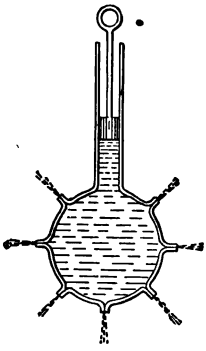


FIG. 87. Force is transmitted equally in all directions.

In other words, **Liquids transmit pressures equally in all directions.**

The student should note carefully that *pressure is force*

per unit area, hence this statement means that **the force on any surface is proportional to its area.**

Thus a 1 kilogram weight pressing on 10 square centimeters in one arm of an apparatus, as shown in Fig. 89,

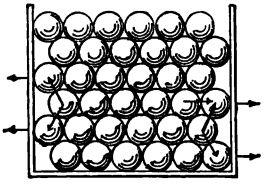


FIG. 88. Each particle transmits force to its neighbors.

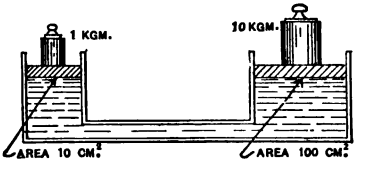


FIG. 89. Force is proportional to area forced upon.

will counterbalance a 10 kilogram weight pressing on an area of 100 square centimeters in the other arm.

These theorems state **Pascal's Law: Liquids transmit pressure equally in all directions, and the forces transmitted are normal to the surfaces pressed upon and proportional to their areas.**

ILLUSTRATIONS OF PASCAL'S LAW. HYDRAULIC PRESS

106. In a city waterworks system (Fig. 90) pressure is frequently maintained by pumps delivering water directly into the mains. If we consider the pressures only when water is not flowing from the faucet, the situation is described by Pascal's Law. The pump piston (P) is exerting a pressure of say 80 lb. per square inch over an area 1000 sq. in. It will be noted that the size of the piston makes no difference in the *pressure*, but

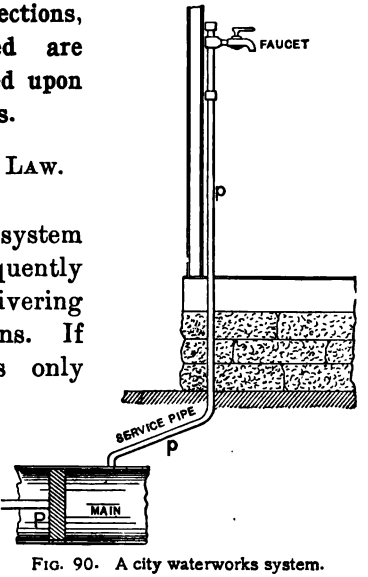


FIG. 90. A city waterworks system.

does make a difference in the *volume* of water delivered. In the small pipe (p) connecting directly with the mains, the pressure is the same as in the mains.

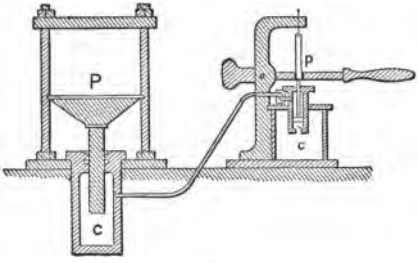


FIG. 91. The hydraulic press.

The reverse of the waterworks system is applied in the hydraulic press (Fig. 91) for the purpose of obtaining sufficient force to make cold steel stampings (Fig. 92), to set heavy fly wheels on their shafts, to bale cotton, to compress steel billets into armor plate, and so on.

The forces on piston (P) and on plunger (p) are evidently in the same ratio as their areas.

$$\text{Hence,} \quad \frac{\text{force on piston}}{\text{force on plunger}} = \frac{\text{area of piston}}{\text{area of plunger}}.$$

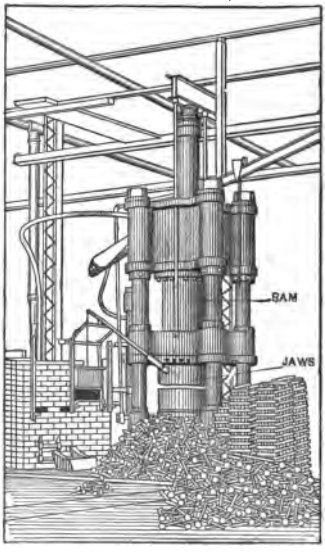


FIG. 92. This press gives a force of 1000 tons weight.

Since the displacement in P (Fig. 91) is produced by transferring water from chamber (c) to chamber (C), if the areas of P and p are in the ratio of 10 to 1, the plunger p would travel 10 in. to make the piston P travel 1 in. In general, then, **the displacements of the pistons are inversely proportional to their areas.**

$$\frac{\text{displacement of } P}{\text{displacement of } p} = \frac{\text{area of } p}{\text{area of } P}.$$

This is equivalent to saying that

the work done by the plunger = the work done on the piston, a conclusion in accordance with the law of the Conservation of Energy.

Pressure due to the Weight of the Liquid

107. It has been shown that the surface of a liquid is horizontal, that the pressure is the same at any point in the same horizontal plane, and that liquids transmit pressures equally in all directions. An inference from these facts is the theorem:

The pressure at any point in a liquid at rest is proportional to the depth.

For, if a horizontal layer, not the surface, is considered, it has to support the weight of all the layers above it. If the liquid is in a vessel with horizontal base and *vertical* sides, the total weight of the liquid is exerted upon the base. At the surface of the liquid the pressure upon the sides of the vessel is zero, and at the bottom of the vessel the pressure is the same as that on the base. At any other point it is proportional to the depth. The *average pressure* on the vertical walls will then be the pressure halfway down. Why? This suggests why dams are built thin at the water surface and very thick at the base, and why leaks are more likely near the bottom of a wooden flume than near the top.

Pressure Independent of the Shape of the Vessel

108. The force which any given liquid exerts on the bottom of the containing vessel depends on the area of the bottom and the depth of the liquid. Experiment shows that the shape of the vessel has no effect upon the pressure as long

as the depth remains unchanged. The theorems already established give the same conclusion. If we consider one

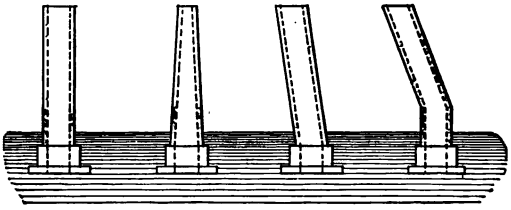


FIG. 93. Force on equal area (10 sq. cm.) is independent of the slope of the vessel.

square centimeter of the base and the column of liquid directly over it, the pressure will be that due to the weight of a column of liquid 1 square centimeter in cross section and say 25 centimeters high. Since the pressure at the

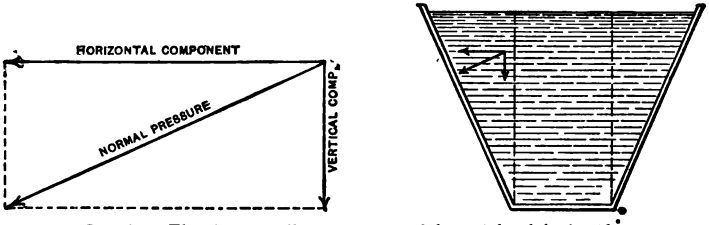


FIG. 94. The sloping walls support part of the weight of the liquid.

same level is everywhere the same, the force on this area is force on unit area × area, and no account has to be taken of the shape of the vessel.

An inspection of Fig. 94 shows how the sloping walls support part of the liquid. The same principle of resolution of forces can easily be applied to vessels of other shapes (Fig. 93).

PRESSURE DEPENDS ON THE DENSITY OF THE LIQUID

109. At a depth of 25 centimeters in water, the pressure will be the weight of a column of water 25 centimeters high and 1 square centimeter in cross section, or 25 grams

weight per square centimeter. At the same depth in mercury, the pressure is the weight of a column of mercury of the same dimensions, but, since 1 cubic centimeter of mercury weighs 13.6 grams, this weight is $25 \times 13.6 =$ 340 grams weight. In every case, the pressure at any point must be computed by multiplying the volume of a column of liquid unit area in cross section by the density. In general, **pressure at any point in a liquid at rest is proportional to the density of the liquid.**

As was stated on page 27, the density of water is 1 gram per cubic centimeter and 62.5 pounds per cubic foot.

PROBLEMS

1. Write a summary of the laws of hydrostatics.

2. Describe how to calculate the pressure on the bottom of a vessel.

3. Describe how to calculate the pressure on the walls of a vessel.

4. How is the force due to a liquid on a given area calculated?

5. A cubical tank with a base 10 cm. on a side is filled with water. What is the weight of the water? What is the pressure on the bottom of the tank? What is the force exerted by the water on the bottom?

6. A conical tank 10 cm. high with a base 100 sq. cm. in area is filled with water. Is the weight of the water greater or less than in problem 5? What is the pressure on the base? Why? What is the force on the base?

7. What effect on the pressure to extend a tube of any cross section filled with water to a height of 200 cm. from the top of the tank in problem 5?

8. The pistons of a hydraulic press are respectively 100 and $\frac{1}{2}$ sq. in. in area. What pressure will be transmitted to the larger piston when 10 lb. weight is placed on the smaller? What force will the larger piston then exert?

9. What is the pressure in pounds per square inch on the bottom of a boat drawing 20 ft. of water? (1 cu. ft. of water weighs 62.5 lb.)

10. The floor of a lead chamber 60 by 150 ft. is covered 6 in. deep with sulphuric acid (density 1.42). What is the total weight of acid on the floor? What is the pressure on the floor?

11. The penstocks of the Niagara Falls Construction Co. are 180 ft. high and 3 ft. in diameter. What is the pressure on the turbines?

12. A canal lock is 40 ft. wide and the water stands 20 ft. deep against the gates. What is the pressure in pounds per square foot on the bottom of the lock? What is the average pressure against the gates? What is the total force exerted by the water upon the gates?

13. A cubic foot of sea water at the surface weighs about 64.5 lb., at a depth of two miles about 65 lb. Is water incompressible?

14. What is the pressure in tons weight per square inch at the depth of 2 mi. in sea water?

15. An hydraulic press gives a force of 300 tons weight on the larger piston 6 in. in diameter. The smaller piston is $\frac{1}{4}$ in. in diameter. What is the force upon it?

16. What is the downward force due to the water on the bottom of a conical vessel filled 10 cm. deep, the area of the base being 800 sq. cm.?

17. The water against a milldam 150 ft. long is 20 ft. deep. What is the horizontal force of the water against the dam?

18. Does the width of the pond make any difference in the *pressure* against the dam?

The Principle of Archimedes. Buoyancy

110. The modern marine engineer, in designing a boat, computes the weight of all the material which enters into its construction. Often the iron used in the hull is actually weighed. Knowing the dimensions of the boat, the designer is then able to say, generally within less than an inch, and before the boat is launched, just how deep she will lie in the water. The simple manner in which this prediction is made will be evident from the following:

Whether a body floats or sinks when placed in a liquid, the body apparently loses weight. But just *how much* weight a body loses under these circumstances was first determined by Archimedes (287–212 B.C.), the founder of our present system of hydrostatics.

It is evident that since the pressure increases with the depth, the bottom of an immersed body will be subject to a

pressure upward which is greater than the pressure downward to which the top of the body is subjected (Fig. 95). **The pressure of the water on a body will,** therefore, **always exert a buoyant force.**

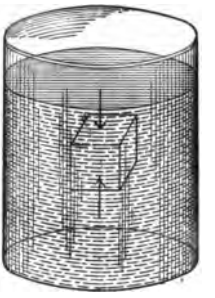


FIG. 95. A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid displaced.

Archimedes saw very clearly, by a simple deduction from experiment, that if from any vessel of liquid (Fig. 96) we imagine a portion of the liquid to be removed, the position of the remainder being unchanged, the upward force exerted by the pressure on the surface so exposed would just be able to support the weight of the liquid removed. Not only so, but it is evident that this upward force would act through the center of mass of the displaced liquid, since, when the liquid is replaced, there is no moment of force tending to rotate it.

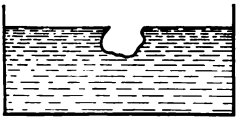


FIG. 96.

By such reasoning he established the following theorem:

A body immersed in a liquid is buoyed up by a force equal to the weight of the liquid it displaces.

SPECIFIC GRAVITY

111. *Definition.* **The specific gravity of a body is the ratio between the weight of the body and the weight of an equal volume of some substance taken as a standard.** The term "specific" means "characteristic," and "specific gravity" means "characteristic heaviness." For instance, the appearance of tin and platinum is quite similar, each having a dull silvery color. The specific gravities, however, are quite different, for tin 7.3, for platinum 21.4.

Therefore, tin and platinum may be distinguished by their "characteristic gravities."

The standard substance for comparing the specific gravities of solids and liquids is water; *i.e.* the specific gravity of water at a temperature of 4° C. is taken as unity. The specific gravities for a number of common substances are given in the following table:

Specific Gravities of Some Substances

Water at 0° C.	0.99987	Granite	2.5 to 3.0
Water at 4° C.	1.00000	Iron (wrought)	7.79 to 7.85
Water at 19° C.	0.99846	Iron (gray cast)	7.03 to 7.13
Water at 20° C.	0.99825	Kerosene	0.778 to 0.804[1]
Water at 21° C.	0.99804	Lead (cast)	11.34
Water at 22° C.	0.99782	Limestone	2.46 to 2.86
Water at 23° C.	0.99760	Marble	2.5 to 2.8
Ice at 0° C.	0.88 to 0.91	Mercury at 0° C.	13.596
Steam at 100° C.	0.0006	Mercury at 20° C.	13.546
Alcohol, absolute at 20° C.	0.789	Paraffin	0.824 to 0.940[1]
Alcohol, 95%, at 20° C.	0.804	Platinum	21.20 to 21.70
Aluminum (cast)	2.56 to 2.58	Quartz	2.65
Brass	8.2 to 8.7	Sandstone	2.2 to 2.5
Copper (cast)	8.80 to 8.95	Silver (cast)	10.4 to 10.5
Glass (common)	2.4 to 2.8	Steel	7.6 to 7.8
Glass (flint)	2.9 to 4.5	Tin (cast)	7.290
Gold (cast)	19.26 to 19.34	Zinc (cast)	7.04 to 7.16

Smithsonian tables except [1] Standard Oil Co.

Methods of Finding Specific Gravity

112. The definition given in the preceding section means that

$$\text{specific gravity} = \frac{\text{weight of the body}}{\text{weight of an equal volume of water}}$$

Any body that sinks displaces its own volume of water, and so, by Archimedes' principle, its loss of weight is the

weight of an equal volume of water, hence

$$\text{specific gravity} = \frac{\text{weight of the body}}{\text{loss of weight in water}}$$

or

$$= \frac{\text{weight of the body}}{\text{buoyant force of water upon it}}$$

Therefore, if we wish to find the specific gravity of any body, we must know

(i) The weight of the body, and

(ii) The weight of a volume of water equal to the volume of the body, which may be determined by its loss of weight in water, namely, the buoyant force of water upon it.

Illustration 1. A piece of brass is found to weigh 40 gm. When immersed in water, its weight is 35.3 gm. What is its specific gravity?

The loss of weight in water is $40.0 - 35.3 = 4.7$ gm. This is the buoyant force of the water upon the brass, and hence the weight of a volume of water equal to the volume of the piece of brass. The specific gravity of this piece of brass is then $\frac{40}{4.7} = 8.5$. Why?

Illustration 2. A piece of pine weighs 24 gm. It floats, so a sinker is attached to hold it below the surface of the water. The sinker in water weighs 100 gm. Sinker and pine together in water weigh 64 gm.

The total downward force before the pine block is submerged is $24 + 100 = 124$ gm. The downward force after the block is submerged is 64 gm. The loss of weight, which is due entirely to the pine, is $124 - 64 = 60$ gm. The specific gravity of the piece of pine is then $\frac{24}{40} = .6$. Why?

Specific Gravity of Liquids

For determining the specific gravity of liquids various methods are employed — the specific gravity bottle, the sinker method, the hydrometer, balancing columns, etc.

FIG. 97. A specific gravity bottle.

The *specific gravity bottle* (Fig. 97) is of convenient size to place on the pans of a balance. It is weighed first when empty, then when filled accurately to some fixed mark with water, and finally when filled to the same mark with the liquid whose specific gravity is desired. Evidently the volumes of liquid taken are equal. Hence the specific gravity may be calculated by dividing the weight of the liquid taken by the weight of water taken. Why?

In the *sinker method* a piece of metal or glass is weighed in air, in water, and again in the liquid whose specific gravity is desired. The volumes of liquid displaced are necessarily equal. The specific gravity is calculated by dividing the loss of weight of the sinker in the liquid by its loss of weight in water. Why?

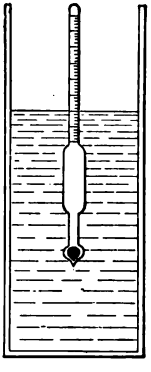


FIG. 98. The hydrometer.

The *hydrometer* (Fig. 98) is the commercial instrument used to determine rapidly the specific gravities of liquids such as alcohol, photographic solutions, acid for batteries, gasoline, etc. Its stem is hollow and uniform in cross section, carrying a scale which is graduated in specific gravities or in arbitrary "degrees Beaumé." It sinks in the liquid until it is buoyed up by a force equal to its own weight; *i.e.* it floats. Depending upon the liquid, the hydrometer sinks to different depths, the specific gravity being read directly upon the stem.

The following problems illustrate in concrete form various methods of determining specific gravity.

PROBLEMS

1. A piece of aluminum weighs 45 gm. in air and 27.7 gm. in water (Fig. 99). What is the buoyant force? What is the specific gravity of the aluminum? What is its volume?

2. (*a*) If a piece of lead weighs 240 gm. in air and 218.76 gm. in water, what is its specific gravity?

(*b*) If the same piece of lead weighs 223.22 gm. in alcohol, what is the specific gravity of the alcohol?

3. Alcohol is in one arm of a Hare's apparatus (Fig. 100) and water in the other. The alcohol rises 65 cm. and the water 51.35 cm. What is the specific gravity of the alcohol?

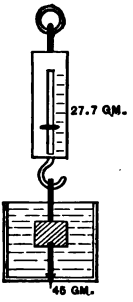


FIG. 99.

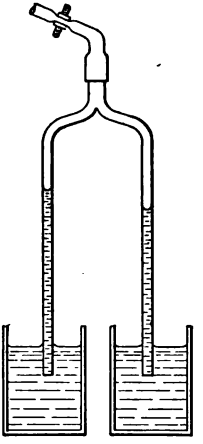


FIG. 100. Hare's apparatus.

4. The dimensions of a piece of oak are $4 \times 6 \times 2$ cm. It weighs 36 gm. What is its density?

5. A block of ice floats in water. One ninth its bulk is out of water. What is its specific gravity?

6. A specific gravity bottle (Fig. 97) weighs 10.5 gm. Filled with water it weighs 25.5 gm. Filled with mercury it weighs 220.0 gm. What is the specific gravity of mercury?

7. If the specific gravity of copper is 8.8 and of mercury 13.6, how much will 1 cubic centimeter of copper weigh in water? in mercury?

8. A rectangular block of oak weighing 1000 gm. floats in water. Its base is 20 by 25 cm. How deep does it sink? Can you calculate its density from these data?

9. A piece of pine is 40 c.c. in volume. Its specific gravity is .38. How much water will it displace when floating? How heavy a sinker would be necessary just to submerge it?

10. A balloon contains 1000 cubic meters of hydrogen (1 liter weighs .0899 gm.) The material of the balloon weighs 500,000 gm. Each cubic meter of the surrounding air weighs 1293 gm. How many kilograms in addition to its own weight can the balloon lift?

11. Why does a soap bubble filled with air from the lungs sink to the floor, while one filled with illuminating gas rises to the ceiling?

12. Calculate the specific gravities of the following substances: —

Common hard brick weighs 125 lb. per cubic foot.
Granite weighs 170 lb. per cubic foot.
Dry hemlock weighs 25 lb. per cubic foot.
Cast iron weighs 450 lb. per cubic foot.
Wrought iron weighs 485 lb. per cubic foot.
Heavy iron weighs 490 lb. per cubic foot.

II. The Phenomena of Surface Tension

113. When two liquids are brought together, either they mix and diffuse, the one into the other, or they do not mix, and consequently exhibit what are known as the phenomena of surface tension. In these phenomena, sometimes called capillary phenomena, the very laws of hydrostatics which we have just been studying *appear* to be contradicted.

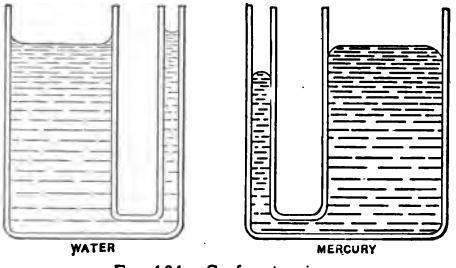


Fig. 101. Surface tension.

Arrange two U-tubes (as in Fig. 101), the smaller arm having an inner diameter of from one to two millimeters.

Partly fill one U-tube with mercury and the other with water: the mercury in the small arm does not rise as high as in the large; while with the water the reverse is true. Either of these phenomena is sufficient to show that, in communicating vessels, liquids do *not always* stand at the same height. As noted above (p. 109), liquids stand at the same height only when the communicating vessels have the same diameter, or when each is large.

Any one who has ever placed a small drop of mercury on a plate of glass knows that its center of mass stands at some distance, perhaps a millimeter, above the surface of the glass. (See Fig. 102.) This phenomenon, *on its face*, would appear to be an exception to the general dynamical principle that the potential energy of a system tends always to a minimum. It will be explained later why the drop does not flatten out, and why its center of mass does not assume a lower position.

FIG. 102. Mercury on glass.

Experiments on Surface Tension

114. Our task now is to show that these contradictions are only apparent, and that the phenomena of the liquids in the U-tube and the phenomenon of the drop of mercury are in perfect accord with general principles already known. **In the discovery of the harmony between any one truth and all other truths lies the essential feature of an "explanation."**

No one can try the following qualitative experiments without being impressed with the fact that liquids in air behave very much as if they were contained in a sac or thin membrane, continually holding them in position.

1. A glass rod dipped into a tumbler of water picks up a drop and carries it on the end of the rod as if a little

rubber bag were tied about the end of the rod and filled with water.

2. A falling raindrop nearly always assumes a spherical shape as if it were inclosed in a skin of some sort, and this skin were continually compressing it into the smallest possible volume.

3. In a camel's-hair brush, when dry, the individual hairs stand apart to some extent, giving it a somewhat bushy appearance. Dip the camel's-hair brush into a tumbler of water; the same bushy appearance remains. Remove the wet brush from the water to the air: the film of adhering water at once binds the individual hairs together, as a band does a sheaf of oats. The liquid in air acts as if it were covered by a film always tending to contract.

4. A piece of clean, fine wire gauze may be bent into the shape of a small open box. Water may be poured into this box to a considerable depth without running through the meshes.

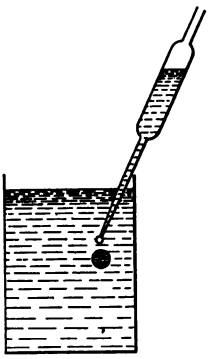


Fig. 103. A drop of oil freed from gravity.

5. A mixture of about 60 per cent of alcohol and 40 per cent of water has very nearly the same density as olive oil. A drop of oil placed in this liquid, as indicated in Fig. 103, will not mix with it; nor will it be affected by gravity. How do you explain the perfectly spherical shape which the oil assumes?

6. One of the simplest methods of observing this phenomenon of liquid surfaces is the following:

Make a strong solution of castile soap. Bend a piece of aluminum wire (about No. 20 or No. 25) into the shape shown in Fig. 104, making the diameter of the ring about two inches. At some point, *A*, of this ring tie a loop of

thread, *B*. If, now, the ring be dipped into the soap solution, a film will spread over it. This film has two surfaces,

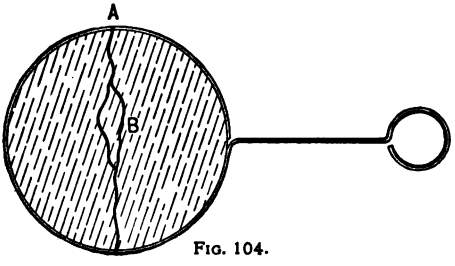


FIG. 104.

inclosing a thin layer of the soap solution between them. The thread floats on the film; now with the point of a lead pencil or a pin puncture the film at some point within the loop of thread, *B*.

Each surface of the film between the wire and the thread now *tends to become as small as possible*, as is indicated by

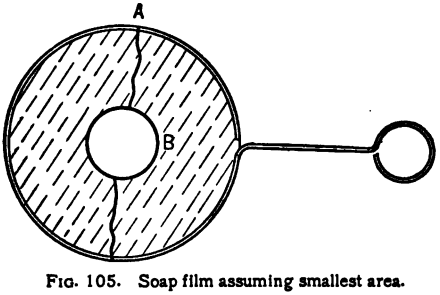


FIG. 105. Soap film assuming smallest area.

the fact that the area within the loop tends to become as large as possible, *i.e.* the loop assumes a circular form, as shown in Fig. 105.

Definition of Surface Tension

115. Such evidence as this proves that if we enlarge a liquid surface we shall have to do work upon it just as truly as if we stretch a rubber band or inflate a bicycle

tire. On the other hand, if a surface contracts, it exerts a force in so doing, and thus does work for us.

Bend a wire into the shape indicated in Fig. 106. Dip this wire into a soap solution to cover it with a film. If, now, a small straight wire, *CD*, is laid across the frame, the film on the *B* side may be broken out with the finger. The two surfaces on the *A* side will exert a certain definite force and draw the bar *CD* quickly to the end of the frame. The contracting film has done work upon the bar. Accordingly **the work which unit area of the liquid film can do, or its potential energy, is called the surface tension of the liquid.** In this case as in all others where the body is at rest, the potential energy tends to become a minimum, or the film contracts.

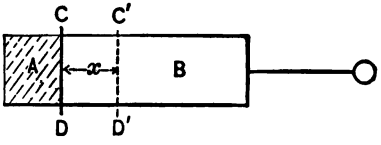


FIG. 106. Work done by a contracting film.

116. And now we see that, when a drop of mercury gathers itself up into a little, round drop, lifting its center of gravity above the plate on which it rests, it does so in order to make its surface as small as possible, and thus reduce its potential energy to a minimum. So far, therefore, from violating our previous experience, this behavior of liquid surfaces is quite in accord with it.

Why does this contraction of the film cause water to *rise* in a tube and mercury to *fall?* Why water wets glass, while mercury does not, is too complex a question to be answered here. It is, however, an experimental fact that *liquids rise in tubes when they wet them and are depressed when they do not.*

PROBLEMS

1. A glass tumbler may be filled with water till the surface of the water rounds up above the rim of the glass. Why does not the heaped-up water spill over?

2. Why does a soap bubble or a drop of water assume a spherical shape?

3. How is it that you can round the end of a glass rod by holding it in a Bunsen flame?

4. How does oil rise in the wick of a lamp?

5. Chemists often use a solid glass rod, instead of a funnel, in pouring a liquid from one vessel to another. Explain.

6. What *mechanical* processes keep a candle burning?

PROPERTIES OF GASES

117. Most of the theorems which we have just been studying under the subject of Hydrostatics are quite as true for fluids in general as for liquids; all of them, in fact, except those which refer to the *free surface* of a liquid. For this free surface is precisely the thing which we do not find in gases. A gas might, indeed, be defined as a body which has no free surface. Thus substituting the word *gas* for *liquid* **Pascal's Law and Archimedes' Principle,** as stated on pages 111 and 117, apply equally to gases and to liquids; in other words, these theorems **are true for all fluids.**

Air Possesses Inertia

118. A sailing vessel under way, a tree bending to a breeze, a bank of sand raised by the wind, are each evidences that air, upon which we are so dependent for life, is something which can transfer momentum to other bodies. Any one who has ever ridden against a head wind knows that air exerts pressure when in motion, and that air, therefore, has inertia. The boy who has blown

up a paper bag for the purpose of bursting it between his hands knows, at least, that the inclosed air is a body.

The lad who has lifted a brick with a piece of wet leather and a string (Fig. 107) knows that he must keep the edges of the leather down snug on the brick, and that, when he does so, the earth's atmosphere, without difficulty, keeps

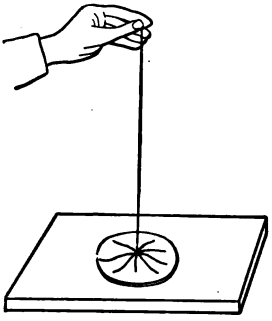


FIG. 107. The "sucker."

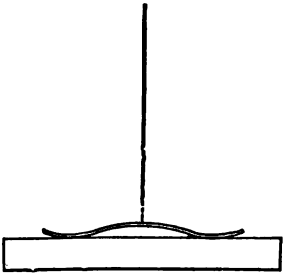


FIG. 108. Section of sucker shown in Fig. 107.

the brick up snug against the leather, except in the middle. Here the leather has been lifted (Fig. 108), and has, in turn, lifted the whole column of the earth's atmosphere, leaving, on top of the brick, a space in which there is practically no air.

Horror Vacui

119. The ancients were, perhaps, essentially as familiar with these facts as we are. At any rate, they were familiar with several kinds of pumps dependent upon these principles. But of the connection between these various facts they had little or no conception. To know a number of different facts is mere knowledge; to know these facts and the connection between them is scientific knowledge.

Before the time of Galileo nearly all the phenomena of pumps were explained by saying that "nature abhors a

vacuum." And this *horror vacui* is a fairly good explanation; it will, in any event, explain a large part of the more familiar phenomena, such as those of air pumps and barometers.

Air has Weight

120. In addition to this horror of a vacuum, it was proved by Galileo that air had weight. This he accomplished by weighing a bottle from which a part of the air had been removed, then refilling the bottle and weighing again.

These were two capital discoveries: (1) **that nature abhors a vacuum, and** (2) **that the air has weight.**

Torricelli's Experiment. Normal Atmospheric Pressure

121. Torricelli (1608–1647) showed, by a justly celebrated experiment, the connection between these two facts; viz. nature abhors a vacuum *because* the air has weight. Torricelli's experiment was as follows: taking a tube more than 76 centimeters long and closed at one end (Fig. 109), he filled it with mercury, and placing his finger over the open end, inverted the tube in a dish of mercury. The column of mercury fell a short distance, but remained standing in the tube approximately at the height of 76 centimeters above the surface of the mercury in the dish.

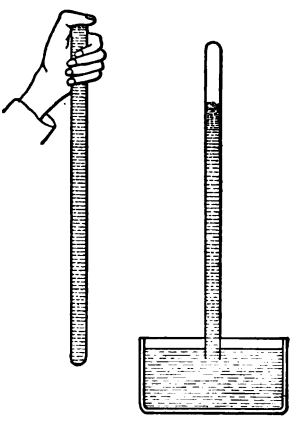


Fig. 109. Torricelli's experiment.

Torricelli ascribed the support of this column of mercury to the pressure of the atmos-

phere, and explained the pressure in turn as due to the weight of the atmosphere. He showed, indeed, that the weight of the earth's atmosphere is very approximately that of an ocean of mercury covering the earth to the depth of 76 centimeters. He also saw that the height of mercury in the inverted tube should diminish as one ascends in the earth's atmosphere. This test was shortly made by Pascal, who carried the inverted tube — such a tube and dish is called a **mercurial barometer** — to the top of a mountain in France, and found that the mercury fell some seven or eight centimeters during the ascent.

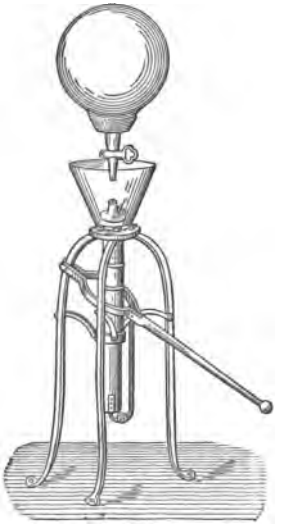


FIG. 110. Guericke's air pump.

The evidence for thinking that we reside at the bottom of an ocean of air, exerting its pressure equally in all directions, was completed by Otto von Guericke, who devised, in 1560, a pump (Fig. 110) exactly like a modern single-acting air pump, only the valves were not automatic. This enabled him to remove the greater part of the air from a glass vessel attached to the pump. In the nearly vacant space thus formed he found that the pressure was much less than that of the outer air, and on opening the valve, air would immediately rush in until the pressure within became equal to that without.

Scientists have agreed to adopt 76 centimeters of mercury as the standard height of the barometer at sea level; and the pressure required to support a column of mercury of this height is known as the normal atmospheric pressure.

Composition of the Earth's Atmosphere

122. More than a century elapsed between the time of Torricelli and the discovery of the principal elements in the air which we breathe. These were first shown by Lavoisier (1743–1794), one of the founders of modern chemistry, to be nitrogen and oxygen, in the proportion four of nitrogen to one of oxygen. In addition to these there is always present a variable quantity of water vapor, about $\frac{3}{100}$ % of carbon dioxide, together with about 1 % of argon, and also very small amounts of other gases.

When we remember that this ocean of air in which we live is invisible, odorless, tasteless, apparently devoid of weight, and almost intangible, it is little wonder that the discovery of its properties came rather late in the history of science. What men have already learned concerning gases is little short of marvelous. And he who thinks that Physics is a subject which has to deal *only* with things that can be seen and felt and heard needs only to read the story of *argon* and *crypton*, as told in Ramsay's *Gases of the Atmosphere* (Macmillan), and the chain of facts from which was drawn the inference that these two new elements are present in the earth's atmosphere.

The Barometer

123. The barometer is simply a U-tube containing mercury, one end of which opens into the earth's atmosphere, and the other into a vacuum. Such a vacuum is called a *Torricellian vacuum.* In case the longer arm were tall enough to extend up beyond the earth's atmosphere (Fig. 111), there would be no need of having either end of the tube closed. The column of mercury would just balance the column of air, and, as now, the ratio of the heights of the two columns would be in the inverse ratio of their

average densities. The simplest conception of the barometer is, then, that of a U-tube, one arm containing mercury only, the other filled with air extending up to the top of the earth's atmosphere. The **pressure which the atmosphere exerts upon a barometer is, then, dependent upon two factors; viz. (1) the height of the atmosphere, and (2) the average density of the atmosphere.** Anything which changes either one of these changes the reading of the barometer; *i.e.* the difference in level between the two horizontal surfaces of the mercury in the barometer.

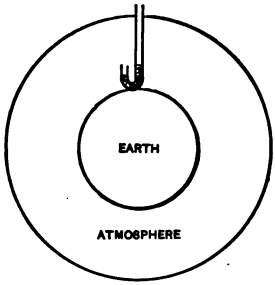


FIG. 111. The barometer conceived as a U-tube.

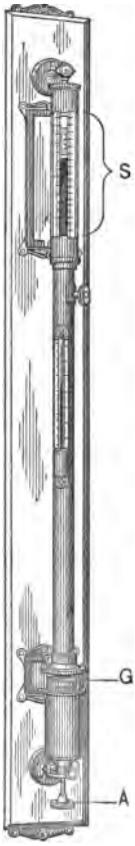


FIG. 112. A weather bureau barometer.

124. In barometers as used by the United States Weather Bureau (Fig. 112), the sealed glass tube containing mercury is inclosed in a brass jacket which is cut away so as to expose the upper end of the mercury column. The lower end of the tube dips into the mercury cistern, *C* (Fig. 113). The level of the mercury in the cistern may be adjusted by the screw, *A*, thus raising or lowering the leather bag which forms the bottom of the cistern. This surface of the mercury is visible through

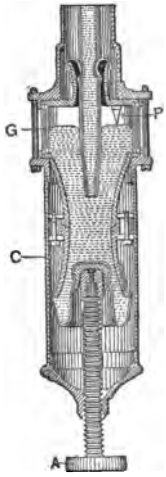


FIG. 113. The cistern of the barometer.

the glass cylinder, *G* (Figs. 112–113). Extending from the roof of the cistern is an ivory pin, *P*. The lower end of this pin is the zero of the scale of the barometer. This scale is not graduated throughout its whole length, but only near the upper end of the mercury column, *S* (Fig. 112).

A reading of the instrument is taken by setting the adjusting screw, *A*, so that the ivory point, *P*, just touches the mercury surface and then reading the position of the top of the mercury column.

Weather and the Barometric Pressure

125. Water vapor when under the same pressure as air has a density which is less than that of air. If, then, the atmosphere contains much water vapor, its density is diminished, consequently its pressure is diminished, and the mercury column which it supports becomes shorter. The barometer is said to "fall." But the same thing happens when the height of the atmosphere is lowered or its pressure is changed by a cyclonic motion. The barometer is, therefore, *not* an instrument for telling whether it is about to rain; but for measuring the pressure of the earth's atmosphere. Nevertheless, barometric observations are of great aid to the Weather Bureau in predicting atmospheric changes, and usually a falling barometer indicates an approaching storm, while a rising barometer means that fair weather may be expected.

PROBLEMS

1. Calculate the normal atmospheric pressure in grams-weight per square centimeter. Density of mercury = 13.6. (Assume the area of cross section of the tube to be 1 sq. cm.)

2. What would be the force in grams-weight exerted by the atmosphere on the surface of a square pond 1 km. wide? (Barometer at 76 cm.)

3. The atmospheric pressure is about 15 lb. weight per square inch. What is the force in pounds-weight exerted by the atmosphere on the outside of a cubical tank 4 ft. on an edge? Why is the tank not crushed?

4. The pressure inside a boiler is 20 atmospheres. What is the difference in pressure between the inside and the outside of the boiler?

5. What is the height of a water barometer when the mercury barometer reads 76 cm.?

6. What is the height of the barometer *in inches*, at the normal atmospheric pressure?

APPLICATION OF PRECEDING PRINCIPLES

The Siphon

126. The siphon is an apparatus for emptying barrels, pails, ponds, and other vessels of liquid not conveniently opened at the bottom. It consists of some form of U-tube (Fig. 114). To start the siphon it is filled with the liquid, the ends closed, and one end placed in the vessel to be emptied while the other end is at some lower level than the surface of the main body of liquid. When the ends are opened, the liquid flows toward the longer arm.

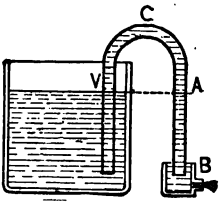


Fig. 114. The Siphon.

The force which moves the water is an excess of pressure in the arm CB. In the U-tube, when originally filled with water, the pressure at V is the same as the pressure at A. But when the siphon is immersed in the tank, the pressure at V is at that of the atmosphere; so also is the pressure at B; while the pressure at A is less than that of an atmosphere by the height of the column AB. Hence the liquid flows from the point of higher pressure V over to the point of lower pressure A. The liquid in AB will fall and its place be taken by liquid from the vessel V until the level in B is the same as that in the vessel.

Can the siphon be used to carry water over any height? What is the limit of its action?

Pumps

127. Pumps of all kinds, whether for gases or liquids, whether to exhaust a vessel of the fluid it contains, or to force fluid into a vessel, are all modifications of the typical form shown in Fig. 115.

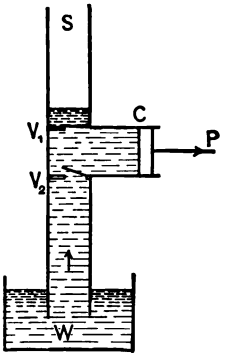


Fig. 115. A typical pump.

S is a tube dipping into a vessel of fluid, W.

P is a piston fitting the cylinder, C.

V_1 is a valve opening out of C.

V_2 is a valve opening into C.

1. **Force pump.** Figure 115 represents in principle the ordinary force pump, such as that employed in the fire-engine and in pumping water into standpipes. When the piston P is moved in the direction indicated by the arrow, the pressure in the region between the valves is diminished. The effect of this is to open the lower valve and close the upper one.

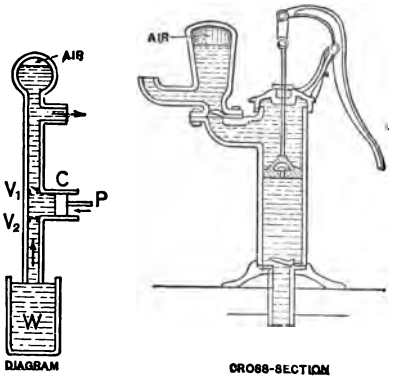


Fig. 116. The force pump.

Not only so, but the pressure of the atmosphere on the free surface of the water in the well begins to raise the water in the tube. And if the lower valve be not too high above the surface of the well,

the water will pour through the valve V_2, and partly fill the space between the valves.

If the motion of the piston be now reversed (Fig. 116), the water inclosed between the valves will be forced through the upper valve into the hose or vessel in which it is desired.

Such a stream, however, would be intermittent; for in the region above the upper valve the pressure would cease on the out stroke of the piston. To avoid this

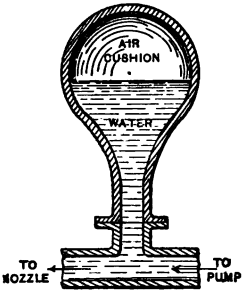


FIG. 117. Fire-engine pump dome.

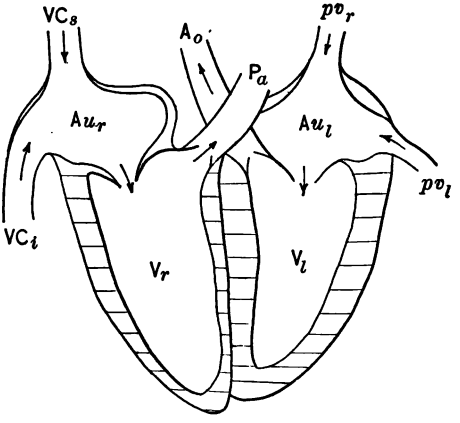


FIG. 118. The human heart considered as a force pump.

difficulty, most force pumps are furnished with an air chamber (Fig. 116). When the pump first begins, this chamber is filled with air. When the water is forced through the upper valve, this air is compressed; energy is stored up in it; and, when the back stroke of the piston occurs, this compressed air supplies enough energy to drive the water partially out of the bulb, and thus keep the stream flowing until the piston begins its inward stroke. The large copper bulb on the fire engine is a familiar sight.

2. **The human heart.** The human heart is equivalent to a force pump like that diagrammed in Fig. 116, only here the space between the valves is no longer furnished with a piston, but has elastic walls, which, by alternate contraction and expansion, effect the same result as the piston. When the pressure in the right ventricle of the heart (V_r, Fig. 118) is relieved, the blood from the veins rushes in by way of the right auricle, Au_r; when the walls of the heart contract, this blood is driven into the pulmonary arteries P_a on its way to the lungs. In like manner, the left side of the heart acts as a force pump, receiving fresh blood from the lungs and driving it to the extremities of the body. The elasticity of the arteries and veins takes the place of the air chamber in the force pump.

3. **The ordinary lift pump.** Here the upper valve is placed in the piston itself, as indicated in Fig. 119. Since the pump has no air chamber, the stream is of course intermittent. On the down stroke the upper valve opens; on the up stroke it closes; the pressure between the valves is diminished; the lower valve opens and the water rushes through. On the next down stroke of the piston it dips into the water, which is held up by the lower valve; and, on the succeeding up stroke, lifts its load of water to the spout. The ordinary lift pump can theoretically raise water about 34

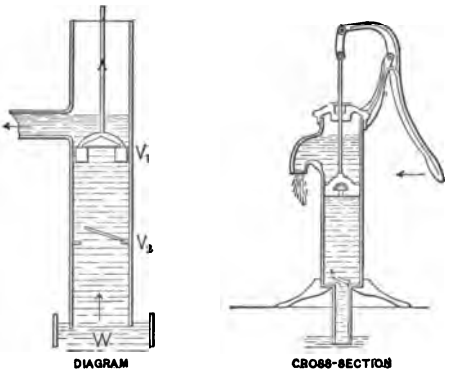


FIG. 119. An ordinary lift pump on up stroke.

feet. Practically, due to leakage in valves and to other causes, the height at which it works is reduced to about 25 feet.

4. **Deep well pumps.** — It is frequently necessary to pump from great depths, either in keeping mines free from water or in providing water supply from wells many

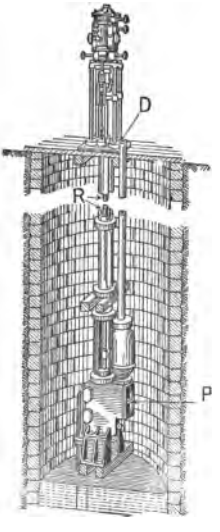


FIG. 120. Deane vertical engine and deep well pump.

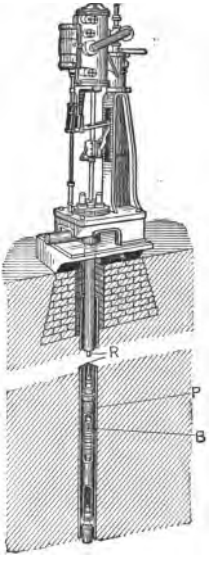


FIG. 121. Deane artesian engine and pump.

hundred feet deep. Evidently this work could not be done by an ordinary lift pump. In mines it may be possible to locate engine and pump within 25 ft. of the level of the water and thus force the water out, or else the engine (Fig. 120) may be placed at the mouth of the mine and connected by a long piston rod, *R*, with the force pump, *P*, which delivers the water at the surface through the pipe, *D*.

Where the diameter of the well is small, a lift pump (Fig. 121) is used. The engine is placed above the well and a long piston rod, *R*, connects with the pump piston, *P*. The valves in the illustration are ball valves, which are much more efficient than the clapper valves in the ordinary pump.

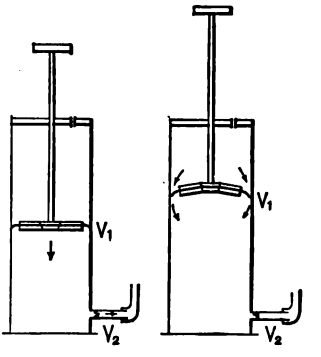


FIG. 122. Single-acting bicycle pump.

5. **The bicycle pump.** This is a force pump, in which air is the fluid transferred. The piston here also contains the valve; is the valve, in fact. The down stroke of an ordinary foot pump is represented in Fig. 122. On the up stroke the air rushes into the cylinder all around the edge of the piston,

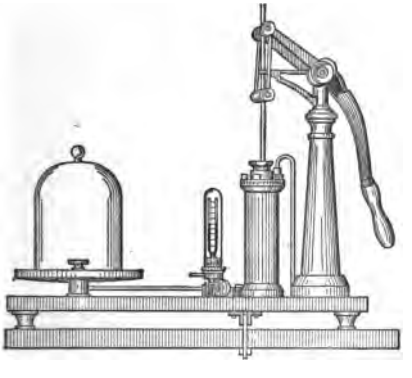


FIG. 123. An ordinary air pump.

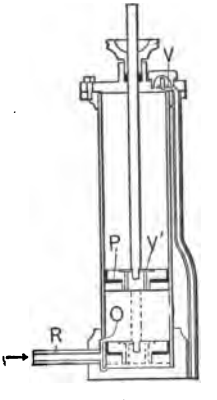


FIG. 124. The action of an air pump.

while the valve, V_2, next the tire closes by the back pressure of the air in the tube.

Why is such a pump so hard to operate when the tire is fairly well inflated?

6. **The vacuum pump.** If we reverse the valves indicated in Fig. 122, we shall have a pump which, instead of pumping air into a tire, will empty the tire. This is the principle of the ordinary air pump (Fig. 123), some form of which is found in every laboratory.

The details of the action of the pump are shown in Fig. 124. As the piston, *P*, travels to the bottom of the

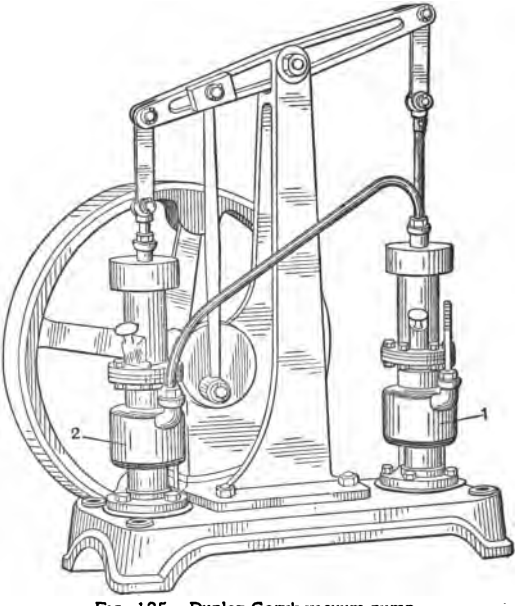


FIG. 125. Duplex Geryk vacuum pump.

cylinder, air from the receiver, *R*, is admitted through the openings, *O*. The piston, at the beginning of its up stroke, closes this opening and compresses the air above it until the valve, *V*, is forced open and the air expelled. On the down stroke the valve, *V*, closes and the valve, *V'*, in the piston is forced open. A fresh volume of air is inclosed above the piston and expelled through *V*. This process is repeated until the receiver is exhausted. Such a pump

rarely gives a higher vacuum than 3 or 4 millimeters of mercury or about $\frac{1}{200}$ of an atmosphere.

Modern mechanical air pumps (Fig. 125) furnish very high vacua comparable with the best mercury pumps. When two pumps, 1 and 2 (Fig. 125), are coupled in tandem, a vacuum of $\frac{1}{5000}$ of a millimeter or $\frac{1}{1000000}$ of an atmosphere may be produced in a very few minutes. The great rapidity of action and high vacua are made possible by the simple device of using oil as a sort of packing for the piston (Fig. 126). When the piston, *P*, comes down upon the oil in the bottom of the cylinder, the air is forced through the valve, *V'*, into the region above the piston. When the piston travels up, this air is forced through the valve, *V*, in the partition, *D*, and air from the vessel to be exhausted expands into the region below the piston. This air is in turn removed on the next stroke of the piston. The oil keeps the valves perfectly sealed.

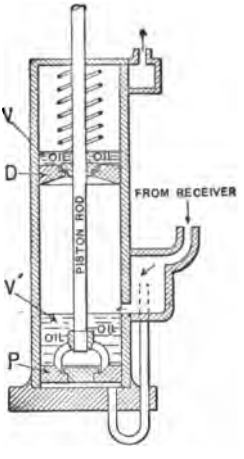


FIG. 126. Cylinder of a Geryk pump.

PROBLEMS

1. Can a liquid be siphoned from an air-tight barrel? Why?

2. How high can mercury be lifted by a suction pump?

3. How high can sulphuric acid whose specific gravity is 1.84 be siphoned?

4. What is the maximum height possible for the arch of a water siphon when the mercury barometer reads 74 cm.?

5. A siphon is working at its maximum height. Describe in detail all changes when the barometer drops two centimeters.

6. Suppose it possible to build a suction pump with perfect valves. How high can water be lifted *by suction* with such a pump when the barometer stands at 73 cm.?

7. Does the downward pressure of the atmosphere *lift* water in a pump or siphon? If so, why is not a perpetual motion thus possible?

8. The barrel of an air pump has a capacity one fifth that of the receiver. How much air will remain in the receiver after 4 strokes? What will be the pressure in the receiver if the original pressure was 760 mm. of mercury?

Boyle's Law

128. Shortly after the time of Torricelli's experiment, Robert Boyle, a distinguished English chemist, invented an air pump, and became greatly interested in the following question:

Gases are easily compressed. A bladder filled with air at ordinary pressure can be reduced in volume without much difficulty, *i.e.* without greatly increasing the pressure. Boyle proposed to himself this query, "Just *how* does the volume of a given body of gas change when the pressure is varied?"

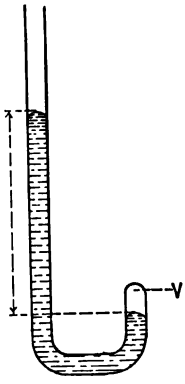


Fig. 127. Boyle's experiment.

To answer this question he inclosed a definite mass of air in the closed arm of a U-tube, as indicated in Fig. 127; varied the volume, V, of this inclosed gas by pouring in more and more mercury; and measured the pressures corresponding.

Now Boyle found that so long as the *mass* of inclosed air remained constant, and so long as its *temperature* remained constant, the **product of the pressure and the volume of the gas** was **approximately a constant.** The law was published in 1660. It may be stated algebraically as follows:

$$PV = K, \text{ a constant.}$$

This summary of fact, known as Boyle's Law, has since been found to be very nearly true for all gases, through a wide range of pressures.

The limitations of this law are important to note; viz.

1. The law is only approximately true for any gas.

2. The law is approximately true only so long as the temperature of the gas remains constant.

3. The law as stated is true only for a given quantity of gas. If during the experiment any gas is added to or taken away from that under consideration, the constancy of the product, PV, does not hold.

Boyle's experiments were mostly performed with pressures greater than one atmosphere (76 cm. of mercury); but Mariotte (1620–1684), a French physicist, shortly afterward showed that Boyle's Law holds when the air in the containing vessel is rarefied; that is, when the pressure is less than one atmosphere.

Computations with Boyle's Law

129. The usefulness of this law of Boyle's can scarcely be overestimated. For, if we know the pressure corresponding to any one known volume, we know at once the value of the constant, K, and can hence *compute* the pressure corresponding to any other volume; or, if it be desired, one can compute the volume corresponding to any other pressure.

For example, suppose that a bottle whose capacity is 100 cubic centimeters contains compressed air, at a pressure equal to that of 180 cm. of mercury. Let us compute the volume this air would occupy if allowed to escape into the atmosphere, at normal pressure. Let P_1, V_1, represent the pressure and volume, respectively, of the air while in the bottle, and P_2, V_2, its pressure and volume after being released. It is required to find V_2. According to Boyle's

Law, the product of pressure and volume remains constant. Hence we may write,

$$P_1V_1 = K = P_2V_2$$

But $P_1 = 180$; $V_1 = 100$; and $P_2 = 76$. Hence, substituting these values,

$$P_1V_1 = 180 \times 100 = 18000 = K$$

$$V_2 = \frac{K}{P_2} = \frac{18000}{76} = 236.8$$

Hence the 100 cubic centimeters of compressed air, if allowed to escape, would occupy 236.8 cubic centimeters.

130. The deviations from Boyle's Law, the extensions of Boyle's Law to solutions, and the evidence which Boyle's Law — together with other properties of gases — offers for thinking that gases are composed of a very large number of exceedingly minute particles, called molecules, form three of the most interesting chapters in the whole of physical science.

PROBLEMS

1. If 1000 c.c. of hydrogen under a pressure of 20 cm. of mercury is subjected to a pressure of 84 cm. of mercury, what will be the resulting volume?

2. A cylinder 40 cm. long is closed at one end by a piston. If the cylinder is filled with air at atmospheric pressure (76 cm.), and the piston is thrust down within 3 cm. of the other end, what will be the pressure of the inclosed air? (It is here assumed that the temperature is the same in each case.)

3. Two hollow spheres of radii 8 and 10 contain equal masses of hydrogen. Compare the pressures.

4. A certain mass of hydrogen measures 97 c.c. when the barometer reads 74.5 cm. What volume will it occupy under normal conditions?

5. Twenty-four cubic centimeters of gas at a pressure of 71 cm. of mercury would have what volume under a pressure of 76 cm.?

6. On admitting a small bubble of air into the top of a barometer tube, the column fell from 75 cm. to 72 cm. Compare the pressure of the air in the tube with that of the outside atmosphere.

7. In calculating the pressure at a point in a liquid exposed to the air, is it necessary to take account of the weight of the air? Explain.

8. What air pressure must be maintained inside a diving bell (Fig. 128) to prevent water from entering? The bottom of the bell is 40 ft. below the surface of the water.

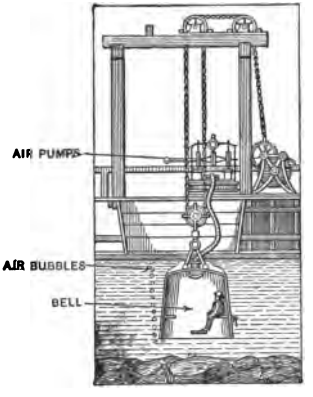


FIG. 128. Diving bell.

9. Why is the building of tunnels and similar underground work frequently carried on under 10 or 15 lb. additional air pressure? Why is it necessary to use higher pressure in soft ground than in rock?

10. Is it necessary, in calculating the force which a liquid exerts upon one side of its containing vessel, to take into account the pressure of the air?

References

LODGE — *Elementary Mechanics, Hydrostatics, and Pneumatics* (Van Nostrand). The student will find constant reference to this book very helpful.

WILLIAMS — *The Romance of Modern Mechanism* (Lippincott). Among descriptions of other devices this volume contains a very interesting account of the hydraulic press as well as of various pneumatic tools.

CHAPTER IV

WAVES

131. The word at the head of this chapter will recall to most minds one or more of the following phenomena: the surface of a lake swept by a wind, the constantly enlarging rings produced by a stone falling into quiet water, the "swell" that follows a storm at sea, the waves which always accompany a moving boat. Many people have seen waves run along a clothesline, or some stretched rope, when struck by a walking stick. Experience shows us that these and all similar cases have two features in common. **First, there is a disturbance at some point in a certain medium. Second, this disturbance is propagated to other points of the medium.**

At a given instant the stone is dropped into quiet water; at a later instant the disturbance may have reached points several meters away.

At a given instant the stretched clothesline is at rest in its position of equilibrium. When one end is suddenly depressed, this depression runs along the line at a definite rate. If the disturbance be an elevation, it is an elevation that runs along the line.

But what is meant by *a disturbance?* Simply this, *a body is said to be* **disturbed** *when it is thrown* **out of equilibrium.**

But when is a body or system of bodies in equilibrium? Not necessarily when it is at rest, but when its potential energy is a minimum. A pendulum is at rest at the end of its swing, but is by no means in equilibrium. A pen-

dulum moves most rapidly at the lowest point in its path; but there is the one point where it is in equilibrium.

A disturbance, then, implies the addition of energy to a system. And as this disturbance is propagated through the medium, some energy is, in general, distributed through the medium.

Definition of Wave Motion

132. We may, therefore, define a wave motion as a state of disturbance propagated from one part of a medium to another.

Note that in this definition nothing is said about any up-and-down motion, such as is seen in the disturbed clothesline. One has only to replace the clothesline by a stretched spiral spring to see that waves may travel without any such up-and-down motion. Such a spring (Fig. 129) should be from two to three meters long, made of brass wire, No. 18 or 20, wound in a spiral of approximately eight millimeters diameter. Fix the ends of the spring to two rigid points, *A* and *B*, so that the spring will be stretched to nearly twice its unstretched length.

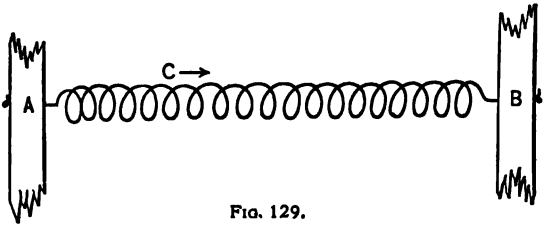


Fig. 129.

If this spring be grasped at any point, *C*, moved in the direction of the arrow, then quickly and gently released, the disturbance thus introduced will run to and fro along the line; but there will be no appreciable up-and-down motion. This very important experiment should be carefully observed.

Note further that, in the definition of wave motion, nothing is said about any transfer of matter. For although in each case experiment shows that the particles of the medium oscillate through a small range, it is evidently not the medium, but the energy, which is transferred.

SOME SPECIAL KINDS OF WAVES

I. Water Waves

133. Of these there are several kinds, and most of them are studied to best advantage in the laboratory.

The source of these waves is anything which elevates or depresses the water at any point. Imagine OX to represent the level surface of the undisturbed water. If by any means the surface of the water be given the form indicated in Fig. 130, the operation will be equivalent to taking the

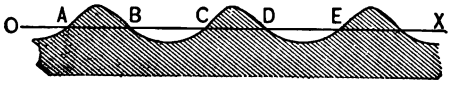


Fig. 130. Waves on surface of water.

water from the "trough" of the wave BC, and lifting it up into the "crest" of the wave AB. Let m be the mass of water which will just fill the trough BC; the amount of water in the crest will then also be m. Let h denote the difference in height between the center of mass of the trough and the center of mass of the crest. The operation of producing a single wave in water previously quiet will require that one lift a mass m through a height h against gravity. The potential energy stored in the wave will, therefore, be mgh. This potential energy tends to a minimum, *i.e.* in this case the center of mass tends to fall, and, since water has inertia, the crest in falling goes on beyond

its position of equilibrium, and thus we have a **continual oscillation due to gravity.**

In addition to this potential energy, it must not be forgotten that waves have considerable kinetic energy. In fact it may be shown that in wave motion in general exactly one half the energy is kinetic and one half potential. For, in general, all the water near the surface is in motion.

Damped Vibrations

134. The disturbance at any point in the water over which a wave is passing is, then, very similar to that in one arm of a U-tube partially filled with mercury when the mercury is in oscillation.

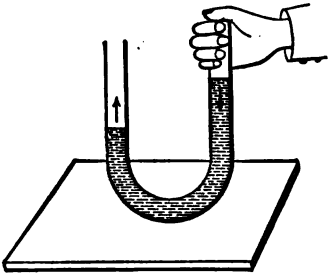


FIG. 131. Damped vibrations.

Take a U-tube of a diameter not less than 1 cm. Pour it half full of clean mercury — water will answer nearly as well. Incline it and close one end with the thumb, as indicated in Fig. 131. Bring the tube into a vertical position and suddenly raise the thumb. A dozen or more oscillations follow. They are such as would occur at any point in an ocean of mercury, or of water, traversed by waves.

Why do these oscillations so quickly stop? Would this same cause be at work in a large lake or at sea to stop the waves?

Waves or oscillations which are made to diminish gradually in amplitude are said to be damped.

Definition of Wave Length

135. A series of waves such as one can start in water by rocking a boat, or by raising and lowering a floating block, is called a **train of waves.** In any train of water waves the distance between two successive crests is called a wave length. The distance between two successive troughs is also a wave length. But the general definition of a **wave length is the shortest distance between any two particles whose motion is in the same phase.**

What would correspond to "two successive crests" in the waves produced by compression in the spiral spring?

Speed of Water Waves

136. A train of waves is travelling *over a point;* how far does this train travel during the period of one oscillation of the water at this point? Evidently one wave length. Let us call this wave length l, the number of vibrations n, and the period of one oscillation T.

If, by any means, direct observation or computation, we can obtain l and n, we have at once the speed of the wave; for, denoting speed by V, we have velocity of wave = number of vibrations per second × wave-length, or

$$V = nl = \frac{l}{T}$$

This may be called the fundamental equation of wave motion; for, since it depends simply upon the defining equation for speed, it is necessarily true of all wave motions.

It may be proved in the laboratory that the speed of waves in shallow water depends only upon the depth of the water, waves traveling faster in deeper water.

Refraction of Water Waves

137. How shall we employ this phenomenon to explain the fact that whenever waves on a pond, or lake, or at the

seacoast approach the shore, the crest of the wave is always more or less parallel to the shore line? In Fig. 132 let the heavy line represent the shore near the entrance to a small bay. A train of waves, *A*, *B*, *C*, *D*, etc., approaches the shore. The dotted lines are called **contour** lines, and indi-

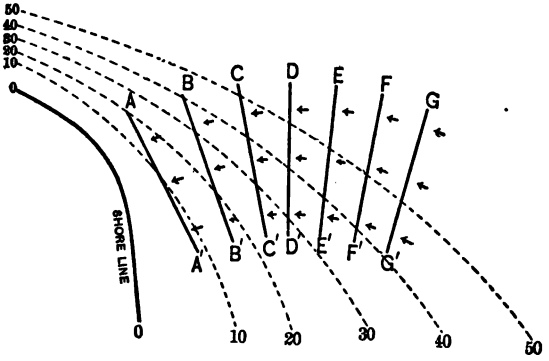


FIG. 132. Refraction of water waves.

cate the depth of the water. Every point on the contour line marked "10" has a depth of 10 centimeters, every point on the line marked "20" has a depth of 20 centimeters, etc. When the waves leave the deep water, the crests have the direction indicated by $\overline{G\,G'}$. The part of the wave which reaches shallow water first travels slower than the part in deeper water. As a result, the crests of the waves gradually swing around so that they are no longer parallel with each other. The end *A* has travelled faster than the end *A'* because it has been less retarded by the shallowness of the water. The nearer the waves approach the shore the more they are turned about. Such a process is called *refraction* and *occurs whenever one part of a wave is compelled to travel at a rate different from the other parts.*

Reflection of Water Waves

138. If a breakwater or solid pier (Fig. 133) is built out into a body of water, one has there an opportunity to see what happens when water waves strike a rigid body.

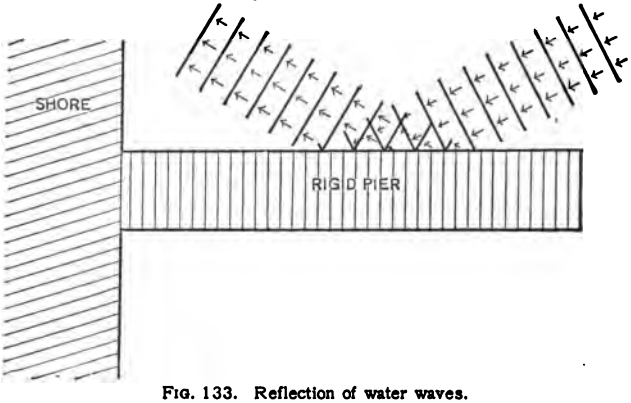


FIG. 133. Reflection of water waves.

In such circumstances it is not difficult to see that the direction of the reflected waves is very different from that of the incident waves.

Stationary Waves

139. From our experience in the laboratory, we know that when a train of waves is started at one end of a wave trough, they no sooner reach the other end than they are reflected back.

These two trains of waves, the incident and reflected, may combine with each other to produce **a series of waves which** appear to **have no motion of translation**, the so-called **stationary waves.** In order that these two trains of waves may produce stationary waves, it is necessary that all the waves should be of one length. A moment's practice with a wooden paddle in a trough of water enables one to pro-

duce such waves. A more detailed study of stationary waves will be made under the head of Waves in Strings.

Summary concerning Water Waves

140. 1. **Medium of propagation** — bounding surface between air and water.

2. **Motion of particles** — small, in paths, nearly circular: through a part of the path the motion of the particle is in the same direction as the motion of the wave; in part of the path it is at right angles to the motion of the wave.

3. **Speed** — in deep water, this depends upon length of wave; but in shallow water, it depends upon the depth of the water. For any single type of wave, $V = nl = \frac{l}{T}$.

4. Water waves are **reflected**, and thus produce **stationary waves.**

II. Waves in Strings

141. A rough method of studying waves in strings is to fasten together two lengths of ordinary rubber tubing, and thus make one piece about 20 feet long. By the aid of a hook in the wall, a heavy mass, or some more convenient means, clamp one end of the tube to the wall, or to the edge

Fig. 134. A simple method of studying waves in strings.

of a table, or even to the floor, as shown at B in Fig. 134. Taking the free end, A, in the hand, one can produce in the tube any tension he desires, and can send trains of waves down the tube in the direction given by the arrow.

Transverse and Longitudinal Waves

142. Consider the motion of any one point in the rubber tube. As a train of waves passes through this point, it oscillates, to and fro, in a direction perpendicular to that of the arrow. **The motion of the particles in the medium is then across the direction in which the waves travel.** Such waves are said to be **transverse.**

In the case of the spiral spring, alternately compressed and elongated (Fig. 135), any one small part of the spring is displaced in the *same* direction as that in which the wave is traveling. **The displacement of the particles is lengthwise.** Such waves are accordingly called **longitudinal.**

FIG. 135. Longitudinal waves in a spiral spring.

In the short time at our disposal we can consider only transverse waves.

Experiments on Transverse Waves

143. Returning now to the rubber tube, stretched upon the floor, observe the following:

1. That you can vary the *length* of the waves you send along the string by varying the *rapidity* with which you move your hand to and fro. Let us denote by T the time occupied by your hand in one complete vibration. During this interval the disturbance travels just one wave length, l, along the tube.

If we had any means of measuring T and l, we could determine the speed of the waves, V; for $V = \frac{l}{T}$.

2. Notice also that by stretching the tube more, but *moving your hand to and fro at the same rate*, the waves become longer; the wave length increases with the stretching force.

How, then, is the *speed* of the wave affected by this increase of tension?

3. Notice that a single disturbance sent down the tube does not in general cease when the wave strikes the fixed end (*B*, Fig. 134). If it does, the wave has been damped out by friction on the floor; stretch the tube a little more, holding it up so as just to clear the floor. The reflected wave can now be easily seen running back in a direction just opposite that of the incident wave.

4. With a moment's practice one can obtain a reflected *train* of waves which interfere with the incident train in such a way as to produce stationary waves, exactly similar to the stationary water waves obtained in the water trough.

Nodes and Loops. Stationary Waves

144. When a single train of waves is passing down a string, *every* point on the string is set in motion; but when we have two trains (an incident and a reflected) traveling in opposite directions, we see that they may combine their displacements in such a way that there are certain points on

FIG. 136. Production of nodes in a string.

the string, such as *A*, *B*, *C*, etc., in Fig. 136, that are not displaced at all from their position of equilibrium. Such points are called **nodes.**

It is important to observe that a single depression sent down the tube is always reflected from the fixed point as an elevation; in like manner, a single elevation sent down the tube comes back as a depression.

With a little practice one can make the rubber tube vibrate in such a way as to show either one, two, or three, perhaps more, of these nodes. The fixed point, *E*, is of

course always a node. **Any portion of the vibrating string which lies between two consecutive nodes is called a loop.** The middle point of a loop is called an **antinode.**

When the hand, by a to-and-fro motion, produces stationary waves in the rubber tube, is the hand at a node or an antinode? Try the experiment again before attempting to answer this question.

One who has observed a vibrating clothesline or a guitar string knows that ordinarily there is no appearance of a wave moving along the string; the vibrations are merely to and fro, that is, stationary. The posts, in the case of a clothesline, are each nodes, places of no motion. The fret and bridge, in the case of the guitar, are nodes for the same reason. The bow of the violin simply starts two trains of waves, which, by their interference, produce the familiar vibrations of the instrument. It it is likewise true of the piano, guitar, and all stringed instruments of the orchestra that their vibrations are produced by two trains of waves running in opposite directions, meeting and interfering.*

Speed of Waves in Strings

145. When the large rubber tube is put in vibration, it is very easily seen that two adjacent loops (Fig. 137) are displaced in opposite directions. Hence it follows that **the length of the wave passing is just twice the distance between two successive nodes.** These nodes are usually well marked; and hence the wave length is an easily measured quantity.

If we could also measure the period of vibration, T (or

* The student is here warned not to think this discussion of stationary waves at all complete. The subject is presented by Professor Tait in a short, readable article under the head of "Waves" in the "Encyclopædia Britannica."

its reciprocal, which is called the **frequency**, and is denoted by n), the speed would be easily computed from $V = nl = \frac{l}{T}$.

If, instead of using our hand, we employ a stiff spring to set the string in vibration, and if we know the period of the spring, we can at once obtain the speed of the waves.

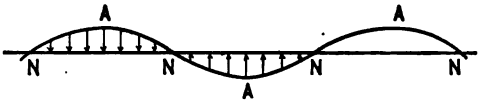


FIG. 137. Showing a wave length as twice the distance between two consecutive nodes.

For under these circumstances the frequency of the string is *forced* to be exactly that of the spring.

So that $$V = nl = 2nL$$

where n = frequency of the spring, and L = mean distance between two successive nodes.

Experiments on Speed of Waves

146. A very convenient spring of this kind is one such as is found in the large electric bells.*

Remove the gong of the bell and perhaps also the hammer; attach a light string to the armature. Carry the other end of the string over a pulley clamped as indicated in Fig. 138; and attach to it a pan for weights as shown, or tie the weight directly to the string.

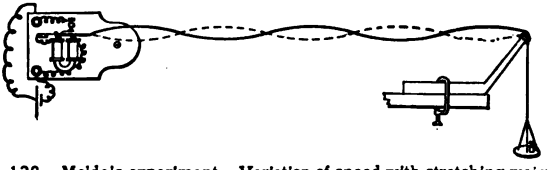


FIG. 138. Melde's experiment. Variation of speed with stretching weight.

* An electrically driven tuning fork, although considerably more expensive, is in other respects much better adapted to this experiment than the electric bell, for it maintains a more constant frequency.

So long as the spring vibrates at a fairly uniform rate, this apparatus is excellently adapted to answer two questions; viz. (1) How does the speed of a wave change when the stretching force of the string is altered? (2) How does the speed of a wave change when a heavy string is substituted for a light one, all other circumstances being the same? These questions should be taken up in the laboratory, where it can easily be shown that the **speed of the waves depends upon two quantities only; viz. the stretching force and the linear density** (*i.e.* mass per unit length) **of the vibrating cord.**

Interference of Waves

147. When two trains of waves pass through any point on a string at the same time, they are said to **interfere.** These interference phenomena are so important in many departments of physics as to demand especial consideration.

When a wave passes any point on a string, the phase of the vibrating point is the phase of the wave at this point. If two waves in exactly the same phase pass any point in the string at the same instant, they will each displace the particle in the same sense; if they are in exactly opposite phases, they tend to displace the particle in opposite sense. The actual motion of the particle will be the resultant of these two motions.

Experiments on Interference of Waves

148. It is very easy to produce two trains of waves in a string such that they will be in exactly opposite phases; *i.e.* their phases will differ by 180°.

To one of the prongs of a tuning-fork attach a light string — silk floss, or a strand of darning cotton, is excellent — and make the other end fast to some rigid object,

as shown in Fig. 139. When the fork starts to vibrate, a train of waves will run down the whole length of the string

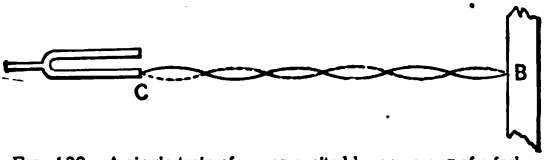


FIG. 139. A single train of waves excited by one prong of a fork.

from C to B; and if the tension of the string is properly adjusted, stationary waves will be produced *throughout the entire length of the string.*

At some point, A (Fig. 140), now tie a second string of length equal to CA; attach the other end to prong D and make the tension the same in each string. Each prong of the fork will now send a wave down the cord. But the prongs of the fork, at any instant, are moving in exactly opposite sense; hence at A the trains of waves in DA and CA are in exactly opposite phases. And since they each tend to displace the point A by equal amounts in opposite sense, the result is that the point A is not

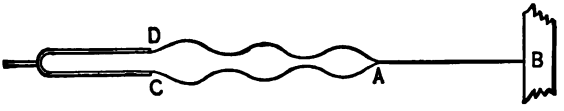


FIG. 140. Wylie's experiment, showing two trains of waves interfering at A.

displaced at all. The wave trains DA and CA are said to **interfere** at A. No disturbance, therefore, can now enter that portion of the string lying between A and B.

Definition of Interference

149. In the case of stationary waves, there are only certain points where the two trains of waves are in opposite phase — these are the nodes. There are other certain points

where the two trains have the same phase — these are the antinodes.

When the solar and lunar tides reach any point on the earth's surface in the *same* phase, they interfere and thus produce the **spring tides.** When, however, they reach any point in *opposite* phases, they interfere and thus produce the **neap tides.**

Interference is, therefore, employed in physics to denote the combination of waves in any difference of phase whatever. It has been suggested that the so-called tidal waves which occasionally sweep the decks of the Atlantic liners are due to the interference of two trains of running waves.

Summary concerning Waves in Strings

150. 1. Medium is linear.

2. Motion of particles is at right angles to direction in which wave travels.

3. Speed of propagation is independent of the length of the string, the material of the string, and the shape of the wave.

Speed depends only on (1) stretching force, F, and (2) the mass of unit length of the string, D. $V = \sqrt{\frac{F}{D}}$

4. Waves in strings are reflected as are water waves.

5. The ordinary vibrations of stretched strings are stationary waves, produced by interference of incident and reflected trains of waves.

Resonance

151. When one attempts to ring a large church bell, he soon learns that he must pull the rope at certain definite intervals if he wishes to ring the bell with ease.

Any heavy object, such as a stone suspended from a

crane, can be set swinging if one's pushes are properly timed. A heavy rowboat, or small sailboat, moored in quiet water can, without difficulty, be set rocking if one throws his weight first on one side, then on the other, at correct intervals.

What is more, every one knows how to discover the correct interval in each of these cases. By watching the amplitude of vibration, he at once times his efforts so that this amplitude will increase. These are all instances of what is known in physics as **resonance** or **sympathetic vibrations.**

Experiments on Resonance

152. The following experiment should be carefully studied:

Mount two heavy pendulums of any kind upon one common wall bracket or frame, as shown in Fig. 141. Heavy lead disks on $\frac{1}{4}$-in. iron rod, suspended by knife-edges at the top, do very well. The lower end of the rod should be threaded for three inches, so that the bob of the pendulum may be held up by a nut, and so that the period of either pendulum may be slightly varied at will. Paint one pendulum red, the other blue.

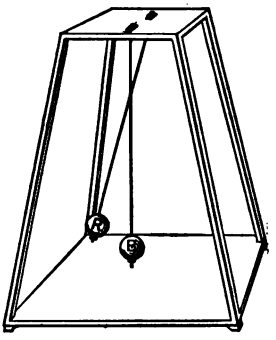


FIG. 141. Illustrating transfer of kinetic energy by resonance.

1. Adjust the pendulums so that they have very nearly the same period. Set the blue one in vibration, leaving the red one at rest. In less than one minute, the red one will pick up sufficient kinetic energy to set itself in vibration with an amplitude not greatly inferior to that of the blue one.

Where did this energy come from? How did it get from the blue pendulum into the red pendulum?

2. Now reverse the order. Set the red pendulum in motion, allowing the blue one to be initially at rest.

3. Take a wooden mallet and tap one of these heavy pendulums in a most irregular way. The result of a dozen such blows is to disturb the pendulum very little.

Now give the same pendulum one half dozen gentle taps, delivering each tap *just as the pendulum passes its position of equilibrium*, going away from you. How do you explain the large amplitude which these six taps produce in the pendulum?

4. Adjust the pendulums so that one is from 8 to 10 % longer than the other. Set the *blue* one in vibration. What is the history of the *red* one for the next five minutes? How do you explain the small amplitude and the complete stoppages of the red one?

These experiments show that one pendulum picks up the vibration of the other only when their periods are very nearly the same. It will be found in every case that **resonance occurs between vibrating bodies only when their periods are approximately equal.**

On the Transfer of Mechanical Energy. Waves and Currents

153. The effects of waves on the seacoast are so well known as to need no description. The energy of waves is here exhibited on a gigantic scale; in some places undermining cliffs, in others, transporting large quantities of sand. **Wave motion is, indeed, one of nature's general methods for transferring energy.**

Every child who has played with a garden hose knows that the jet of water can be used to dig a considerable hole

in the ground. In California these jets are employed on a large scale for mining. The process is known as **hydraulic mining**. Currents of air are employed for running windmills, thus transferring the energy of the wind to the water, which is pumped into a tank, say, in the attic of a house. One may transfer energy to a body by throwing a stone at the body or by firing a shot at it. All of these are simply special kinds of currents.

Currents of matter may be called the second general method which nature employs for transferring energy. The moving matter, in these currents, may be air, water, stones, shot, etc.

Our study of the subjects which follow, Sound, Heat, Electricity, Light, we shall find to be largely a study of the transfer of energy by one or the other of the two general methods, *Currents* and *Waves*.

CHAPTER V

SOUND

154. We have not been in this world very long before we learn to attract the attention of our friends by the use of our voice. At a very little later period we have learned to distinguish between the voices of friends. We can soon produce sounds which are more or less loud according to our wish. The ability to "carry a tune" marks a period when we are able not only to distinguish one note from another as having a different pitch, but also to *produce* notes of different pitch. Not many years later, and we have learned to distinguish the flute from the piano, the piano from the guitar, the guitar from the human voice, and so on. In childhood we discover the echo. In early youth we observe that a distant locomotive sends up a column of white steam some little while before the sound of the whistle reaches our ears; and we infer that it takes time for sound to travel the intervening distance. There is no longer any difficulty in distinguishing, in the popular sense, between a noise and a musical sound; and, when several notes are sounded together, we decide, at once, which combination is harmonious and which is unpleasant to the ear.

Something like the foregoing is a part, at least, of our common "stock in trade" of information on the subject of sound; but the *arrangement* of these facts in our minds is perhaps quite as chaotic as the above presentation of them. The object of the present chapter is to aid the student in rearranging this large, but disordered, heap of

valuable information into a well-ordered whole. This will involve the discovery of many new relations among many familiar facts.

Typical Case

155. Imagine two boys out in the woods, out of sight of one another, yet each within earshot of the other. When one calls and the other hears we have a sort of wireless telegraphy, a process in which there are three essential features, namely:

1. The boy calling — the "sender," so to speak.
2. The air or elastic medium, which we may consider as replacing the wire used in ordinary telegraphy.
3. The listening ear or receiver.

Of these three processes — sending, transmitting, and hearing — it is only the first two that we consider in the science of Physics. The sensation of hearing and the perception of sound are very complex and difficult subjects, studied under the head of Physiology and Psychology.

ANALYSIS

156. That part of the subject which belongs to Physics will be considered in the following order:

1. Evidence for thinking sound a wave motion of the air.
2. Character of waves in air.
3. Graphical representation of waves in air.
4. Distinction between a noise and a musical note.
5. The three distinguishing features of a musical note, { Pitch, Loudness, Quality.
6. Relation of musical tones to each other — the musical scale.
7. Application of preceding principles to musical instruments, { 1. Wind instruments, 2. Stringed instruments, 3. Vibrating membranes.

I. Production of Sound

157. Of the multitudinous sounds which reach our ears during any single day, there are very few which we are not able to *locate* at once. And when we find the source of any sound, we find ourselves in the presence of some vibrating body.

When one string of a guitar is emitting a note, there is no difficulty in telling which one; we have only to look at the strings. A piece of thin steel — say, part of the blade of a hack saw — clamped in a vise gives a note only when set in vibration. Does the sound disappear as the vibrations *die down?* The edges of a tuning fork become hazy in appearance as it begins to emit a note. Why?

Try the following experiment, which was suggested by the late Professor A. M. Mayer.

"Get a match and spreading two of your fingers apart rest it upon both. Hold the tuning fork in the right hand and strike the end of one prong squarely and smartly on the knee or on a piece of thick paper folded over the edge of the table. Now bring the fork up under the match. The instant the match is touched it flies into the air as if knocked away by a sudden blow."

We have already seen that in addition to the vibrating body, there is one other essential feature in the production of every sound; viz. an elastic medium — practically always air — between the vibrating body and the hearer.

The sound of a watch, a bell, or a clock, placed under the receiver of an air pump (Fig. 142), is much diminished when the air is even partially removed. It is very difficult to make the sounding body quite inaudible in a vacuum, for the reason that the body must rest on some kind of support; and this support, being necessarily somewhat elastic, will transmit some vibration to the receiver,

and hence to the outside air. But we are perfectly justified in inferring that, if no elastic medium whatever lay between the source of vibration and the listening ear, no sound would be heard.

For the hearing of a sound, then, three things are always required:

1. A vibrating body, rapidly disturbing its surrounding medium.
2. An elastic medium connecting the vibrating body and the listening ear.
3. A sensitive ear.

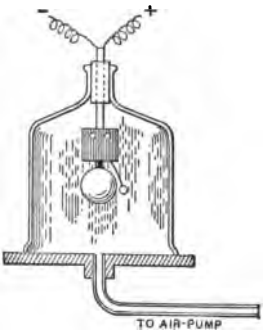


FIG. 142. An electric bell ringing in a vacuum.

II. Character of Waves in Air

One of the shortest cuts to a clear and accurate conception of sound waves is through the following series of experiments. Indeed, the whole philosophy of air waves and the propagation of sound is wrapped up in them.

Disturbance in the Vibrating Body

158. A round brass rod, $\overline{AB}$, from 8 to 10 millimeters in diameter and from 80 to 100 centimeters in length, is clamped to a table in an ordinary vise or by means of an iron clamp, as shown in Fig. 143. A small ivory ball or glass marble or hollow glass sphere is suspended by a string, as indicated, so

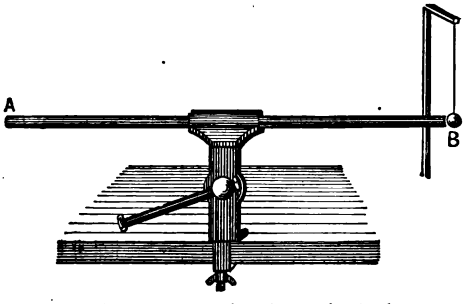


FIG. 143. Experiment to show longitudinal vibrations of brass rod.

as just to touch the end of the rod, *B*. When rubbed with a dry cloth, upon which a little powdered rosin has been sprinkled, the rod will emit a very high musical note. It is not the easiest matter in the world to get this note at first, but every one succeeds after a few trials; a fairly firm grasp and a good steady pull will generally accomplish the end.

When this note sounds, it is observed that the ivory ball rebounds, often several inches, *showing that it has received a blow from the direction AB.*

The rosin on the cloth has evidently taken hold of the rod and elongated it, then let it go, taken hold of it again, suddenly let it go again, and so on. The vibrating body here is evidently the brass rod. The motion of the particles in the rod is evidently in the direction $\overline{AB}$. This is shown by the fact that they are originally displaced in that direction by the rosin and by the fact that the momentum which they communicate to the ball is in the direction $\overline{AB}$. Such vibrations we have (p. 154) called longitudinal.

When the brass rod is elongated by a pull and suddenly let go, would you expect, from analogy with other bodies, that it would go back to its position of equilibrium and stop there, or would you expect its inertia to carry it beyond? Would you consider it necessary, for the production of free vibrations, that the vibrating body have *inertia* as well as *elasticity?*

Are the ends of the vibrating rod nodes or antinodes? The middle point must be a node because it is held fast by the iron clamp.

Character of Disturbance in Air

159. Let us now suppose that the nature of the vibrations in the rod is clearly understood. What sort of disturbance does the rod produce in the air? The displacement of the

particles in the rod was so small and rapid as to be invisible; we were obliged to get our evidence by means of the ivory ball. In air, not only is the disturbance small and rapid, but the whole vibrating mass is invisible; so that, even if the disturbances are large and slow, we do not ordinarily *see* them.

FIG. 144. Showing the compressions and rarefactions which accompany a rifle ball in its flight.

However, Töpler of Dresden has shown that by a certain disposition of apparatus it is quite possible to see waves in air; while Mach of Vienna has succeeded in photographing the stationary waves which accompany a Mauser rifle ball in its flight through the air. These are shown in Fig. 144.

Kundt's Method

160. On account of the invisibility of the air the following method was devised by Professor Kundt* in 1865.

To one end of the brass rod described in § 158 attach a disk of cork, *B*, which fits loosely into a glass tube about 30 millimeters in diameter and a meter in length.

By the aid of a cork and a file, you can, in a moment, make enough cork dust † to lightly cover the bottom of the tube.

At the end, *D*, of the glass tube, fit another easily moving piston; so that the distance between the two pistons, *D* and *B*, can be varied.

* Late Professor of Physics in the University of Berlin. Died in 1893.

† Avoid the lycopodium powder which is sometimes recommended for this purpose. The slightest bit of breath or moist air introduced into the tube will make the dust adhere firmly to the walls of the tube.

A glass rod is more easily excited into longitudinal vibrations than a brass rod, so it may be well to substitute for the brass rod, *AB* (Fig. 145), one of glass.

Such an arrangement is called a **Kundt's tube.**

Set the rod *AB* into longitudinal vibration as before.

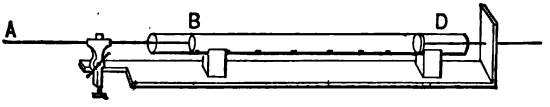


FIG. 145. Kundt's device for studying waves in air.

If the cork piston at *B* is free and does not rub on the walls of the glass tube, the elastic air will be disturbed very much as the ivory ball was.

As the rod elongates, the air between the two pistons must be suddenly compressed, for it has not time to escape — the compression is too quick for that; when the rod contracts, the inclosed air is suddenly rarefied. The

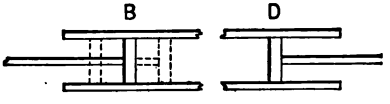


FIG. 146. Showing action of piston in Kundt's tube.

rod thus starts a train of waves down the tube exactly analogous to those which we produced in the long spiral spring (pp. 147, 154). When this train of waves strikes the fixed piston, *D* (Fig. 146), we have all the conditions necessary for reflection. And, if reflection occurs, we shall have a train of waves starting back in the direction from *D* to *B*. We should then expect this train of reflected waves to interfere with the incident train and produce stationary waves with their characteristic nodes and loops.

Sound is a Longitudinal Wave Motion

161. We have then the following evidence for thinking that the phenomena which we have just predicted are the actual phenomena which occur in the tube: —

1. Air is a fluid. It has, therefore, no elasticity of shape. It cannot, therefore, transmit transverse vibrations.

2. There are certain equidistant points in the tube where the cork dust is not at all disturbed. These correspond to the nodes predicted.

3. Between these points of rest there are certain regions of very great disturbance. These correspond to the predicted loops.

4. By slightly adjusting the fixed piston, the nodes and antinodes become much more sharply marked. And this is exactly what one might expect if the air waves had a definite wave length determined by the vibrating rod and *not* by the inclosing glass tube.

5. To these reasons for thinking sound waves longitudinal must be added the very cogent evidence furnished by the manometric flame, which will be considered later, § 165.

From testimony of this kind one is led to think (i) that sound consists of a wave motion in the air, (ii) that the motion of the particles of air is to and fro in the same direction as that in which the sound is traveling, (iii) that this longitudinal wave motion can be reflected, and (iv) that two trains of sound waves may interfere to produce stationary sound waves.

What really travels through the air, then, is a series of compressions, alternating with a series of rarefactions. This is fairly represented in Fig. 147 for the tuning fork and in Fig. 148 for Kundt's tube.

Just how far apart these successive rarefactions and compressions are, depends entirely upon how fast the piston works. Every time the piston moves forward, a compression occurs; this compression starts out on its journey, traveling at a definite rate. When the piston

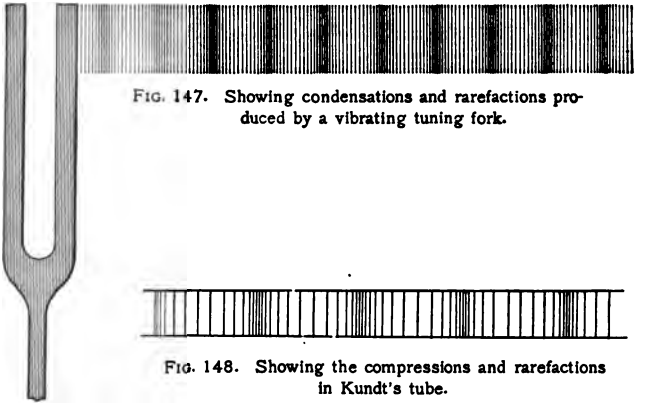


FIG. 147. Showing condensations and rarefactions produced by a vibrating tuning fork.

FIG. 148. Showing the compressions and rarefactions in Kundt's tube.

goes back, a rarefaction starts out. These disturbances travel in the same direction and with the same speed. The distance through which a compression (or a rarefaction) travels while the piston makes one complete oscillation to *and* fro is called a wave length. And this corresponds exactly with the **definition of wave length** given above (p. 150); viz., **the distance between any particle and its nearest neighbor that moves in the same phase.**

Sound Waves in Three-dimensional Space

162. Hitherto we have been considering waves in air as they travel through tubes, say through a speaking tube; but in ordinary conversation we stand in an open space, surrounded by air on all sides; our voice is heard by those at our back, also by those in front, and by those at our side. The waves produced by our voice evidently

travel away from us in all directions. As the pebble dropped into smooth water sends out rings of disturbance, our voice sends out spheres of disturbance. These expanding spheres are propagated in the same manner as waves in tubes; they travel at the same rate; the displacement of the air particles is longitudinal. These spherical waves are, therefore, fairly represented in Fig. 149.

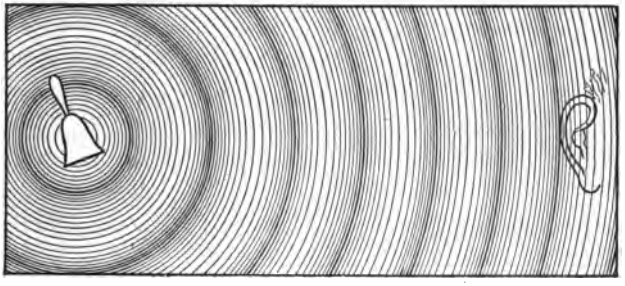


FIG. 149. Sound waves in air.

As an illustration of a wave motion propagated through a volume (*i.e.* a three-dimensional wave) we may mention the remarkable explosion which occurred at Krakatoa on the 26th and 27th of August, 1883.

In this volcanic eruption a great column of gas was shot up to a height of perhaps 20,000 feet. When this great mountain of air and other gases settled down, it sent out an air wave through the rest of the earth's atmosphere. This air wave expanded in circles. It girdled the whole earth, converging at the antipode of Krakatoa. Passing over any point on the earth's surface it compressed the underlying air; the height of the barometer was accordingly increased; the wave of depression following had just the opposite effect. In this way all the recording barometers on the earth were affected. And by such means this great disturbance was followed through three

trips from Krakatoa, near the island of Sumatra, to the antipodes. The entire circuit of the globe was performed in the almost incredible interval of 36 hours.

Compute the speed of this wave and compare it with the speed of sound as given in the immediately following pages.

Speed of Sound

163. Even primitive man must have observed the interval of time which elapses between the fall of a distant tree and the hearing of the sound. But the first accurate measurement of the speed of sound was made under the direction of the French Academy of Sciences in 1738, by measuring the time elapsed between the firing of a cannon and the hearing of the report. Many determinations have since been made with the conclusion that sound travels in air at the rate of about 332 meters per second; that is, about 600 miles an hour, or, say, 15 times as fast as an express train.

The following table shows the speed of sound in some of the more important substances:

SPEED OF SOUND

SUBSTANCE	TEMP. C	SPEED IN METERS PER SECOND	SPEED IN FEET PER SECOND
Air	0°	332	1090
Brass		3500	11480
Carbon dioxide . . .	0	261.6	858
Glass		5000 to 6000	16410 to 19690
Illuminating Gas . . .	0	490.4	1609
Iron and Soft Steel . .		5000	16410
Water	13.7°	1437	4714

Smithsonian Tables.

Experiment shows that the speed of sound in air increases at the rate of about 0.6 m. per second for each degree Centigrade through which the temperature rises.

III. Graphical Representation of Sound Waves

164. As to the proper mode of representing water waves, there can be no question. We have only to draw a vertical section through the body of water at right angles

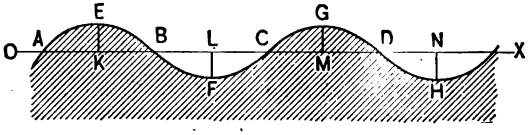


Fig. 150.

to the crest of the wave. Such a section is shown in Fig. 150. The distance of any particle above or below the level surface $\overline{OX}$ measures the displacement of the particle.

Nor can there be any hesitation as to the proper way of representing waves in strings. Imagine the vibrating string at any instant to lose its flexibility: let it instantaneously become a rigid body, and we have at once a picture of the wave. Such a diagram is shown in Fig. 134.

The vertical distance, KE, of any particle from the position of equilibrium, $\overline{OX}$, in Fig. 150, gives the displacement of that particle. This is the distance which we have (p. 4) called the ordinate of the curve. From such a curve we can instantly point out those particles which are in the same phase, or in opposite phase. Point out on such a curve a series of points which differ in phase by 180°; and another series which differ in phase by 90°.

But suppose that we are dealing with a compression running along a spiral spring. Here the displacements of the particles all lie in the direction of the stretched spring, *i.e.* along the line $\overline{OX}$, which represents the position of

equilibrium. How, then, shall we draw these displacements? And the same question arises when we attempt to picture a sound wave.

The whole question is this: How shall we represent the state of disturbance at any, and every, point in the air when a train of waves is passing through it?

The method is as follows: We agree to represent all waves in one way, viz. after the manner of the transverse wave in Figs. 134 or 136. To every particle of air in the line along which the wave travels corresponds a point on the curve. But when the wave is *longitudinal*, the vertical distance of any point on the curve from the line $\overline{OX}$

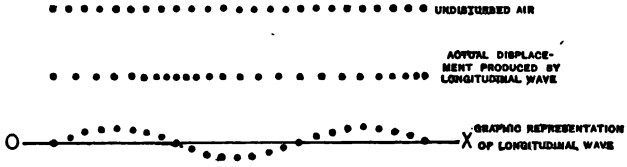


FIG. 151.

(Fig. 150) represents the displacement of the corresponding particle *along* the line $\overline{OX}$. **Displacements, then, are represented by lines drawn at right angles to the actual displacement.**

Longitudinal waves are the only ones that offer any difficulty in representation. Hereafter we shall draw *all* waves as transverse; and it will be understood from the context or the title of the figure whether the wave is transverse or longitudinal.

The Manometric Flame

165. So far as concerns a picture of the disturbances which sound waves produce in the air, nothing can surpass in beauty and simplicity the **manometric flame**, which was invented and perfected by Rudolph König, of

Paris. The construction of the apparatus will be clear on consideration of Fig. 152.

Two blocks of wood, *R* and *S*, are bored out. Between them is clamped a diaphragm of very thin goldbeater's skin. This diaphragm divides the whole apparatus into

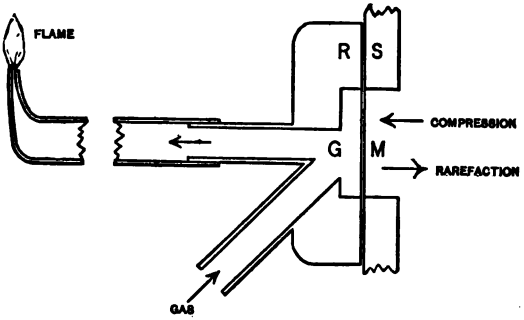


FIG. 152. König's device to show variation of pressure produced by sound waves.

two parts. On the left-hand side of the diaphragm illuminating gas is admitted, as indicated by the arrows, and so long as the diaphragm is undisturbed, the flame is apparently undisturbed. But if the disturbances of the flame were very rapid, as is the case with acoustical disturbances generally, we might not be able to detect by the eye any changes in the flame. Accordingly, a rotating mirror is placed near the flame (Fig. 153). A crank enables the observer to give this mirror a moderate angular speed, say two or three turns a second. For every new position which the mirror assumes the image of the flame takes a new position. The effect of rotation on the mirror, then, is to separate the images of the flame at successive instants.

The space on the right of the diaphragm, *M*, is one end of a speaking-tube, a sort of *cul de sac*, into which all kinds of sound waves can be sent for examination and study.

The effect of a sound wave (if our previously formed conception of sound waves be correct) upon the diaphragm will be to drive it alternately forward and backward. When a compression arrives at the diaphragm, the membrane will be thrust slightly inward. The gas space, *G*, will suddenly be made smaller in volume; the pressure of the gas will as suddenly rise; the flame will immedi-

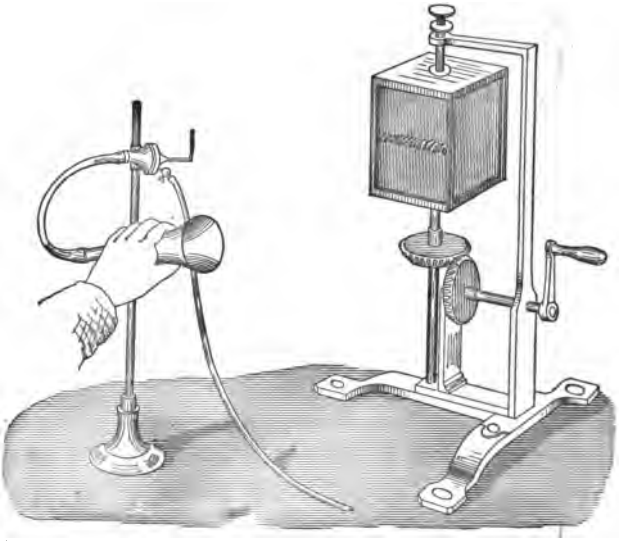


FIG. 153. Illustrating how a manometric flame is examined by means of a rotating mirror.

ately become higher — just as if more pressure had been put on at the gas works.

But this compression in the speaking-tube will be followed by a rarefaction. The pressure outward diminishes, the membrane moves outward, the pressure in the gas chamber, *G*, suddenly diminishes; the flame as suddenly diminishes in height.

The essential features of the instrument are, then:

1. A membrane which responds at once to any displacement of the air particles adjoining it, *i.e.* to any wave of compression or rarefaction.

2. A gas flame so arranged as to respond to every motion of the membrane.

3. A mirror that can be rotated so as to separate the successive images of the flame; and thus allow us to detect the variations of pressure at the membrane due to any disturbance of the air.

Since, now, the height of the flame measures in a general way the displacement of the particles adjoining the membrane, we need only draw an outline of the image of the flame in the rotating mirror, and we have at once a graphical representation of a sound wave. Such are the following diagrams. (See Fig. 154.) They merely record the variations of pressure produced by three different trains of waves at the membrane of the manometric flame. Such a flame is simply a quick-acting barometer, responding to changes of pressure which occur at rates even as high as several hundred per second.

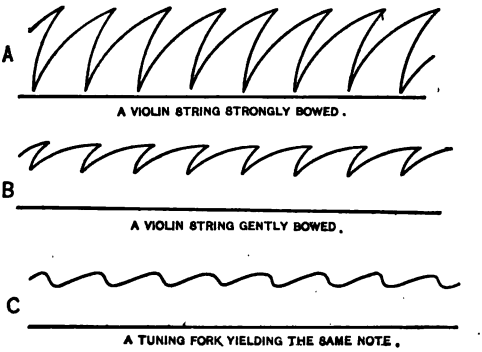


FIG. 154.

IV. Distinction between a Noise and a Musical Sound

166. Having mastered a method for clearly representing sound waves, we are, for the first time, in a position to distinguish between noises and musical tones. The differ-

ence, *in sensation*, between these two is something with which we are very early familiar.

A wagon rumbling over cobblestones, the splash of water, footsteps on the pavement, the slam of a door, are all instances of those irregular, fitful sounds which we call **noises.** The note yielded by a single wire of a piano is a **musical tone.** The same is true of a guitar string, a flute,

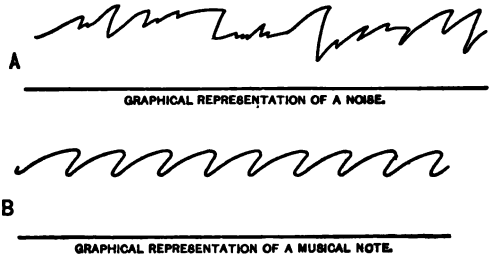


FIG. 155.

an organ pipe. The sensation produced by a musical tone is uniform so long as the note continues.

Corresponding to these two different sensations, the manometric flame shows two different kinds of disturbances in the air. These are exhibited fairly well in Fig. 155. The noise is made up of many small parts of trains of waves, all jumbled together. But the flame shows that a musical note is produced by *one* train of uniform waves.

It shows also that vowels, as employed by singers, are musical tones; while consonants are essentially brief noises.

"**The sensation of a musical tone is due to a rapid periodic motion of a sonorous body; the sensation of a noise to a non-periodic motion.**" — HELMHOLTZ, "Sensations of Tone," p. 8.

V. Three Features by which Musical Tones are distinguished from Each Other

A. Loudness

167. When the key of a piano is struck gently and again with a hard blow, we recognize that the two notes thus produced are alike except that the second is *louder* than the first. The vibrations of a guitar string when first plucked are sufficiently large for us to see them; but as the vibrations diminish in amplitude, we observe that the note diminishes in *loudness.*

When a note is sung into the tube of a manometric flame, it is at once evident that the indentations in the ribbon of flame are deeper in proportion as the note is louder. (See Fig. 156.) That is, the louder the sound, the greater the displacement of the air particles which lie against the membrane and the greater the energy which the membrane is handing on to the adjacent air.

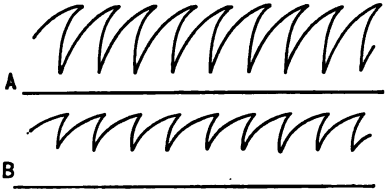


Fig. 156. *A* and *B* are two notes alike in all respects except that *A* is louder than *B*.

Fig. 157. *A* and *B* are two strings alike in all respects except that *A* is vibrating with larger amplitude and energy and hence greater loudness than *B*.

From these, and similar experiments, it is concluded, therefore, that **the loudness, or intensity, of a sound increases or diminishes as the amplitude of the vibrating air particles at the ear increases or diminishes.**

B. Pitch

168. On clamping a piece of steel or brass spring, say a piece of clock spring, in a vise, one observes not only that the spring vibrates more rapidly as it is made shorter, but also that *it emits a higher note.* (See Fig. 158.)

In the case of a rubber band stretched between two nails driven into a board there is no doubt but that it vibrates more rapidly the more it is stretched; we observe also that the note it emits is higher and higher the more it is stretched.

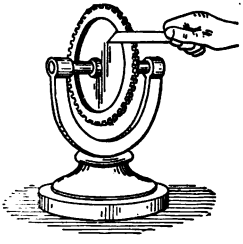


FIG. 158. Illustrating the fact that pitch varies with frequency.

From such experience we might suspect the pitch of a note to depend upon the frequency (p. 157) of the vibrating body. But we need more direct and conclusive evidence to thoroughly establish such a conclusion. A card held against a rotating toothed wheel will furnish us just such evidence. Here we compel the card to make one vibration every time it passes a tooth. The frequency of the vibrating card is the number of teeth passed per second. When the wheel slows up, the pitch of the note produced by the card falls; when the wheel increases in speed, the pitch of the note rises.

If still further evidence be desired, try the following experiment:

Into the speaking tube of a manometric flame sound two notes of different pitch in rapid succession. Or, what is better still, use two manometric flames placed near each other and viewed in the same rotating mirror. It is then easily seen that when the flames are excited by notes of the same pitch, the notches in the image of each flame are the same distance apart.

But if either flame be excited by a note of higher pitch, the notches on this flame are closer together. And if one note be an octave above another, we see that the notches are exactly twice as close together in the high note as in the low. (See Fig. 159.)

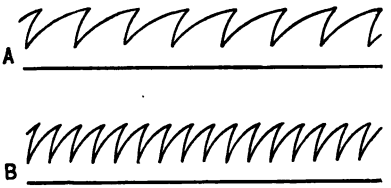


FIG. 159. *A* and *B* are two notes alike in all respects except pitch.

Such evidence leads us to think that **the pitch of a note depends simply upon the frequency of the vibrating body which produces it**; or, what is the same thing, upon the period of the vibrating body.

The frequency of a vibrating body is simply the number of vibrations it makes per second; or if its period is T, its frequency, n, is $\frac{1}{T}$.

The numerical value of any pitch is the frequency, n.

A tuning fork which makes 300 vibrations per second is said to have "a pitch of 300."

C. Quality. Fundamental. Overtone

169. But two notes may have the same pitch, and may have the same loudness, and yet be quite different. Two tenor voices singing the same note with equal loudness may be easily distinguished; for, as we are accustomed to say, they have different **qualities.** Strike the same note upon a piano and upon an organ — let them be equally loud; no one finds any difficulty in distinguishing the piano from the organ. Two of our friends may converse in the same key and with equally loud voices, but we have not the slightest hesitation in telling their voices apart, even though they stand at our back or perhaps in the dark. Their voices have a different quality. Differences

of quality are most strikingly illustrated in the various instruments of the orchestra, where we recognize with great ease just which instruments are being used. A guitar string when plucked at its middle emits a note of different character from that of the same string when plucked near the end.

How shall we explain these differences which are so familiar to every one? Difference of amplitude produces difference of loudness. Difference of frequency produces difference of pitch. But what produces difference of quality?

The most direct answer to this question is obtained by means of the manometric flame. Into the speaking-tube of such a flame pronounce, as nearly as possible with the same pitch and loudness, each of the vowels of the alphabet. It is observed at once that the ribbon of flame seen in the mirror is different in each case; to each particular vowel corresponds a wave of different shape. The notches in the ribbon are not the same for any two vowels. **It is, then, the form of the wave which determines the quality of any note.**

In Fig. 160 is shown the appearance of two flames each excited by a note of the same frequency and the same loudness, but of different quality. In this case we see not only that the form of the wave in *A* is different from that in *B*, but we see just *how* it differs.

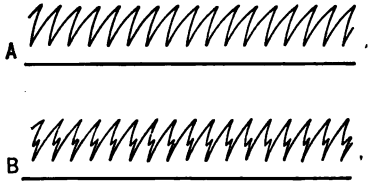


FIG. 160. Two notes of the same pitch and loudness, but of different quality.

In *B* there is present a small wave having twice the frequency of that of *A*. So that *A* represents the fundamental, while *B* represents a note in which both the fundamental *and* the octave are present.

In the case of a piano wire the quality (wave form) is determined largely by the presence of vibrations which are more rapid than the fundamental, **the fundamental being defined as the lowest note which a body can give out.** These other and higher notes are called **overtones** or **harmonics** (Fig. 161).

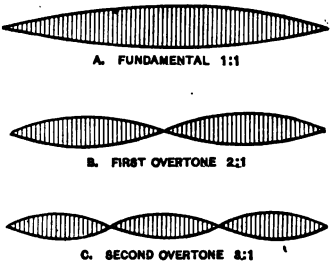


FIG. 161.

In the case of a string which is emitting the first overtone as well as the fundamental, the actual appearance of the string is more nearly represented by compounding *A* and *B* of Fig. 161 as shown in Fig. 162.

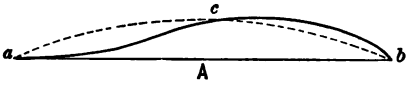


FIG. 162. Appearance, at one instant, of string emitting fundamental and octave.

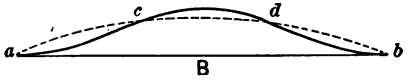


FIG. 163. Appearance, at one instant, of string emitting fundamental and second overtone.

Or if a string is emitting its fundamental and its second overtone this state of affairs will be represented by Fig. 163.

Beats

170. When two notes differ slightly in pitch, they have slightly different wave lengths in air. The consequence is that they reach our ears with a difference of phase which is constantly changing. At one instant, they interfere in such a way as *almost* to produce silence; at the next instant, they combine in such a way as to produce a sound louder than that of either alone.

Most people are familiar with the peculiar rise and fall

that attends the sound of a large bell after the clapper has struck it for the last time. This is due to the interference of two segments of the bell, such as *A* and *B* in Fig. 164. The same phenomenon is easily shown with a glass bell jar or an iron ring.

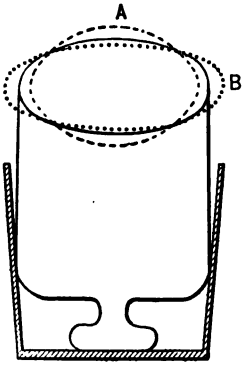


FIG. 164. A vibrating bell jar: really equivalent to four different vibrating bodies.

Two tones which interfere in this way to produce this alternate rise and fall of intensity are said to **beat.** These alternations of loudness are called **beats.**

This phenomenon is most readily shown by means of two tuning forks which are in unison. If the forks are in exact unison to begin with, a little wax placed on the prong of either one will slow it up; for the wax adds to the inertia of the fork, and does not add to its elastic force. Consequently, whatever difference of phase may exist at any instant between the two forks, this difference will continuously change, and, hence, will follow a series of beats. The more wax placed on the prong, the more rapid the beats. How do you explain this? Two guitar strings are easily adjusted to unison. Tighten either one ever so little and they produce beats.

In Fig. 165 two waves which "beat" are represented by the dotted line and by the fine solid line. The resultant

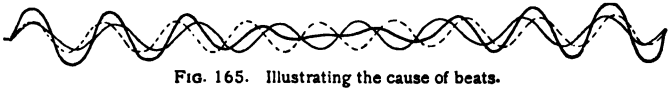


FIG. 165. Illustrating the cause of beats.

wave — the effect produced at the ear — is represented by the heavy solid line. Note how its amplitude varies.

It is easily seen from the theory of waves in general that the number of beats per second is numerically equal to the difference in frequency between the notes beating. For the interval between two successive beats is simply the time taken for one train of waves at any point, say, at the listening ear, to gain 360° in phase over the other train. Each train of waves travels at the same speed; but as these trains pass any one point their difference of phase is continually changing. If one note makes 500 vibrations per second and another 502 per second, they will be in the same phase twice each second, and in opposite phases twice each second. They will therefore beat twice each second. **In general, if the frequency of one note is n and of another n', the number of beats per second will be $n - n'$.**

Helmholtz likened the effect of beats on the human ear to that of a flickering light upon the eye. The sensation is quite unpleasant. When, however, the beats become *very* rapid, they cease to be disagreeable, — just as an electric light alternating in intensity ceases to be painful when the alternations are very rapid. Helmholtz showed that beats are most disagreeable when they occur at the rate of 30 per second; but they can be detected even when they run as high as 132 per second.

VI. Relation of Tones to Each Other. The Musical Scale

Up to the present moment we have been studying the nature of a single train of waves in the air, the propagation of a single musical tone. We now proceed to consider some of the relations of musical tones to each other. Here we find ourselves already in the domain of **Music.** What we seek is, in short, the scientific basis of music.

Definition of Musical Interval

171. The interval between two notes is defined as the ratio of their frequencies. Two notes having the same pitch, whatever their quality or intensity, are said to be in **unison**; the interval in this case is not zero, but unity. If two notes have the ratio 2 : 1, the interval is said to be an

Octave

A note whose pitch (frequency) is 400 is, then, an octave *above* a note whose pitch is 200. A note whose pitch is 100 is an octave *below* one whose pitch is 200. The impressions produced upon the human ear by a note and its octave are strikingly similar. So alike, indeed, are the two sensations that many fairly well trained musicians sometimes mistake the one for the other.

The Major Chord

172. Still more remarkable is the following fact concerning the human ear — a fact which has been well known for more than two thousand years, a fact which is fundamental in all music, but which, up to the present century, had baffled all attempts at explanation.

The fact is that any three tones whose frequencies are as 4 : 5 : 6 always make a pleasing impression upon the ear. This combination of notes is known as the **major triad**; or, taken together with the octave of the fundamental, is called the **major chord**. The universal judgment is that three such notes, whatever their frequencies, produce harmony when sounded together.

The explanation of this fact we owe to Helmholtz: for it was he who first showed that the reason notes whose frequencies bear the simple ratios 4 : 5 : 6 are always pleasant is that neither between the fundamentals, nor between the

fundamentals and the overtones, nor between the overtones, themselves, are there any disagreeable beats.

The Major Scale

173. The notes which make up the octave are denoted by the letters of the alphabet, and are built up out of three major triads, as follows:

$$C : E : \quad G = 4 : 5 : 6.$$
$$F : A : 2\ C = 4 : 5 : 6.$$
$$G : B : 2\ D = 4 : 5 : 6.$$

Let us call the lowest note of the octave C, and denote its frequency by "m"; these proportions then amount to a series of equations which we may solve to find the frequency of any other note in the octave. Thus,

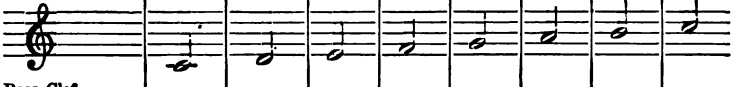
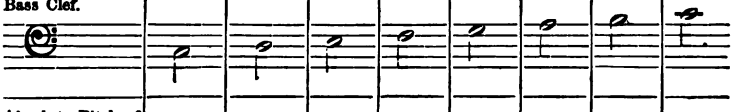
$$C : E = 4 : 5. \qquad \therefore E = \tfrac{5}{4} m.$$
$$C : G = 4 : 6. \qquad \therefore G = \tfrac{3}{2} m.$$
$$F : 2\ C = 4 : 6. \qquad \therefore F = \tfrac{4}{3} m.$$

In like manner, we compute the frequencies and the intervals for the entire octave.

The following table indicates also the ordinary musical notation, a very beautiful form of shorthand. The character employed tells the reader the time for which any note is to be sounded, while the vertical position of the note among the lines and spaces gives its pitch.

It can here be added only that the ***C* represented by the first line above the bass is identical with the *C* on the first line below the treble.** This note is called **Middle *C*** on the piano; its frequency is about 259.

TABLE OF MUSICAL NOMENCLATURES

NAME OF NOTE.	C	D	E	F	G	A	B	2 C
Frequency in terms of fundamental.	m	$\frac{9}{8}\,m$	$\frac{5}{4}\,m$	$\frac{4}{3}\,m$	$\frac{3}{2}\,m$	$\frac{5}{3}\,m$	$\frac{15}{8}\,m$	$2\,m$
Intervals.	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	
Name of note in vocal music.	do	re	mi	fa	sol	la	si	do
Treble Clef.								
Bass Clef.								
Absolute Pitch of treble clef as used in international Scale.	259	290	326	345	388	435	488	517
Scientific Scale	256	288	320	341	384	427	480	512

VII. APPLICATION OF PRECEDING PRINCIPLES TO VARIOUS INSTRUMENTS

Wind Instruments

174. Of these the organ pipe is perhaps the simplest and most typical. There are two kinds, *open* and *closed.* In each of them two trains of waves interfere to produce stationary waves. By an **open pipe** is meant one which is not closed at either end. By a **closed pipe** is meant one which is closed at one end only. These two types are shown in Fig. 166.

One train of waves is produced by blowing against the lip, *l*, of the pipe; another train is produced by reflection from the upper end of the pipe.

In the case of the open pipe, reflection occurs when the incident train strikes the quiet air at the end of the pipe; in the case of the closed pipe, reflection occurs when the wave strikes against the wooden end of the pipe. In either case, the whole behavior of the air in the organ pipe is exactly that of a small portion of the air in a Kundt's tube (p. 170).

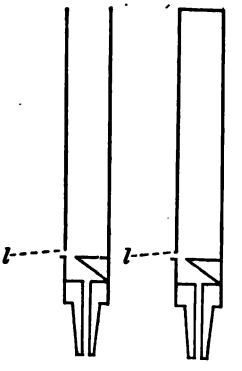


FIG. 166. Sections of open and closed organ pipes.

In the case of the open organ pipe, it is evident that each end is an antinode; *i.e.* a place where no compression occurs, but where great motion is always possible. Consequently, an open organ pipe is simply that portion of a Kundt's tube which lies between two consecutive antinodes, as shown in Fig. 167, where *N* denotes a node and *A* an antinode. It is evident, therefore, that **the total length of an open pipe is one half the wave length of its fundamental.**

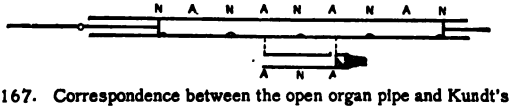


FIG. 167. Correspondence between the open organ pipe and Kundt's tube.

In the case of the closed pipe, no longitudinal motion of the air is possible at the stopped end; consequently, this end must be a node, while the lower (*i.e* the open) end is an antinode. The correspondence is indicated by Fig. 168.

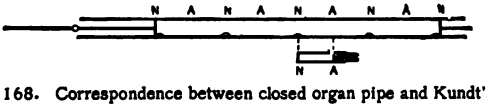


FIG. 168. Correspondence between closed organ pipe and Kundt's tube.

It is at once evident that **the length of a closed pipe is one quarter of a wave length of its fundamental.**

To this class of instruments belong also the **flute**, the **clarinet**, the **cornet**, and the **trombone**. In the first three of these instruments the vibrating column of air is made to break up, by means of tone holes, to yield the fundamental or overtones at the will of the player. The opening of a hole at any point on the tube insures an antinode at that point. In the trombone the player controls the pitch by varying the *length* of the air column.

Stringed Instruments

175. The **guitar** is an instrument of six strings. The range of the instrument is about three octaves. The lowest string yields the note E, the highest open string $4\,E$, or when held at the middle $8\,E$. Three methods of varying the pitch are illustrated in the guitar:

1. To vary the pitch, n, of any one string a screw is employed to change the stretching force. As the tension increases, the force restoring the string to its position of equilibrium becomes greater. The frequency therefore increases.

2. To pass from one note to another on the same string, "frets" are employed. The finger holds the string down over the fret and thus changes the length of the string L. The shorter the string the higher the pitch and hence **the frequency, n, is inversely as the length of the string.**

$$V = nl = 2\,nL = \text{a constant.} \quad \therefore n \propto \frac{1}{L}.$$

3. In passing from one string to another, the pitch of the lower strings is diminished by making them heavier. This is accomplished by wrapping them with wire. The strings would be too clumsy and too rigid if the same mass were given to the string by use of catgut.

The wrapping of wire adds nothing to the restoring

force, but does increase the mass to be moved, and hence diminishes the frequency.

In the **violin** the length of the string is no longer determined by the position of the fret, but is under the entire control of the player. The violin is very responsive to the will and to the skill of the player.

The **piano** is an instrument having a scale of 88 notes. Each note is produced by a felt hammer striking a set of strings. The sound thus produced is reënforced by the sounding board. The instrument is tuned by adjusting the tension of the strings.

Vibrating Membranes. The Human Voice

176. Of this class, the **drum** is the well-known example. But by all odds the most interesting and most marvelous instrument of this, or indeed of any class, is that which produces the **human voice.** Here, in general, three factors enter:

(*a*) The vibrations of the so-called vocal cords.
(*b*) The vibrations of the tongue.
(*c*) The vibrations of the lips.

The vocal cords consist of a pair of membranes situated one on each side of the throat (larynx). By changing the muscular tension on these one easily changes the pitch of his voice. By altering the form of the mouth one easily changes the overtones, and hence the quality of the tone. In this manner the various vowel sounds are produced.

PROBLEMS

1. Approaching a large building at night, I stamp my foot on the pavement and 0.6 sec. later I hear the echo. How far away is the building?

2. What is the pitch of sound whose wave length *in air* is 33.2 cm.?

3. Explain how the violinist varies the pitch of any one string by placing his finger upon it.

4. A visiting card is held against a toothed wheel which is rotating the rate of 120 revolutions per minute. The wheel contains 64 teeth. What is the pitch of the note emitted by the visiting card?

5. A Kundt's tube is filled with hydrogen, and it is then found that the distance between the successive dust heaps is 3.8 times as great as when the same note is sounded in the tube filled with air. From this compute the speed of sound in hydrogen. *Ans.* 1261.6 m. per second.

6. The brass rod used to excite a certain Kundt's tube is just 1 m. long; the dust heaps which it produces in air are 99 mm. apart. What is the speed of sound in brass? *Ans.* 3353 m. per second.

7. An open organ pipe 166 cm. long will yield a note of what pitch?

8. A closed organ pipe 62 cm. long will yield what note?

9. Two tuning forks are sounded together. One has a frequency of 128.25 per second; the other vibrates at the rate of 128.75 per second. How many beats per minute will be heard?

10. Two telegraph sounders are placed on the same electric circuit so that they click each five times a second. One of these sounders is fixed; the other I can carry off to some distance. How far away shall I have to take the movable one in order to hear its nth click at the same instant that I hear the $(n + 1)$th click of the fixed sounder? This is the "unison" method for measuring the speed of sound. *Ans.* 66.4 m.

11. One sound is transmitted through water to a distance of one mile; another sound is carried through the iron portion of a gas main to a distance of 4 miles. Which of these two requires the longer time?

12. How far will the sound travel through the illuminating gas in a gas main while it travels 4 miles through the wall of the main itself?

13. Find the frequency of a note which is two octaves above middle C on the piano.

14. The pitch of a certain guitar string when held down over a certain fret is 440. Find its pitch when held down over a fret which is only $\frac{2}{3}$ as far away from the lower end of the string.

References

TYNDALL — *Lectures on Sound* (Appleton). Price, $2.00.

BLASERNA — *Theory of Sound.* International Scientific Series. Price, $1.50.

BARNES — *Practical Acoustics.* The third volume of Stewart and Gee's *Practical Physics* (Macmillan). Price, $1.10.

Encyclopedia Britannica — Art. "Music." Section on "Scientific Basis of Music," pp. 102–106, by Professor Bosanquet.

CHAPTER VI

HEAT

177. In spite of the fact that heat is something which we can neither see nor hear, we are, nevertheless, all exceedingly familiar with the sensations described by the words, *hot*, *warm*, *cool*, and *cold;* while the other thermal effects with which the average person is acquainted would fill a volume. These include everything from the hatching of an egg to the running of a locomotive. But, so far as is known, heat has no weight; indeed, as we shall presently see, there is strong evidence for thinking that it is not a substance at all, but rather a form of energy. What, now, is the method of dealing with this invisible, intangible something called *heat?* What is the nature of the evidence for thinking it a kind of energy and *not* a kind of matter? To answer these questions, and not to introduce a new catalogue of phenomena, is the object of the present chapter.

We shall consider the subject under the following sections:

I. Distinction between Heat and Temperature.
II. Measurement of Temperature.
III. Transfer of Heat.
IV. Some Effects of Heat.
V. Quantity of Heat.
VI. Nature of Heat.
VII. Heat Engines.

I. Distinction between Heat and Temperature

178. Most people will agree that a cup of water taken from a boiling kettle has, at the instant it is dipped out, the same temperature as that of the boiling water. Most people will likewise agree that for heating purposes the large kettle is much more efficient than the small cup of water. A water bag or a water bottle that holds a gallon will give out more heat and give it out longer, other things being equal, than one which holds only a pint. If the temperatures of the two vessels of water are the same, the larger is said to contain the more heat. If a bath tub be half filled with cold water, one can heat it more by pouring in a gallon of boiling water than by putting in a quart of boiling water; for, although the temperatures are the same, the gallon contains more heat than the quart. These simple facts are cited merely to show that popular notions concerning the distinction between heat and temperature are perfectly clear. They are also correct. But before the word *temperature* can be admitted to the rather select vocabulary of physics, it must be defined in unmistakable English. This we now proceed to do.

Imagine three vessels of water, A, B, C (Fig. 169), each containing a different quantity of water. If A and B are placed side by side, in contact, and B thereby gains heat,—be it ever so little,—A has imparted heat to B; and A is said to have a higher temperature than B.

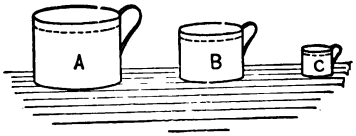


Fig. 169.

Put C in contact with B; if B thereby loses heat,—be

it ever so little,—heat has passed into C; and C is said to be at a lower temperature than B.

When, in general, one body is placed in contact with another, **the difference of temperature between the bodies is that which determines which way the heat flows.** If the heat flows from A to B, A has the higher temperature; but if the heat flows from B to A, then B has the higher temperature.

Now heat the water in the smaller vessel, C, nearly to the boiling point, while the water in A has about the temperature of the room. Place a thermometer in each vessel; and then put the vessel C into A, without allowing the liquids to mix. Does the temperature of C fall as much as that of A rises?

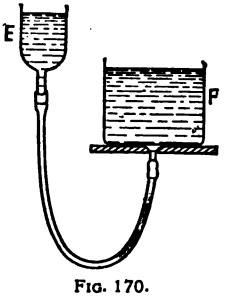


FIG. 170.

Consider two vessels of water, E and F (Fig. 170), connected by a rubber tube. The water does not necessarily flow from the large to the small vessel but does always flow from the higher to the lower level.

Point out clearly the analogy between temperature and level; also between change of temperature and change of level.

Definitions

179. The following definitions will now be clear to every one who has mastered the foregoing facts:

"**Definition of Temperature.** — *The temperature of a body is its thermal state considered with reference to its power of communicating heat to other bodies.*

"**Definition of Higher and Lower Temperature.** — *If when two bodies are placed in thermal communication, one of the*

bodies loses heat, and the other gains heat, that body which gives out heat is said to have a higher temperature than that which receives heat from it.

"**Corollary.** — *If when two bodies are placed in thermal communication neither of them loses or gains heat, the two bodies are said to have equal temperatures or the same temperature. The two bodies are then said to be in thermal equilibrium.*

"**Law of Equal Temperatures.** — *Bodies whose temperatures are equal to that of the same body have themselves equal temperatures.*" — MAXWELL, "Theory of Heat," Ch. II.

The point in this definition of temperature which calls for emphasis is the fact that temperature is a state of a body.

In estimating temperatures, the student must be warned against drawing incorrect inferences from his sensations. In judging the temperature of the atmosphere, it is well known that our estimate is greatly influenced by any wind that may be blowing and by the presence of moisture in the air. If the hand be dipped into hot water and immediately afterward into hydrant water, the hydrant water will feel distinctly cold. But if a piece of ice be held in the hand for a moment, the same hydrant water will immediately after feel distinctly warm.

The indications of thermometers such as are usually employed in the laboratory are much more independent of their previous history than are the sensations of the human hand.

II. MEASUREMENT OF TEMPERATURE. THERMOMETRY

180. The mercurial thermometer is an instrument which belongs quite as much to the household as to the physical laboratory. But the principles upon which it is constructed are not so well known as the instrument itself.

Since heat itself is invisible and intangible, and can be perceived only through its effects upon bodies, we must employ some one of these effects for the measurement of heat. For this purpose we have chosen *change in size*, which always accompanies a change in temperature. We might take a brass rod, 10 cm. long, and then define one degree as that difference in temperature which is required to lengthen the brass rod by $\frac{1}{1000}$ mm. Such an instrument would be portable and would give the same reading for the same temperature; but it would be impossible to measure such a minute elongation except by means of elaborate apparatus.

If a glass bulb fitted with a hollow stem were partially filled with water, we might define one degree as that difference of temperature which would increase the volume of water by $\frac{1}{1000}$ of itself. But, unfortunately, water does not always *increase* in volume as its temperature rises. Moreover, each particular volume of the water does not correspond to one particular temperature. As we shall see presently, a body of water at 6° has the same volume as at 2°. A water thermometer would, therefore, be ambiguous in its readings. Hence water is never used for making thermometers.

But there is one substance, viz. mercury, which has shown itself eminently adapted for use in thermometers. Among the advantages of mercury may be mentioned the following:

(*a*) It is easily prepared in a pure state.

(*b*) It does not wet glass or stick to it.

(*c*) It expands rapidly with rise of temperature, so that its changes in volume are easily read.

(*d*) To each particular volume corresponds a definite temperature.

(*e*) It does not freeze except at temperatures compara-

tively low, and does not boil except at temperatures comparatively high.

(*f*) Compared with water and with most solutions, it requires a very small amount of heat to raise its temperature through a given range.

(*g*) It is a good conductor of heat and so responds quickly to change of temperature.

The indications of temperature which are given by the mercurial thermometer hinge upon the fact that mercury expands more than glass. If, therefore, a glass tube having a bulb blown at one end be partially filled with mercury and immersed in a bath at higher temperature than its own, the mercury will rise in the tube. If the instrument is immersed in a bath of lower temperature, heat will flow from the mercury to the bath, and the mercury will descend in the tube. When the mercury is neither rising nor falling, it is fair to suppose that the thermometer is at the same temperature as the bath, or, at least, the part of the bath immediately about the thermometer.

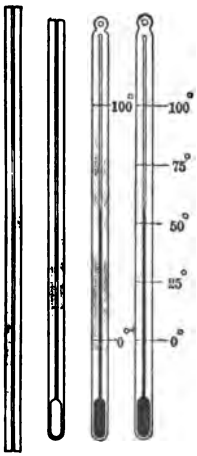


FIG. 171. Showing four stages in the manufacture of a mercurial thermometer.

Manufacture of a Thermometer

181. The steps in the process are as follows (Fig. 171):

1. The selection of a piece of thick-walled capillary tubing of very uniform bore.

2. A bulb is blown on the end of this tube.

3. The bulb is filled with mercury. It is then highly heated and "sealed off."

4. The tube of the thermometer is graduated. This last step is one which demands further consideration.

182. Fixed Points of a Thermometer.

It so happens that there are two temperatures which can be easily produced and reproduced at almost any place and at any time: one of them is the melting point of ice, the other is the boiling point of water.

It has been agreed upon by the whole scientific world that these two temperatures — the melting point of ice and the boiling point of water — should be called zero and one hundred degrees respectively. Together they are known as the fixed points of the thermometer.

The interval between these two fixed points is divided into 100 steps — degrees — in such a way that the apparent change in volume of the mercury between any two successive steps is exactly $\frac{1}{100}$ part of the total apparent change in volume between 0° and 100°. This is known as the **centigrade scale,** and is generally indicated by writing a "C." after the numerical value of the temperature. Thus "48° C." means "48 degrees on the centigrade scale."

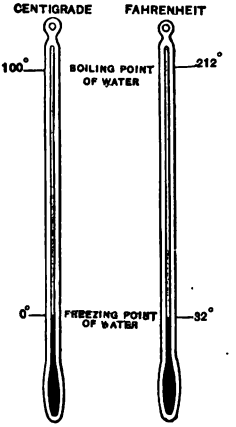


FIG. 172. Illustrating the fact that 100 centigrade degrees are equal to 180 Fahrenheit degrees.

The first step in the graduation of a thermometer is, then, to determine its fixed points, and mark them on the tube. Next the 50° point is so chosen that the volume (not the length) of the tube between 50° and 100° is equal to the volume between 50° and 0°. In like manner, the 25° point must halve the volume between 0° and 50°.

The manner in which these points are determined will be considered in the laboratory.

In the scale proposed by Fahrenheit (1686–1736), the volume between the fixed points is divided into 180 parts; while the zero is placed at a point which is 32 of these degrees below the melting-point of ice.

It is important to notice that these two thermometers, the centigrade and Fahrenheit, differ from one another in no essential respect. They are identical in principle, mode of construction, and use; the only difference is in the size of the degree employed.

183. Essential Features of a Good Thermometer.

1. It must be easily portable.
2. It must be permanent.
3. It must always give the same reading when subjected to the same temperature.
4. It must be possible for the user to test the correctness of its graduation, and to determine any errors in its fixed points.
5. It must be relatively small, so that when placed in contact with a second body the temperature of the second body will not be seriously affected.

III. Transfer of Heat

Having defined temperature as that which determines the *direction* of flow of heat, we next consider the various *modes* by which heat is transferred or diffused. Of these only two general methods are known; viz. (1) Conduction and (2) Radiation. Convection is frequently regarded as a third method, though, strictly speaking, it is the transfer of a heated substance, not a transfer of heat itself.

1. *Heat Conduction*

184. The handle of a spoon may become very hot when the bowl of the spoon is placed in a cup of tea. The

process by which the heat is transferred from the lower to the upper part of the spoon is called **conduction.** The same spoon, when used with ices, may become very cold. Here, again, the process by which the heat is transferred, this time from the handle to the bowl of the spoon, is called conduction.

It is to be carefully observed at this point that we are none the wiser for having given a name to this phenomenon. A name is a mere convention, in this case a matter of agreement among scientific men. It is very convenient, but it explains nothing.

Very little, indeed, is known about the mechanism by which the particles of a solid body hand on the heat from one to another. A great deal else, however, is known about heat conduction. At any rate, we know that the process does not depend upon the visible motion of ordinary matter. Again, every one knows that he can hold the stick of a burning match for 30 or 40 seconds without difficulty; but a copper wire of the same size and shape held in a match flame for that length of time becomes unbearably hot.

An iron and a copper rod, each of the same length, say half a meter, and of the same cross section, each having one end placed in the same flame, will become unequally heated at equal distances from the junction. (See Fig. 173.) For, if at equal distances from the flame on each rod be placed either bits of wax or matches, those on the copper will melt or burn earlier than those on the iron. A series of bicycle balls

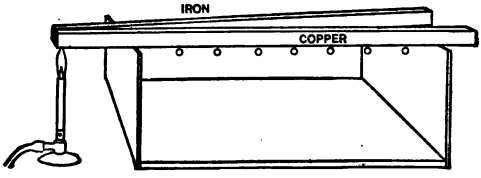


FIG. 173. Illustrating the fact that copper is a better conductor than iron.

attached with wax, three centimeters apart, along each bar will melt off in such a way as to show the progress of temperatures in the bars.

It is here to be borne in mind that the rapidity with which the iron and the copper heat up at any point depends not only upon the rate of conduction, but also upon the amount of heat required to raise the temperature of equal masses of iron and copper through one degree; *i.e.* upon what we shall call, later, the specific heats of iron and copper. In the present experiment, however, these two values are so nearly equal that the relative conductivities of iron and copper are fairly represented.

It is a very interesting fact that all substances which are good conductors of electricity are also good conductors of heat.

Table of Relative Heat Conductivities

Copper	1.108	Water	0.0014
Zinc	0.307	Glass	0.0005–0.002
Brass	0.302	Paraffine	0.00014
Iron	0.164	Wool	0.00012
German silver	0.109	Paper	0.000094
Ice	0.0057	Air	0.000049
Fire brick	0.0017		

Were we to attempt to melt ice by means of a copper bar one end of which is held in a flame at constant temperature, the other end on the ice, we should find the *rate* at which the ice is melted depends upon the dimensions as well as the composition of the bar. If the bar were made twice as thick, the heat would be transferred nearly twice as rapidly. But if the cross section remained constant while the bar were made twice as long, we should find the ice melting only about half as rapidly as before.

By such experiments it has been found that the rate at which heat *flows* through a solid depends upon four things only, viz. (i) **the material of the bar**, (ii) **the length of the bar**, (iii) **the cross section of the bar**, and (iv) **the difference in temperature between the ends of the bar.** In fact, we may regard a difference of temperature as producing a flow of heat very much in the same way that a difference of level, or of pressure, will produce a flow of water.

PROBLEMS

1. Why is it that cork handles on a bicycle are cooler in summer and warmer in winter than metal handles?

2. What is the principle of the miner's safety lamp devised by Sir Humphry Davy?

3. Express in readings on the Fahrenheit scale the following temperatures: 15° C., 50° C., 10° C., 75° C., 150° C., − 12° C.

4. Why are steam pipes packed in glass wool or asbestos?

5. Express in readings on the Centigrade scale the following temperatures: —

Melting point of paraffine, 104° F.
Melting point of sulphur, 833° F.
Melting point of copper, 1943° F.
Melting point of pure iron, 2737° F.
Boiling point of oxygen, − 295° F.
Melting point of mercury, − 38° F.

6. Three rods, one of brass, one of iron, and one of German silver, have the same dimensions. Which one would you undertake to hold in a flame for the longest time without burning yourself?

7. Why is it that when frost forms on a board walk the spaces immediately about the heads of the nails are the last over which the frost forms and the first over which it melts?

8. What is the purpose of the iron projections which are placed on the outside of the cylinder of a motor cycle? Just how do they accomplish their purpose?

9. Why are "soldering irons" made of copper? Why are they made very large compared with small tip which carries the solder and touches the work?

10. Why does a light snow in the autumn generally melt off the concrete sidewalk much sooner than off the grass sod on either side of the walk?

11. Tillet, a Frenchman, remained in a baker's oven (heated to a temperature of 320° F.) for nearly an hour without being burned. Explain. This temperature, 320° F., is sufficient to cook a beefsteak in a few minutes.

Convection Currents

185. If a cup of tea be too hot to drink, we may cool it by adding cream or cold water. Here the heat of the tea is transferred to the cold water by the process of conduction; but the hot and cold portions of the tea do not need to be stirred: they will mix, although a little more slowly, of themselves. **This automatic mixing, which is a mere mechanical process, is called convection.**

The hot air of a chimney rises, mixes with the outside air, and gives some of its heat to the outside air. The hot air rises because its density is less than that of cold air. This process of carrying the hot air up the chimney is, again, convection. A beaker of water to which a flame is applied on one side, as in Fig. 174, becomes equally heated all through. First of all, the water just over the flame becomes hot by *conduction* through the glass. Then, by *convection*, the hot water just over the flame is displaced by the colder water which is heavier and therefore sinks to the bottom, as indicated by the arrows. This cold water, in turn, becomes heated by *conduction* through the glass. **These currents, produced by differences of density, are called convection currents.** They are exhibited on a gigantic scale in the equatorial regions of the earth, where the lower

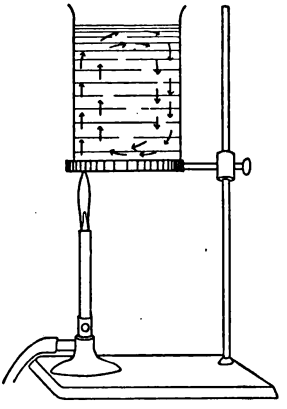


FIG. 174. Convection currents in water.

layers of the atmosphere, in contact with the water of the oceans or with the land, become highly heated by *conduction* and then rise by *convection*. The air rushing in from north and south constitutes the trade winds.

Perhaps the best illustration of convection currents is obtained by the simple experiment indicated in Fig. 175.

B is a glass tube with both ends fitting into a cork in the bottom of a lamp chimney. The chimney, *A*, is filled with colored water; but the tube, *B*, is filled with clear water. The circulation becomes very evident in a few seconds after a flame is applied to one side.

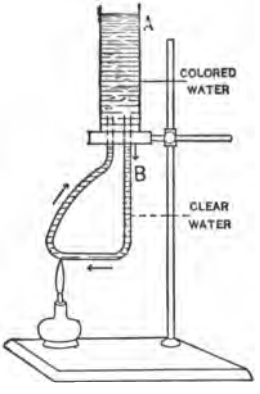


FIG. 175. Circulation in tubes, brought about by convection currents.

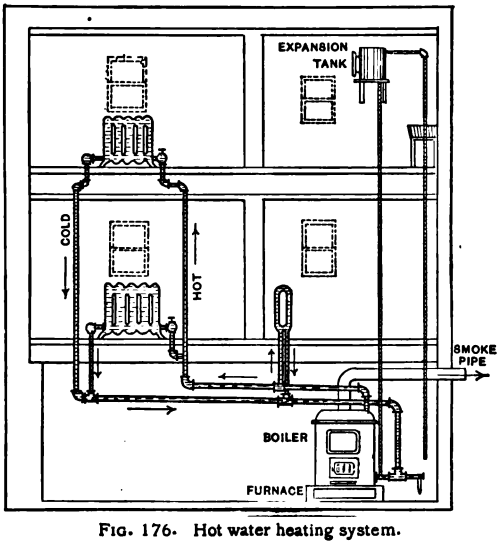


FIG. 176. Hot water heating system.

Observe that here again the *heat* is communicated to the water by *conduction* through the glass, while the *water* is transferred from one part of the vessel to another by the mechanical (not thermal) process of *convection*.

Figure 176 will indicate the manner in which

a furnace in the basement may be employed to warm a house by means of convection currents of water.

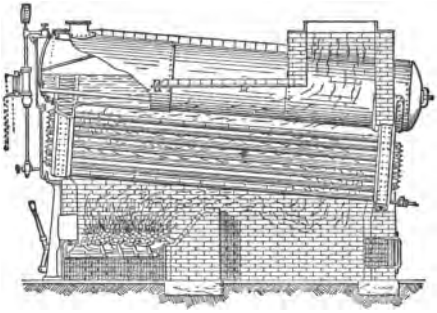


FIG. 177. A water tube boiler.

Figure 177 shows how convection currents of hot gases circulate over the tubes of a steam boiler.

Liquids are Poor Conductors

186. From the preceding it will be evident that in fluids we have, in general, both processes, conduction and convection, always at work; while in solids, convection is necessarily impossible. When, however, heat is applied to the upper layers of a fluid, convection may be reduced to a minimum.

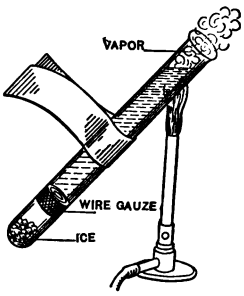


FIG. 178. Illustrating the fact that water is a poor conductor of heat.

To illustrate, put a small piece of ice, say two grams, in the bottom of a test tube and hold it at the bottom by means of a little coil of wire or wire gauze. (See Fig. 178.) The tube may be filled with water, and held over a flame until the water in the top of the tube boils. The heated portions, being lighter than the cold, remain on top, while water is such a poor conductor of heat that the ice in the bottom is not easily melted.

In all the varied processes of heat conduction, whether aided by convection or not, it will be observed that the heat is transferred from places of higher temperature to places of lower temperature. *In this respect* heat behaves like steam and all other fluids in flowing from points of higher pressure to points of lower pressure. We shall later find electricity behaving in the same way.

2. *Radiation of Heat*

187. When the hand is held some inches at the side of, or underneath, an incandescent electric lamp, the sensation of heat is distinctly recognized. The same is true of the hand placed beneath a heated metal ball. (See Fig. 179.) We hold our hands before an open grate fire to warm them. How does the heat pass from the fire to the hands? Certainly not by conduction. For air is one of the very poorest conductors known. (See Table, p. 204.) Nor is it a case of conduction aided by convection currents. For, in each of the three cases cited above, the currents due to convection are such as to cool the hand. The process by which heat crosses space in each of these and other similar cases is called **radiation.** The medium by which it is transmitted is clearly not the air, for one of the earliest observations made after the discovery of the air pump was that **both heat and light pass through a vacuum with the utmost ease.** There is every reason for believing that the space which separates us from the sun is more nearly a perfect vacuum than any other known; yet across this vast,

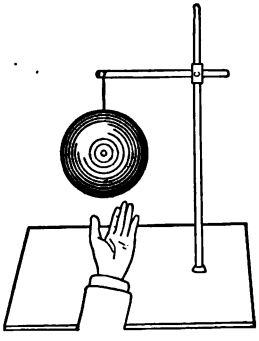


FIG. 179. Radiation of heat downwards from a hot iron ball.

and so-called empty, region the earth daily receives enormous quantities of heat by the process which we call radiation. But what is radiation?

Radiant Heat is a Wave Motion

188. The question here raised can be answered (but only very imperfectly) by anticipating some of the results of optics and thermodynamics. At this point we must content ourselves with saying that **there is most excellent evidence for thinking that light is a form of energy, that light consists in a wave motion, that this wave motion is propagated in a medium (called the *ether*)** which permeates all bodies and fills all interstellar space, and that this wave motion travels in free space (*i.e.* space devoid of matter) at the rate of 300 million meters per second, but always more slowly in space which is not devoid of matter.

Now the experimental evidence for thinking that **radiant heat is of exactly the same nature as light** is overwhelming, and if we once take for granted the evidence for thinking radiant heat a wave motion we may immediately predict for it resonance, interference, reflection, refraction, and other properties of waves.

Kirchhoff's Law of Radiation

189. In this connection the following experiment should be seen by every one:

A narrow line is marked with white chalk upon a small electric light carbon as indicated in *A*, Fig. 180. In a moment, the carbon can be heated to redness in the flame of a blast lamp. If this red-hot carbon is now viewed in a room well darkened, the white belt which before heating sent to the eye the most light is now quite dark and sends to the eye less light than any other part of the carbon, *B*, Fig. 180.

The same fact has been shown to be true for rays of heat which do not affect the eye, but which can be detected by other means.

The black carbon rod represents a body which absorbs heat and light very well. A crayon of chalk is a body which reflects light and heat very well. This particular experiment illustrates the general fact that **good absorbers of heat are also good radiators of heat, while good reflectors are poor radiators.** Kirchhoff, an eminent German physicist (1824–1886), gave this statement a simple quantitative form: hence it is known as **Kirchhoff's Law.**

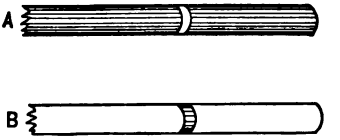


FIG. 180. Illustrating Kirchhoff's Law.

It follows from this law that a well-designed teakettle should be highly polished everywhere except on the bottom. So also a stove would be more efficient for heating a room if blacked than if nickel-plated.

IV. EFFECTS OF HEAT

The effects of heat are so various and so numerous that their mere mention would fill a large volume. But those which are most important, from the beginner's point of view, may perhaps be grouped under three heads, viz.:

1. *Change of Dimensions.*
2. *Change of Molecular State.*
3. *Change of Temperature.*

CHANGE OF DIMENSIONS

1. *Expansion of Solids*

190. When the blacksmith kindles a fire in a ring about the size of a wagon wheel, and places on the fire an iron

tire so as to expand it and make it large enough to slip over the wooden part of the wheel, it may generally be observed that the small boy of the neighborhood has time to stop and watch the process. When cold the tire is just too small to be forced on the wheel; when heated it has become large enough to be driven on; but when cold again the tire has a firm grip on the wheel. This process is known as "shrinking on" a tire.

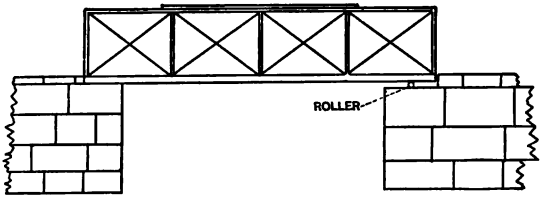


FIG. 181. Roller providing for expansion of iron bridge.

Exactly the same effect is observed in the uncompensated pendulum, *i.e.* in a clock pendulum made of one single kind of metal. In hot weather the pendulum becomes longer, its period becomes greater, and the clock runs more slowly. In cold weather the clock runs faster.

When next you cross a girder bridge made of iron, observe that one end rests, not directly upon the pier, but upon an iron roller, as shown in Fig. 181. The bridge is thus free to elongate and contract without damage to itself or to the pier.

Coefficient of Linear Expansion

191. For any particular material **the change in length per unit length per degree is called the coefficient of expansion** of that material. The value of the coefficient for a number of common substances is entered in the following:

Table of Coefficients of Linear Expansion

(Centigrade Scale)

Glass	0.0000085	$= \frac{1}{120000}$
Platinum	0.0000085	$= \frac{1}{120000}$
Steel	0.000012	$= \frac{1}{85000}$
Brass	0.000018	$= \frac{1}{55000}$
Copper	0.000019	$= \frac{1}{52000}$
Aluminum	0.000023	$= \frac{1}{43000}$
Zinc	0.000029	$= \frac{1}{34000}$
Invar	0.00000096	$= \frac{1}{1000000}$

Study this table to discover why platinum wire is fused into the glass of an incandescent lamp, instead of copper or any other cheaper wire. Also show from this table how one may construct a gridiron pendulum from alternate rods of brass and steel, so that the length of the pendulum will not vary with temperature. (See Fig. 182.) In the case of many alloys the coefficient of expansion varies rapidly with the percentage of the metals which go to make up the alloy. Taking advantage of this fact, the French physicist Guillaume has succeeded in producing a nickel-steel alloy — about 36 % nickel — for which the linear coefficient is less than one millionth. The size of a body constructed of this material is practically *invariable* with temperature; and hence the substance has been called "invar." The usefulness of such a metal for making clock pendulums, surveyor's tapes, and other measuring instruments is at once apparent.

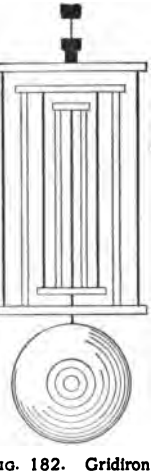


Fig. 182. Gridiron pendulum: black lines indicate steel; double lines indicate brass.

PROBLEMS

1. If the diameter of an iron disk is 120 cm. at 0° C., how great will it be at 50° C., if the coefficient of linear expansion is 0.000012 ?

2. If the coefficient of linear expansion of brass is 0.000019, and that of steel is 0.000012, what will be the difference in length at 20° C. of a brass meter rod and a steel meter rod, each of which is correct at 15° C. ?

3. An iron rail 30 ft. long is laid in a railroad track in winter when the temperature is 0° C. If the highest temperature of summer is 35° C., how much space should be left between the ends of the rails ?

4. Since glass, copper, etc., expand on heating and contract again on cooling, why not employ a rod of definite length made of some one of these materials to measure temperatures ? To each length of the rod would correspond a definite temperature. What practical objections ?

5. Explain, by diagram, just how a strip of zinc soldered to a strip of sheet copper, as indicated in Fig. 183, can be employed to announce (by the ringing of a bell) when a room is too hot and when too cold.

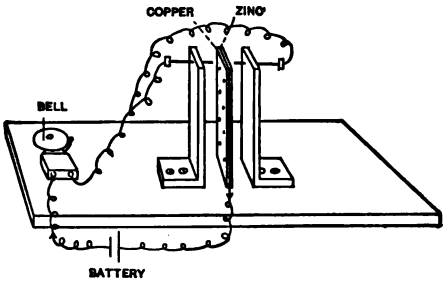


FIG. 183. Device for announcing change of temperature.

6. Explain by a diagram the convection currents which produce the "draft" in an ordinary open fire place and chimney.

7. Long horizontal steam pipes are always supported in such a way that they are free to move at one end. Why ?

8. A bar which is exactly 5 m. long at 10° C., is heated to a temperature 210° C., when its length is found to be 5.08 m. What is its coefficient of expansion ?

9. An iron steam pipe is 100 ft. long at 0° C. What will be its length when live steam passes through it ?

10. The brass scale on a barometer is correct at 0° C. At 20° C. the height of the mercury column appears to be 76.5 cm. What correction must be applied to this reading, on account of the expansion of the brass?

Change of Dimensions

2. *Expansion of Liquids*

192. Since liquid bodies are defined (p. 104) as those which have no shape of their own, but assume the shape of the containing vessel, it is evident that a liquid body has no characteristic edges or lines, and hence no coefficient of linear expansion. But liquid bodies have characteristic volumes, and these change when the liquids are heated.

Definition of Coefficient of Expansion of a Liquid

Just as we defined coefficient of linear expansion as change of length per unit length per degree, so also **coefficient of cubical expansion is defined as change of volume per unit volume per degree.**

Of all liquids, water and mercury are perhaps those of most importance in the physical sciences. We shall briefly consider these two.

Mercury

193. The expansion of mercury was first accurately measured by Regnault (1810–1878), a skillful French physicist.* Probably the most accurate value for the average expansion of mercury between 0° and 100° C. is that obtained by Regnault's method at the Reichsanstalt, the great national physical laboratory of Germany, namely, 0.00018245. For nearly all purposes we may use the value 0.00018.

Fig. 184. Apparent expansion.

Of course, we might take a single tube of mercury, Fig. 184, and measure its height at two different temperatures; but here the

* The explanation of Regnault's method may be found in Crew's *General Physics*, pp. 277–279.

difficulty is that the glass vessel containing the mercury expands with the heat so that the volume of the mercury is no longer proportional to the height of the column. In this way we should obtain only the **apparent expansion** of mercury. **It is the apparent expansion which is used in glass and mercury thermometers.**

Water

194. From many points of view, water is a remarkable substance. This is strikingly exhibited in its behavior under heat. In general, warm liquids are less dense than the same liquids cold; and this is true of water except between the temperatures 0° C. and 3.98° C. The manner in which water expands under heat is clearly set forth in the curve, Fig. 185, from which it will be seen that

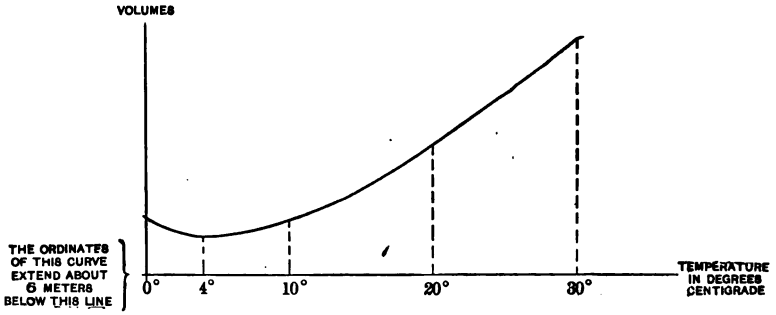


FIG. 185. Expansion of water.

when water at the temperature of melting ice is heated, its volume decreases (*i.e.* its density increases) until a temperature of 4° is reached.

Above 4° C. the volume increases with temperature, *i.e.* the density diminishes as the temperature rises. From Fig. 185 it will be clear why the bottom of a deep lake never freezes. The coldest water remains at the top,

and when it freezes forms a sheet of ice which protects the warmer and denser water below. Unless the lake is so shallow as to freeze solid, the water on the bottom will never be colder than 4° C.

The coefficient of expansion of water also has recently been determined at the Reichsanstalt by the method which Regnault employed for mercury. The results are summarized in the following table:

Density of Water

Temperature	Density	Volume
Centigrade		
0°	0.999867	1.000132
3.98	1.000000	1.000000
10	0.999727	1.000272
15	0.999126	1.000874
20	0.998229	1.001773
25	0.997071	1.002937
30	0.995673	1.003345
35	0.994057	1.005977
40	0.992241	1.007819

The preceding figure (185) is merely a graphical description of the results contained in this table. The first column in the table gives the abscissas, and the last column the ordinates, of the curve.

Change of Dimensions

3. *Expansion of Gases*

195. Perhaps the simplest method of showing the pressure of the earth's atmosphere is to partially exhaust the air from a vessel (Fig. 186) having one opening covered with sheet rubber such as dentists use. When the air is removed from the inside, the excess of pressure on the

outside shows itself in the concave surface of the rubber. In exactly the same way, we may detect the effect of heat upon a body of gas inclosed in a wide-mouthed bottle or Florence flask. If we carefully cover the mouth of the bottle with a sheet of rubber, as shown in Fig. 187, a slight rise in temperature is sufficient to make the rubber convex outwards. From this what inference do you draw concerning the pressure inside? What effect is produced on the rubber when the bottle is cooled in water? What inference do you draw concerning the pressure inside? How is the *volume* of the gas affected by the heat? Is the *mass* of the gas altered by the heat?

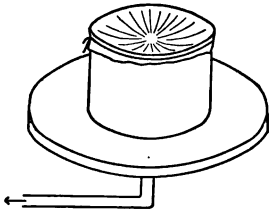


FIG. 186. Stretched membrane used to indicate pressure.

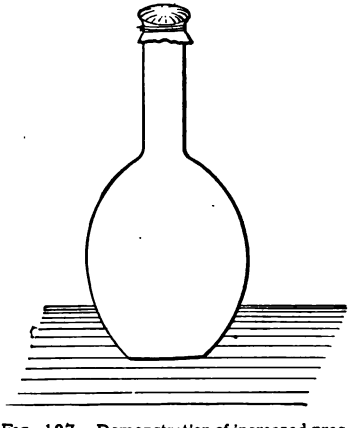


FIG. 187. Demonstration of increased pressure produced by heat.

The same phenomena are illustrated in a more marked way by the use of a glass bulb with a stem of rather small bore as shown in Fig. 188. The bulb is first warmed by holding in the hand for a moment: it is then inverted in a flask of colored liquid with the stem under the surface of the liquid. When the bulb cools to the tem-

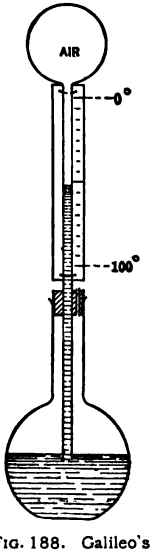


FIG. 188. Galileo's thermometer.

perature of the room, the liquid rises in the stem. This instrument is sensitive to the touch of a finger.

This is the earliest form of thermometer, and was devised by Galileo. He showed it in his lectures at Padua not later than 1603, and also employed it as a clinical thermometer, to detect fever. For this latter purpose a scale of 100 parts was placed on the stem.

In 1699 Amontons substituted mercury for water in the thermometer of Galileo and discovered that the change of pressure produced by heating the bulb through a range of 100° was the same whatever the amount of air inclosed in the bulb.

Law of Charles

196. By definition, the volume of a gas is always the volume of the containing vessel. Boyle's Law states the experimental fact that to each volume of gas there corresponds a definite pressure, provided the mass and the temperature remain constant. If, however, the temperature varies, the effect may be

(1) a change of volume while the pressure remains constant; or

(2) a change of pressure while the volume remains constant; or

(3) a change of *both* pressure and volume.

To determine the effect of heat on a gas, we must then measure either

(1) the ratio of increase of volume to original volume in a gas kept at constant pressure; or

(2) the ratio of increase of pressure to original pressure in a gas kept at constant volume,

in each case for a rise of one degree in temperature.

These two quantities were first roughly measured by Charles, afterwards more accurately by Gay-Lussac.*

The capital discovery was thus made that

(*a*) these two quantities are the same for any one gas;

(*b*) they have very approximately the same numerical value for *all* gases, namely,

$$0.003665 = \tfrac{1}{273};$$

or, **the pressure remaining unchanged, a gas expands or contracts $\frac{1}{273}$ of its volume at 0° C. for a change of 1° C.** (*Law of Charles*).

The actual measurement of these quantities is a somewhat difficult matter, and may well be reserved for the student's second or third year in physics. But the algebraic description of Gay-Lussac's *results* is a simple matter.

According to the experimental law as stated above, 1 cubic centimeter of gas at 0° C. becomes $\left(1+\frac{t}{273}\right)$ cubic centimeters at t° C.; 5 cubic centimeters of gas at 0° C. occupies 5 times the volume of 1 cubic centimeter, or $5\left(1+\frac{t}{273}\right)$ cubic centimeters, and, at t° C., V_0 cubic centimeters of gas at 0° C. occupies $V_0\left(1+\frac{t}{273}\right)$ cubic centimeters. Therefore the volume, V_t, occupied by V_0 cubic centimeters of gas when the temperature is changed from 0° C. to t° C. is

$$V_t = V_0\left(1+\frac{t}{273}\right) = V_0\left(\frac{273+t}{273}\right).$$

At any other temperature, as s, the volume would be

$$V_s = V_0\left(\frac{273+s}{273}\right).$$

* Gay-Lussac, a French chemist, born 1778, died 1850.

Comparing these two volumes of gas,

$$\frac{V_t}{V_s} = \frac{\dfrac{273 + t}{273}}{\dfrac{273 + s}{273}} = \frac{273 + t}{273 + s}.$$

Now, 273 + *the temperature on the centigrade scale is called temperature on the ideal gas scale*, and is denoted by T. Therefore the Law of Charles may also be stated as follows: **When the pressure remains unchanged, the volume occupied by any mass of gas is proportional to the temperature on the ideal gas scale.**

By similar reasoning, **when the volume remains unchanged, the pressure of a given mass of gas is found to be proportional to the temperature on the ideal gas scale.**

Since both temperature and pressure may change at the same time, an expression for the laws of Boyle and Charles combined is very useful. At any two temperatures, $t°$ C. and $s°$ C., the following relation is found to be true:

$$\frac{\text{pressure} \times \text{volume at } t° \text{ C.}}{273 + t} = \frac{\text{pressure} \times \text{volume at } s° \text{ C.}}{273 + s};$$

or,

$$\frac{PV}{T} \text{ at } t° \text{ C.} = \frac{PV}{T} \text{ at } s° \text{ C.} = \text{a constant.}$$

PROBLEMS

1. The volume of hydrogen in a balloon is 10,000 cu. ft. at 17° C. when the barometer stands at 75 cm. of mercury.

(*a*) What volume will it occupy if the temperature rises to 27° C. and the barometer also rises to 76 cm.?

(*b*) What volume will it occupy if the temperature falls to − 3° C. and the barometer drops to 72 cm.?

2. The air in an auto tire is compressed 6 atmospheres at a temperature of 12° C. What will be the pressure if the temperature rises to 37° C.? Suppose the volume to remain the same.

3. The pressure inside an incandescent light bulb is 0.0001 cm. of mercury at 22° C. What will be the pressure when the temperature rises to 117° C.?

IV. Effects of Heat

2. *Change of Molecular State*

197. There are probably not many people whose memories reach back to years so early that they recall their surprise at first seeing a piece of ice melt in their hands and apparently disappear. The change from water to steam which occurs in a boiling teakettle is equally familiar. These two instances of **melting** and **boiling** are so typical of many similar cases met with in nature that they deserve close attention.

In the first place, it is well known that water and ice have the same chemical composition and the same chemical properties. A similar statement concerning solid and liquid paraffine would be true.

In the second place, a glass of water so long as it contains ice and is stirred does not become either hotter or colder on standing. The ice may melt away; but, so long as there is any ice left, the water will remain approximately at what we call *the temperature of melting ice.*

Thirdly, the temperature of melting ice can be changed by placing the ice under pressure. The most convenient method of showing this is perhaps the following:

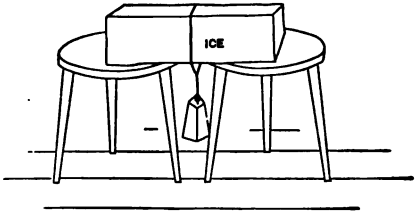


Fig. 189. Bottomley's experiment, showing the effect of pressure upon the melting point of ice.

Choose a block of ice having a section of from 10 to 15 centimeters square. Support it, as indicated in the figure, upon two stools or boxes. Over the ice tie a loop of steel piano wire about one half millimeter in diameter. Hang a mass of about 25 kilograms on the wire. In this

way you can subject the ice to a pressure of several hundred kilograms per square centimeter. The block of ice will, in general, be not far from its melting temperature; but the melting will all occur at the surface until pressure is put on the wire; **it will then be observed that the ice just underneath the wire melts also and lets the wire cut its way through the block.** But the water behind the wire freezes at once, so that the ice remains a solid block although it has just been cut in two. **The effect of pressure is, then, to produce melting, *i.e.* to lower the freezing point of water.**

Our experience with ice and all other solids may be summarized in the following statement:

Experimental Law of Melting

"The pressure remaining the same, there is a definite melting point for every solid; and (provided the mass be stirred) however much heat be slowly applied, the temperature of the whole remains at the melting point till the last particle is melted." — TAIT, *Heat*, Ch. VIII.

TABLE OF MELTING AND BOILING POINTS

SUBSTANCE	CENTIGRADE	FAHRENHEIT
Liquid air	—191	—312.7
Liquid hydrogen	—253	—423
Liquid nitrogen	—194	—317
Liquid oxygen	—182	—296
Freezing mercury	—39	—38
Melting ice	0	32
Boiling point of water at 760 mm. pressure	100	212
Boiling point of aniline at 760 mm. pressure	184	363
Boiling point of sulphur at 760 mm. pressure	445	833
Melting point of lead	327	620
Melting point of gold	1064	1947
Melting point of pure iron	1503	2737
Melting point of platinum	1753	3191

The temperature of the positive crater of the electric arc is probably between 3600° and 3900° C.; while the temperature of the sun appears to be about 7800° C.

Law of Boiling

198. In like manner, the chemical composition and the chemical properties of steam do not differ from those of water. However much you boil the water in the teakettle its temperature does not change after boiling has once begun. One can easily satisfy himself of this with an ordinary mercury thermometer. But the temperature of boiling water is very much changed by a change of pressure.

This is most easily proved by boiling in a kettle of water a bottle half filled with water. If this bottle be corked while still boiling, it may be removed with so little air inclosed in it that the pressure on the inclosed water becomes greatly less than that of the atmosphere. Under these circumstances, the water in the bottle will continue to boil long after it has reached a temperature not uncomfortable to the hand.

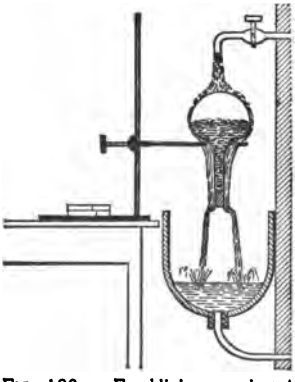


FIG. 190. Franklin's experiment showing how the boiling point is lowered by diminution of pressure.

In the laboratory this phenomenon is most commonly shown by boiling over a Bunsen flame a round bottom flask half filled with water. Cork the flask while boiling. Remove it at once to a support previously prepared and cool with cold water. Violent boiling will occur during the cooling process.

To show that to each different pressure corresponds a definite boiling point, a very simple method is that indicated in Fig. 191. Through the cork

of a large-necked flask insert a thermometer, a U-tube that can be partially filled with mercury and used as a pressure gauge, also an escape pipe, *e*.

The escape pipe is fitted with a tip of rubber tubing and a pinchcock. By a little manipulation of the pinchcock one can produce and maintain any desired height, *h*, in the pressure gauge. In like manner, to observe the effect of diminished pressure, one can connect the escape pipe, *e*, with an air pump. For each different *h*, the thermometer registers a different boiling point. It will thus be seen that the boiling point of water is much more sensitive to changes of pressure than is the melting point of ice. Not only so, but the effects of pressure in these two cases are exactly opposite: the melting point of ice is lowered by pressure; the boiling point of water is raised by it.

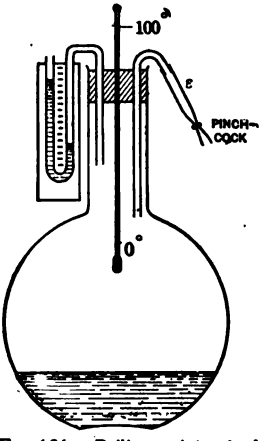


FIG. 191. Boiling point raised by increase of pressure.

Our experience concerning boiling may be summarized as follows:

"The pressure remaining the same, there is a definite boiling point for the free surface of every liquid; and (provided the mass be stirred) however much heat be applied, the temperature of the whole remains at the boiling point till the last particle is evaporated." — TAIT, *Heat*, Ch. IX.

Definition of Boiling Point

199. It can be shown that liquids are undergoing evaporation at all temperatures. But the region over the free surface of a liquid may contain so much vapor from the liquid that condensation will occur as rapidly as new

vapor is formed. The atmosphere over the liquid is then said to be **saturated.**

The process of **boiling** occurs when the temperature has reached a point where vapor is formed in the interior of the liquid, and rises to the top. But unless the pressure of this hot vapor were equal in pressure to that at the free surface of the liquid, the bubble of vapor would collapse in the liquid.

The **temperature of boiling (what we ordinarily call the boiling point) is then simply that temperature at which the saturated vapor of the liquid exerts a pressure equal to the pressure at the free surface of the liquid.**

The following table, due to Regnault, shows how the boiling point of water changes with the pressure:

Pressure of Saturated Water Vapor

Temperature Centigrade	Pressure in Cm. of Mercury	Temperature Centigrade	Pressure in Cm. of Mercury
0°	0.5	90°	52.5
10°	0.9	100°	76.0
20°	1.7	110°	107.5
30°	3.2	120°	149.1
40°	5.5	140°	271.8
50°	9.2	160°	465.1
60°	14.9	180°	754.6
70°	23.3	200°	1168.9
80°	35.5	220°	1739.0

Dew Point. Humidity

200. There is always some water vapor in the atmosphere which will condense in the form of rain, snow, dew, or frost if the temperature falls low enough. **The temperature at which the water vapor present in the atmos-**

phere begins to condense is called the "dew point." The "humidity" at any time and place is the ratio between the amount of moisture actually present in the atmosphere and the amount that would be present if the atmosphere were saturated.

PROBLEMS

1. When one is "caught in the rain" and allows his clothing to "dry on him," from what source comes the heat which evaporates the moisture in the clothing?

2. Why is it that alcohol poured on one's hand gives the sensation of cold?

3. Why can water and alcohol be separated by distillation? The boiling point of alcohol is about 70° C.

4. Why is a sudden drop of temperature in summer time apt to be accompanied by rain? In general, at what temperature does the water vapor in the earth's atmosphere begin to condense into rain?

5. Why are cooking vessels used in high altitudes frequently provided with steam-tight lids?

6. A pressure of one atmosphere is defined as a pressure that will support a column of mercury 76 cm. high. If we take the acceleration of gravity as 980, how many dynes to the square centimeter will be equivalent to a pressure of one atmosphere? Density of mercury = 13.6.

7. The average reading of the barometer at the top of Mount Hamilton (the Lick Observatory) is 65 cm. At what temperature, therefore, does water boil on Mount Hamilton? *Ans.* 95°.3 C.

8. At the top of Pike's Peak water boils at a temperature of 86°. What, therefore, is the average reading of the barometer on Pike's Peak? *Ans.* 45.68 cm.

9. The pressure of the atmosphere is approximately 15 lb. weight to the square inch. The boiler of an Atlantic liner carries a steam pressure of 300 lb. weight to the square inch. What is the temperature of the water in this boiler? See Regnault's table. *Ans.* 212°.3 C.

10. How do you explain the fact that as a vessel of sap from the maple tree is boiled away the percentage of sugar in the solution left behind continually increases?

IV. Effects of Heat

3. *Change of Temperature*

201. Everybody knows that, *in general*, the effect of imparting heat to a body is to make it hot, *i.e.* to increase its temperature. Two exceptions to this rule we have just studied under the head of *melting* and *boiling*.

As we shall see presently, heat is a form of energy; and when heat energy is imparted to a body it may disappear as heat energy and assume some other form of energy. This is the case with the heat employed to melt a piece of ice.

Or the heat energy imparted to a body may remain as heat energy, in which case the temperature of the body rises. It is this latter effect which we now propose to study.

Heat as a Quantity. Unit of Heat

202. Since more heat is required to change the temperature of a large body than to produce the same change in a small body, and since by duplicating any heat-giving process we can continue to add equal amounts of heat to a body, it is evident that we are entitled to speak of heat as a quantity. And if it is a quantity, and measurable, we shall need a **unit of heat.** For this unit it is customary to use **the quantity of heat necessary to raise the temperature of one gram of water through one degree Centigrade.** This unit is called a **calorie.**

Engineers more frequently employ the British Thermal Unit (written "B.T.U."), which is the amount of heat required to raise the temperature of one pound of water through one degree Fahrenheit.

Definition of Specific Heat

203. It was early discovered that, if equal amounts of heat are added to equal masses of different substances, the rise in temperature is not the same. If to a gram of lead you add the same amount of heat as to a gram of water, the effect is to increase the temperature of the lead 33 times as much as that of the water. The temperatures of equal masses of lead and mercury are almost equally affected by the same amounts of heat.

The ratio between the amount of heat required to raise the temperature of any body one degree and that required to raise the temperature of an equal mass of water one degree, is known as the **specific heat** of the body.

The *numerical value* of the specific heat of any substance will be the quantity of heat (number of calories) required to change the temperature of one gram of the substance by one degree.

From the definition of the unit of heat it is clear that the specific heat of water is unity, and that the heat in calories taken in or given out by a certain mass of *water* is the

mass in grams × change of temperature in degrees C.

In the case of any body the product of its mass and its specific heat is known as its **water equivalent.** It is **the mass of water that has the same capacity for heat as the given mass of the substance.** Thus the water equivalent of 500 gm. of aluminum is $500 \times .214 = 107$ gm. Therefore the heat in calories taken in or given out by a certain mass of *any substance* when the temperature is changed is

mass in grams × specific heat × change of temperature in degrees C.

Table of Specific Heats

Substance	Specific Heat	Substance	Specific Heat
Aluminum	0.214	Lead	0.030
Copper	0.092	Mercury	0.033
Glass (flint)	0.117	Paraffine	0.694
Graphite	0.160	Quartz	0.174
Ice	0.505	Silver	0.056
Iron	0.109	Zinc	0.094

Smithsonian Tables.

V. Measurement of Heat

204. We know from experience that twice as much heat is required to change the temperature of 200 gm. of lead as is required to change, by an equal amount, the temperature of 100 gm. of lead. We have also seen that by definition the amount of heat required to change the temperature of any substance varies directly as the specific heat of that substance. Furthermore, the amount of heat required to alter the temperature of a body varies directly as the amount of the change in temperature.

Other than these three things [(i) mass, (ii) specific heat, and (iii) change of temperature], there is nothing upon which the amount of heat depends, *provided the molecular state of the body is not changed; i.e.* provided the body does not change from solid to liquid, liquid to gas, or *vice versa.*

The Heat Equation

205. We have now learned how to measure the quantity of heat added to or taken away from a body, namely, multiply the water equivalent of the body by its change in temperature.

We also know, from the Law of the Conservation of Energy, that when two bodies are brought into contact, whatever heat is gained by one body is lost by the other, *provided, of course, there is no loss or gain of heat to outside bodies.* If, then, we can isolate two bodies which are placed in contact, we may equate the heat gained by one to the heat lost by the other. This is often called the **heat equation.**

Heat of Fusion

206. It has been found by experiment that a definite amount of heat is required **just to melt one gram of a substance,** *i.e.* to change it from a solid to a liquid **without changing its temperature.** This quantity of heat is, in general, different for each different substance, and is called the **heat of fusion** of the substance. For the last hundred years, this quantity has been known as the "Latent Heat" of the substance. This name is, however, rather misleading; for, as we shall presently see, it is exactly during the present century that men have learned that heat is a form of energy which, when employed to melt a body, disappears as heat *of any kind*, and assumes some other form of energy concerning which we know comparatively little.

In the case of ice, the heat of fusion is found to be very nearly 79 calories per gram.

Heat of Vaporization

207. In a strictly analogous manner, experiment shows that it takes a definite amount of heat **just to change one gram of any liquid into vapor without changing the temperature.** This quantity, for any particular substance, is called its **heat of vaporization.**

In the case of water boiling under standard atmospheric pressure (760 mm. of mercury), it is found that 536 calo-

ries are required to evaporate one gram without changing its temperature. The heat of vaporization of water is, therefore, said to be 536 calories per gram.

PROBLEMS

1. A beaker contains 400 gm. of water at 20° C. How much heat must be added to the water before it begins to boil?

2. What is the water equivalent of a copper vessel weighing 250 gm.?

3. The specific heat of copper is 0.092. Find how many calories will be required to heat 200 gm. of copper from 10° to 70° C.

4. Two pounds of boiling water are poured into a vessel containing 6 lb. of water at 18° C. Find the temperature of the resulting mixture.

5. What is the water equivalent of 10 lb. of mercury? Of 1000 gm. of lead?

6. The specific heat of lead is 0.03. One thousand grams of boiling water are poured over 1000 gm. of shot at the temperature 20° C. Find the common temperature of the shot and the water.

7. A vessel contains 40 gm. of ice at the temperature 0° C. How much boiling water must be poured over it to just melt the ice without changing its temperature?

8. One hundred grams of lead shot at 100° C. are emptied into 100 gm. of water at 20° C. Assume the water equivalent of the containing vessel as 20, and find the resulting temperature of the lead and water.

9. The heat equivalent of fusion of ice is 79 calories. 120 gm. of ice at 0° C. are added to 500 gm. of water at 20° C. What is the resulting temperature?

10. The specific heat of air is 0.237. A certain room contains 20 kgm. of air. How much will the temperature of this room be raised by allowing a tub of 60 kgm. of water at 80° C. to cool to 20° C. and give all its heat to the air of the room?

11. In a radiator 2 kgm. of water per hour are condensed. The steam is at a temperature of 100° C., the condensed water at 50° C. How many calories are given out per hour?

12. Steam at 100° C. is passed into 1 kgm. of water at 15° C. How many grams of steam must condense to raise the temperature of the water to 100° C.?

13. Why does moisture usually condense upon the outside of a pitcher of ice water?

14. Why is there no dew deposited during a windy night?

VI. The Nature of Heat

Mechanical Equivalent of Heat

208. What, after all, is this something we call heat? What is this which changes the size or pressure of a body, alters its temperature, melts and vaporizes it?

Up to the beginning of the present century it was thought to be a form of matter, something like air, only of much greater tenuity, and without appreciable weight. It was considered so thin that it would penetrate solid bodies with the utmost ease. This substance was called "caloric."

That no such substance exists was proved by the joint labors of Count Rumford,* Sir Humphry Davy,† and Mr. James P. Joule.‡ The enticing history of this subject must be left for advanced study, while we pass to the consideration of the experiments by which Joule, about the middle of the nineteenth century, discovered the exact relation between *Heat* and *Work*.

He arranged a paddle-wheel in a vessel of water (Fig. 192) in such a way that the paddle-wheel could be kept in rapid rotation by means of a descending clock-weight. By placing a thermometer in the water, he found that the more the water was stirred, the warmer it became. But, what is more important, he measured the **work done by the descending weight** and the corresponding **heat produced by the paddle** turning in the water, and found that **in every case the ratio between these two quantities was the same.**

* *Rumford* (1753–1814). By birth, an American; by residence, an Englishman and a German successively. His most interesting experiments were performed while he was in charge of the Royal Arsenal at Munich.

† *Davy* (1778–1829), an English chemist.

‡ *Joule* (1818–1889), an English physicist.

The discovery of Joule was that for every unit of mechanical energy which *disappears* as the weight descends, a definite and **constant** quantity of heat *appears* in the

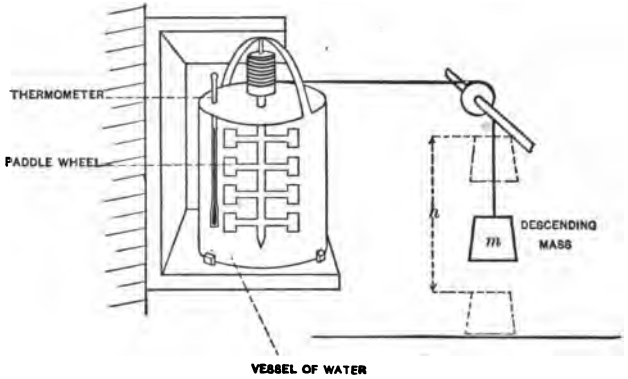


FIG. 192. Joule's experiment to determine the mechanical equivalent of heat.

stirred water. He determined the numerical value of this constant, and found it to be very nearly 42,000,000 ergs per calorie.

The physical meaning of this constant, which is called the **Mechanical Equivalent of Heat**, is as follows, viz. **42 million units of work must be done in order to produce one unit of heat; in like manner, when one calorie of heat disappears, some other form of energy equivalent in amount to 42 million ergs of work always appears.**

In engineer's units the mechanical equivalent of heat has the value 778: which means that 778 foot-pounds of work are required to raise the temperature of 1 pound of water 1° Fahrenheit.

The inference, then, is, since work disappears as heat appears, and *vice versa*, that **heat is a form of energy.** This principle is known as the **First Law of Thermodynamics.** The transformation of heat into forms of energy

other than mechanical will be discussed in following chapters.

Having shown heat to be a form of energy, we see that **the first law of thermodynamics is merely a special case of the Conservation of Energy.**

The Kinetic Theory of Matter

209. Since heat is required to change a solid into a liquid, or a liquid into a gas, work must be done in order to effect these changes of state. To explain these and other facts, it has been assumed that matter is made up of very small particles called molecules. In solids one may imagine these molecules to be so close together that they have comparatively little freedom of motion; that in liquids they are free to move over one another; and that in gases each molecule is practically independent of its neighbors.

It is further assumed that these gaseous molecules are always in very rapid motion. Then the pressure which a gas exerts on the walls of its containing vessel is the average momentum per unit of time which the particles deliver to each unit area of the walls. As the gas is heated, the motion of the particles is quickened, the number of impacts on the wall in a given time increases, and the pressure rises. Expansion of liquids and solids with rise of temperature may be attributed to increased amplitude of vibration of the molecules. Evaporation may be regarded as the escape of molecules from the liquid surface. Since work in the form of heat must be done upon each molecule before it escapes, each escaping molecule carries heat away from the main body of liquid, and *evaporation is a cooling process;* similarly condensation and solidification are warming processes.

Thus the kinetic theory of matter explains the laws of boiling and melting, Boyle's Law, the Law of Charles, expansion and contraction, besides many other phenomena, such as the diffusion of gases and the osmosis of liquids, which cannot be described here.

PROBLEMS

1. Explain how one can warm his hands by rubbing them together.

2. Explain the fact that a piece of iron is heated by hammering. What kind of energy is here transformed into heat?

3. Explain the fact that when a piece of copper wire, at any point, is rapidly bent first in one direction, then in the other, it becomes hot.

4. How is it that when the tire of a bicycle is being pumped up,
(*a*) the barrel of the pump becomes warm?
(*b*) the rubber hose connecting the pump and the tire becomes warm?

5. At a place where the acceleration of gravity is 980, a bullet whose mass is 8 gm. is allowed to fall from a height of 1000 cm. What will be the kinetic energy of the bullet in ergs just before it strikes the ground? To what fraction of a calorie will this mechanical energy be equivalent? *Ans.* 0.186 calorie.

6. Suppose, in the preceding problem, that when the bullet strikes the ground one half its kinetic energy goes to heat the bullet, the other half to heat the ground. If the specific heat of lead is $\frac{1}{30}$, by how much will the temperature of the bullet be raised? *Ans.* 0°.34 C.

7. (*a*) The height of the American Falls at Niagara is 50 m. How much kinetic energy will a gram of water acquire in falling through this distance? ($g = 980$.)
(*b*) What will be the equivalent of this energy in heat units?
(*c*) By how much then will the temperature of the water at the bottom be increased over that at the top?

8. Which requires the greater amount of heat, to raise 1000 gm. of water from 97° to 99° C., or to raise the same amount of water from 99° to 101° C. under ordinary atmospheric pressure? Why? (Princeton Entrance Exam.)

9. If heat be applied uniformly to a body of ice initially at − 10° C. until its temperature has risen to 10° C., what different effects will be

noticed and what relative *times* will have been occupied in the three processes; namely,

(i) raising the temperature of the ice,
(ii) melting the ice,
(iii) raising the temperature of the water?

10. How far would 1 lb. of water have to fall if it is to be heated 1° F. by the work done upon it by the earth?

11. How much work in the form of heat will be obtained from condensing one ton of steam at 100° C. into water at the same temperature?

VII. Applications. Heat Engines

The Steam Engine

210. We have seen how Joule transformed mechanical energy into heat by means of his raised weight and pad-

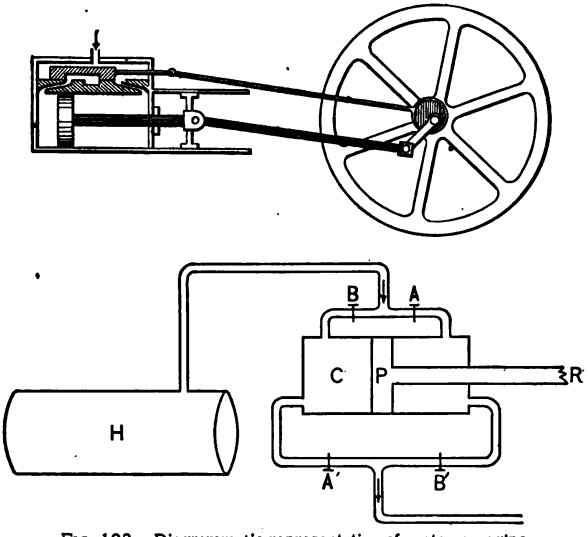


Fig. 193. Diagrammatic representation of a steam engine.

dle. The steam engine is a machine devised to accomplish the reverse operation, viz. to transform heat into mechanical energy.

The essential features of the ordinary **steam engine** are the following:

1. A **boiler,** *H*, in which heat, furnished by combustion of fuel, is employed to evaporate water.

2. A **cylinder,** *C*, in which slides a close-fitting **piston head,** *P*, and a **piston rod,** *R*, as indicated in Fig. 193.

3. A **steam chest,** *S*, containing a **slide valve,** as shown in Fig. 195.

4. A crank and **connecting rod** by which the to-and-fro motion of the piston rod is transformed into a circular motion of the shaft and pulley.

The principle of the steam engine is most easily understood from the diagram in Fig. 193; the actual working parts of a modern engine are represented in Fig. 194.

In Fig. 193 *A'* and B' are valves which serve to connect or disconnect either end of the cylinder and the boiler. In like manner *A* and *B* are valves which serve to connect or disconnect either end of the cylinder with the open air.

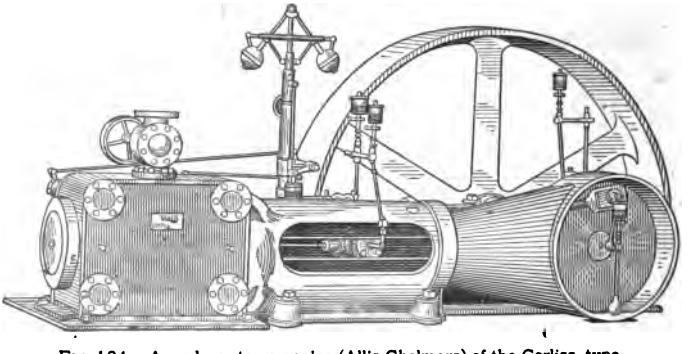


FIG. 194. A modern steam engine (Allis-Chalmers) of the Corliss type.

Let us imagine that the water in the boiler has been heated until the inclosed steam is exerting considerable pressure. With the piston in the position shown in the

figure, imagine the valves A and A' to be opened at the same time. Steam will rush in at the right of the piston head and will push it to the left.

If now A and A' are closed, while B and B' are each opened, the steam at the right-hand end of the cylinder will escape to the open air; fresh steam from the boiler will rush through B and drive the piston back to the other end of the cylinder. B and B' are now closed, while A and A' are again opened; the piston is again driven to the left; and so indefinitely.

If only we had a means of opening and closing these four valves automatically, the whole process would be smoother, more simple, and more economical. And this is exactly the function of the **slide valve** which is shown in one of its older and simpler forms in Fig. 195.

The steam from the boiler always enters at D, and escapes to the open air through the pipe E, which is called the **exhaust** or **escape pipe**.

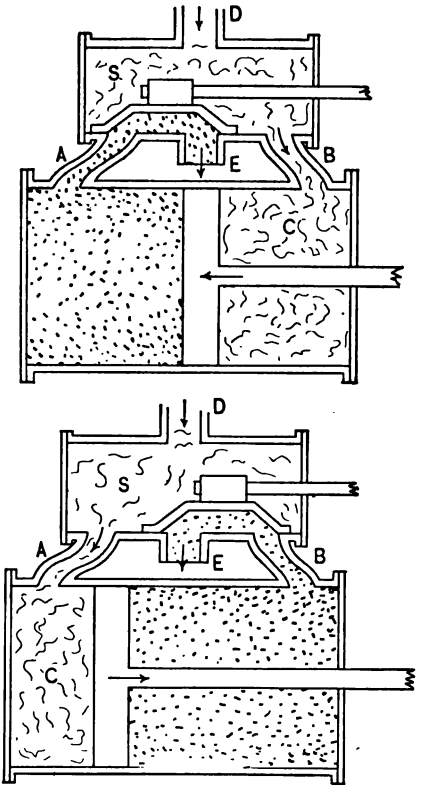


FIG. 195. Slide valve shown in two typical positions.

The pipes A and B connecting the steam chest and

cylinder are called **ports**; they serve alternately to let the steam in and out of the cylinder.

As will be seen from the two accompanying figures, the slide valve is worked to and fro. At one end of its stroke it connects *A* with *E* and *B* with *D*; the piston is then driven to the left; at the other end of the stroke, *A* is connected with *D* and *B* with *E*. The piston is then driven to the right, as indicated by the arrow. This motion of the slide valve is produced by a crank or an eccentric on the shaft, which is driven by the piston; consequently the machine is automatic.

The essential feature of this automatic device was introduced in 1713 by Humphrey Potter, an English boy employed to open and close the valves by hand as had been the custom up to that time.*

In marine engines (and in many others also) the exhaust steam is collected, condensed into water, and again returned to the boilers. The same water is thus used over and over again. The steam is not used up, but is merely a "working substance," a carrier to transfer the energy of the combining coal and oxygen in the furnace to the machinery which is doing mechanical work.

The Steam Turbine

211. On account of the to-and-fro motion of the piston, the machine just described is called a "reciprocating engine." But there has recently come into use a sort of modified water-wheel in which the driving fluid is steam. Here the axis of the driving shaft is the axis of the steam

* The student who would know more of the history or the theory of the heat engines should consult the very readable article of Professor Ewing on this subject in the "Encyclopædia Britannica"; also the biographical sketch of James Watt, there given; or Thurston, "Growth of the Steam Engine" (Appleton).

cylinder. This driving shaft carries a series of blades upon which the steam acts both by impulse and by expansion, imparting a motion of continuous rotation to the shaft. The steam turbine has few moving parts, runs very smoothly, and is very efficient in driving such machines as the propeller shaft of a steamship or the armature of a dynamo.

The Gas Engine

212. The two preceding engines derive their energy from fuel which is burned outside the cylinder. But there has been perfected within recent years a type of machine which burns its fuel (either gas or a spray of oil) inside the cylinder itself. When the proper fuel is used the products of combustion — the ashes so to speak — are entirely gaseous, so that the cylinder keeps itself clean. These are the little engines so familiar to us on automobiles. In most of them there are four steps through which the "working substance" — the gas — is taken. These are indicated in Fig. 196, and are called the "Otto cycle" after the German engineer, Otto, who did much to perfect the modern gas engine.

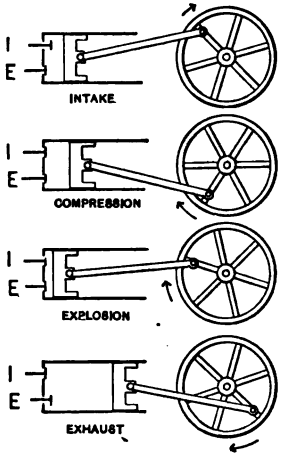


FIG. 196. The four cycle gas engine.

In these engines the piston does work only when moving in one direction; they are therefore said to be "single acting" to distinguish them from the ordinary reciprocating steam engines which do work on both the forward and backward strokes, and are hence said to be "double acting."

The use of the intake and exhaust valves, *I* and *E*, shown in Fig. 196, will be sufficiently clear from that diagram. The four steps in the Otto cycle are:

1. The intake valve admits the explosive mixture, say, air and gasoline vapor, during the forward stroke.
2. Compression then occurs during the back stroke.
3. Explosion is produced by an electric spark, and useful work is done during the forward stroke.
4. The products of combustion are swept out through the exhaust during the following back stroke, and the engine is left exactly where it was at the beginning of the cycle.

PROBLEMS

1. Assuming that the specific heat of iron is 0.12, find the number of heat units required to raise the temperature of 300 gm. of iron from 20° C. to 360° C.

2. Why is it that alcohol or ether poured on one's hand gives the sensation of cold?

3. How much heat must be given to 12 gm. water at 40° C. in order to make it boil?

4. Why are damp clothes apt to make one chilly?

5. Ten pounds of water at 0° C. are mixed with 40 lb. of water at 50° C. Find the temperature of the mixture.

6. Give five illustrations of the transformation of some form of energy into heat.

7. What is meant by a "land breeze"? By a "sea breeze"? How do you explain these?

8. Write an examination paper of five questions on the subject of heat.

9. A harvesting machine, made partly of wood and partly of iron, is allowed to stand overnight in an open field. In the morning, which parts will be covered with dew, the iron or the wood? Why?

10. What are clouds, and how are they produced?

11. Why is it that in winter ice which is covered with dirt or ashes melts sooner than clean ice?

12. Why are teapots often provided with wooden handles?

13. Fifty cubic centimeters of water are heated from a temperature 15° C. to 35° C. Find the change in volume thus produced.

14. A steel rail 10 m. long will change its length by how many millimeters when heated from 0° C. to 50° C.?

15. Why do you pour hot water on the neck of a bottle when you wish to loosen the glass stopper?

References

MAXWELL — *Theory of Heat* (Longmans, Green & Co.), pp. 343. Price, $1.25. First four chapters especially valuable for the beginner.

TAIT — *Heat* (Macmillan Co.), pp. 368. Price, $2.00. Exceedingly lucid. Covers the whole subject.

TYNDALL — *Heat as a Mode of Motion* (Appleton), pp. 529. Price, $2.00. A series of popular lectures, marked by their wealth of experimental illustration.

HELMHOLTZ — *Popular Lectures, First Series* (Appleton). Price, $2.00. Seventh Lecture recommended.

KELVIN — Art. "Heat" in *Ency. Brit.*, pp. 554–565 are especially recommended to beginners.

PRESTON — *Theory of Heat* (Macmillan Co.), pp. 719. Price, $5.00. A treatise recommended to the teacher as an excellent compendium of both experimental and mathematical results.

EWING — *The Steam Engine and Other Heat Engines* (Cambridge Press). Price, $3.75.

TYNDALL — *The Forms of Water* (Appleton). A series of popular lectures dealing with the forms of water in snow and rain, ice and glaciers.

GRIFFITHS — *Thermal Measurement of Energy* (Cambridge Press, 1901), pp. 135. A series of four admirable lectures. Elementary.

THURSTON — *Growth of the Steam Engine* (Appleton).

CHAPTER VII

MAGNETISM

213. Any body which attracts iron filings is said to be magnetized, and is called a magnet.

One kind of iron ore is found to be already magnetized as it comes from the earth. If a piece of this ore (which mineralogists call **magnetite** or **lodestone**) be dipped into a box of iron filings, it is seen that certain regions of it attract the filings very strongly, while the remaining parts attract scarcely at all.

Those parts which attract the iron filings most strongly are called **magnetic poles.**

Leading Facts of Magnetism

214. The following nine facts, presented in essentially the order in which they were discovered, are the most important phenomena connected with this subject.

FIG. 197. The mariner's compass. A magnetic needle is suspended above this card.

1. If a piece of magnetized iron or a piece of lodestone be freely suspended, it will always set itself so that a certain direction in it makes a fixed angle with the geographical meridian; *i.e.* with the north and south line.

This fact is illustrated by the ordinary compass and constitutes its fundamental principle. A long, slender piece of steel is magnetized

by drawing it over another magnet or lodestone, and is then mounted on a sharp pivot so that it is free to turn in any direction. This idea of the compass, which dates from a time certainly not much later than 1200 A.D., is the first important discovery in the science of magnetism.

In the case of a long, slender magnet, such as is used for a compass needle, there is, in general, one pole near each end. **That pole which turns toward the north, when the magnet is free to rotate, is called the north-seeking pole, or simply the north pole; that pole which turns toward the south is called the south pole. The north pole is taken as +; the south pole as −.**

The direction of the line joining the poles is called the **axis** of the magnet.

The direction assumed by the axis of a freely suspended magnet is called the magnetic meridian.

215. 2. The discovery of Columbus. Columbus used the compass on his first voyage to America. Up to this date it had been supposed by European navigators that while the suspended magnet in the compass did not always point exactly north and south, it did always point in the same direction; *i.e.* they supposed the angle between the geographical and magnetic meridians to be constant. But Columbus, much to his astonishment, found that as he proceeded west, the needle pointed to a part of the sky which lay more and more to the west.

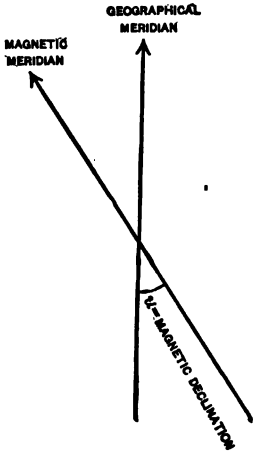


FIG. 198.

The angle between the magnetic

and geographical meridians is called the magnetic declination (Fig. 198). It is generally expressed in degrees, minutes, and seconds.

What Columbus discovered, then, was that the **magnetic declination varies from one point to another on the earth's surface.** This we may call the second important discovery in magnetism.

The following table shows the values of the declination in different parts of the United States, to the nearest degree. W. and E. indicate west and east declinations, respectively.

Magnetic Declinations (1902)

Augusta, Me.	16° W.	New Orleans	5° E.
Boston	12° W.	Kansas City	8° E.
New York	9° W.	Omaha	10° E.
Cleveland	2° W.	Denver	14° E.
Knoxville, Tenn.	0°	San Francisco	17° E.
Chicago	3° E.	Tacoma, Wash.	23° E.

The Magnetoscope

216. In the laboratory one has frequently occasion to use a compass needle of some sort. A most convenient and easily prepared form is the following: A wooden block about one decimeter square is bored halfway through by an auger (extension bit), which has the same diameter as the glass chimney of an Argand lamp. The chimney is then waxed into the block, and a small steel magnet is suspended by a fiber through a cork at the top. Such an instrument (Fig. 199) is not disturbed by currents of air in the room. It is called a **magnetoscope**, because it indicates by the

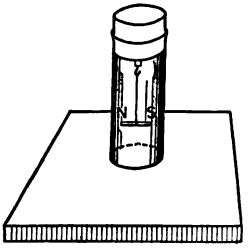


FIG. 199. Simple form of magnetoscope.

deflection of the magnet the presence of a magnetic body in the neighborhood.

A less sensitive form of compass needle, but one very convenient in the laboratory, is shown in Fig. 201. It consists of a small bit of steel wire supported on the point of a needle. Small compasses, like those made for watch charms, are useful. (See Fig. 203.)

217. 3. The magnetic dip. Not long after the time of Columbus it was found by Hartmann (1489–1564) that if an iron needle be suspended so as to lie *horizontal* before magnetization, it takes up a different position after magnetization. Imagine the needle before it is magnetized to be horizontal. After magnetization it is found that the north end "dips" down as if it had become heavier than the south end. But Norman (1576), the inventor of the dipping needle, represented in Fig. 200, weighed the iron needle, and found that it was no heavier after magnetization than before. As we shall see later, this dip is due to the fact that the earth itself is a great magnet. **Only the direction, therefore, and not the weight, of the needle is changed by magnetization.** In the southern hemisphere it is the south end of the magnet which "dips."

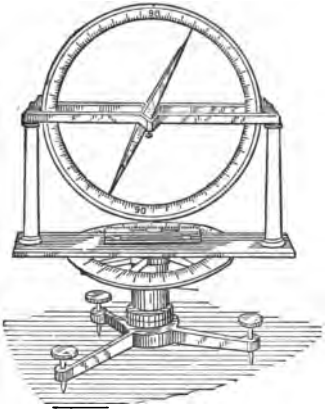


FIG. 200. Dipping needle.

In the United States it is customary to attach a small weight to the south end of a surveyor's compass needle in order to counteract the effect of the dip.

The phenomenon of magnetic dip might be called the *third* important discovery in this subject.

218. 4. **Like poles repel each other; unlike poles attract each other.** The evidence for this may be shown in a variety of ways; perhaps the most convenient method is the following: Mount two magnets, as shown in Fig. 201. Determine the north end of each, and slip bits of paper over them as indicated. When one magnet is brought near the other, the repulsion between the two north poles or between the two south poles is very evident; likewise the attraction between a north and south pole.

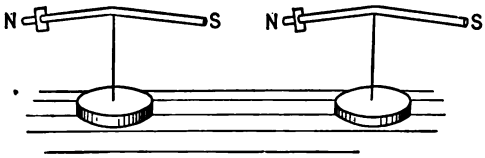


Fig. 201. Showing attraction between north and south poles.

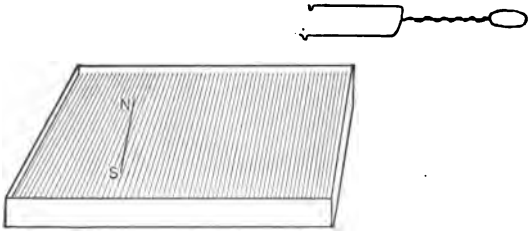


Fig. 202. Floating magnet and wire for placing it on the water.

Norman magnetized a sewing needle and floated it on water. By bringing into the neighborhood another magnet, he had a delicate means of showing attraction and repulsion of the poles.

The Magnetic Field. Lines of Force

219. If a sheet of white paper be laid over a magnet, and iron filings be sprinkled over the paper, one obtains very striking evidence that the region about the magnet is very different from other parts of space. The iron filings arrange themselves in curved lines; and these curved lines remain the same, however many times we repeat the experiment.

"We next introduce a method of conceiving and describing magnetic actions which was invented and much used by Faraday. Since a magnet acts upon a magnetic needle placed anywhere in the surrounding space, we call that space the **magnetic field** of the magnet. Neglecting the earth's magnetism, we may map out this field as follows: Conceive any plane drawn through the axis of the magnet and place it so that this plane shall be horizontal. Then, at any point in this plane, place a very small magnetic needle, and note the direction which its axis assumes under the action of the magnet; then proceed to move the center of the needle in the direction in which its north pole points, and continue the motion so that at each point the center is following the direction indicated by the north pole.

"The line thus traced will at last cut the surface of the magnet at some point lying toward the south pole; and if we continue the line backward by following the direction continually indicated by the south pole of the needle, it will cut the surface of the magnet at some point lying toward the north pole. Such a line is called a **line of magnetic force**; and since one such line can be drawn through every point in the plane and any number of planes can be drawn through the axis of the magnet, we can conceive the whole magnetic field filled with such lines." — CHRYSTAL, "Ency. Brit.," Art. "Magnetism."

Figure 203 indicates the process of mapping these fields. **Lines of force are assumed to run out of the north-seeking pole and into the south,** and the direction of the line of force at any

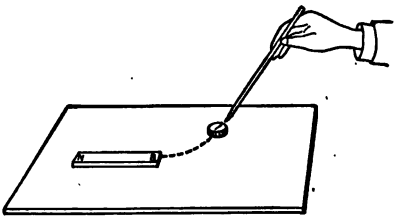


FIG. 203. Method of mapping magnetic fields.

point is the direction in which a north-seeking pole placed upon it would move.

Figure 204 shows the field about a strong bar magnet. The effect of the earth's field is here negligible.

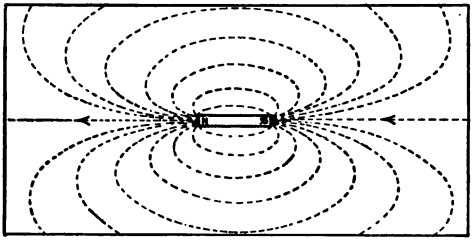


FIG. 204. Field about a straight magnet.

Figure 205 represents a small portion of the field due to the earth alone.

A magnetic field as actually mapped in the laboratory

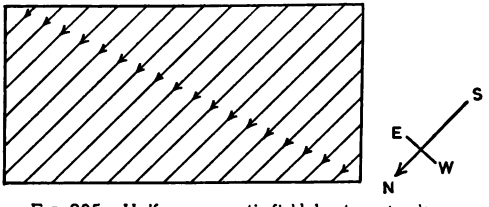


FIG. 205. Uniform magnetic field due to earth alone.

will always be the resultant of the earth's field and the field of the magnet.

220. 5. The earth itself a great magnet. We have seen that at any one point on the earth's surface the compass needle points in a definite direction, generally not far from north. We have seen that, in the northern hemisphere, the north end of the needle dips; while in the southern hemisphere, the south end dips.

But this is exactly the manner in which a small com-

pass needle would behave in the presence of a large iron sphere which had been magnetized. Accordingly Dr. Gilbert* was led to suggest that the earth itself was a large magnet. And since his time this theory has been confirmed by navigators who have located the north magnetic pole of the earth on Boothia Peninsula not far from Baffin's Bay, and the south magnetic pole definitely between Australia and the south geographical pole.

If we map the magnetic field of the earth by means of compass and pencil as indicated in the preceding paragraph, we shall find that **the lines of force due to the earth are, in any limited region, practically parallel.**

221. 6. **The earth's action a couple.** In a preceding chapter (p. 9) we have considered the distinction between a motion of translation and one of rotation. To a system of forces which can produce rotation only, the name **couple** has been given. Norman offered the following three experiments to show that **the earth does not exert any force of translation upon a magnet**, or, as we might say, three reasons for thinking that the force which the earth *does* exert upon a magnet is a *couple*.

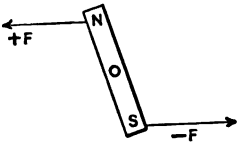


FIG. 206. Illustrating the couple which the earth's magnetic field exerts upon a pivoted magnet.

"First, he weighed several small pieces of steel in a delicate balance, and then magnetized them, but could not detect the slightest alteration in their weight, 'though every one of them had received virtue sufficient to lift up his fellow.'

"Secondly, he pushed a steel wire through a spherical piece of cork, and carefully pared the latter so that the

* *Dr. William Gilbert* (1540–1603), the leading man of science in England during the reign of Queen Elizabeth.

whole sank to a certain depth in a vessel of water and remained there, taking up any position about the center indifferently. After the wire was magnetized very carefully, without disturbing its position in the cork, it sank to the same depth as before, neither more nor less, the only difference being that now the wire set itself persistently in a definite fixed direction parallel to the magnetic meridian, the north end dipping about 71° or 72° below the horizon.

"Thirdly, he arranged a magnetized needle on a cork so as to float on the surface of water, and found that, although it set in the magnetic meridian, there was not the slightest tendency to translation in any direction.

"He concludes that there is no force of translation on the magnet, either vertical or horizontal."

—CHRYSTAL, "Ency. Brit.," Art. "Magnetism."

222. 7. Magnetic quality disappears at red heat. Any substance which is attracted or repelled by a magnet is said to possess **magnetic quality.** It has been found that the only substances which exhibit magnetic quality to any great extent are iron, nickel, cobalt, and an alloy of copper, aluminum, and manganese known as Heusler's Alloy.

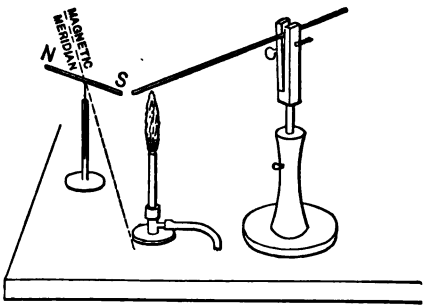


FIG. 207. Illustrating the disappearance of magnetic quality at red heat.

Iron is the only really important magnetic substance known. But if a piece of iron or steel be magnetized and then heated until red hot all over, it will be found to have lost its magnetism.

As soon as the iron has cooled again, however, it will be found to have regained its magnetic quality, for it will now attract *either* end of a compass needle. There is, then, a marked difference between a piece of iron which is magnetized and one which is not. The iron which is not magnetized still possesses *magnetic quality* at all ordinary temperatures; but it does not possess any *magnetic poles* or any *magnetism*.

The evidence for thinking that magnetic quality disappears at high temperatures is simply shown as follows:

Mount a compass needle some five or six inches long on a single pivot as shown in Fig. 207. Bring near it in a wood clamp a piece of unmagnetized iron wire, say five or six inches long. The compass needle will magnetize the wire and will in turn be deflected from the magnetic meridian so that one end of it will swing about and touch the wire.

If now the iron wire be heated in the flame of a Bunsen burner until it becomes a bright red, the compass needle will "let go" and will swing back into the meridian, showing that the hot iron has lost its magnetic quality. On removing the flame the iron will cool and one end of the compass needle will be seen to swing around into contact with the iron, showing that the wire on cooling has regained its magnetic quality.

223. 8. **Magnetization a molecular property.** The evidence furnished by the preceding experiment, together with the fact that a magnet — say a piece of magnetized watch spring — may be broken into an unlimited number of pieces (Fig. 208),

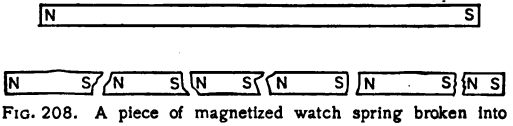


FIG. 208. A piece of magnetized watch spring broken into fragments, each fragment remaining a complete magnet.

each of which remains a complete magnet, with north and south pole, leads us to think that, whatever magnetism may be, it is something which belongs to the smallest particles of which the body is made up.

The experiment of breaking up the magnetized watch spring should be tried by each student for himself. The fragments are easily examined by use of the magnetoscope.

A tube of iron filings represented in Fig. 209 may be magnetized by placing it across the poles of a strong permanent magnet. It may be again demagnetized by shaking. And it is not improbable that when we heat a magnet red hot we produce such violent motions in its smaller parts as to continually "shake out" any magnetization that might be introduced.

FIG. 209. A tube of iron filings magnetized.

A long wire of soft iron — say 70 centimeters in length and one or two millimeters in diameter — may be strongly magnetized; but if it receive even a slight jar, its magnetization will almost entirely disappear. This also would lead us to think magnetization not a property of the surface of the body, nor of the body as a whole, but of its innermost structure; that is, its molecular structure.

This we may call the *eighth* important discovery in magnetic science.

224. 9. **The phenomena of magnetic induction.** If we bring a piece of soft iron near a magnet, we shall find that the iron acquires magnetic poles; *i.e.* it exhibits magnetization as well as magnetic quality.

A wire nail is made of soft iron, and hence shows this phenomenon very easily, as indicated in Fig. 210.

If the iron nail be tethered at its lower end, it will remain suspended in midair. That it is a magnet is at

once shown by the fact that its lower end will attract small nails or iron filings.

A wire nail held near *either* end of a magnetoscope will *attract* the needle. But if the magnet, *NS*, be brought up near the nail, the latter will attract one pole of the magnetoscope and *repel* the other.

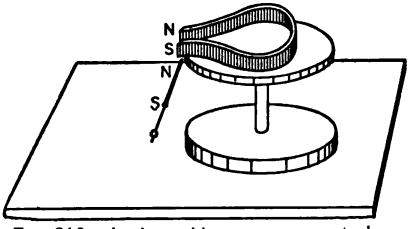


FIG. 210. A wire nail becomes a magnet when brought into a magnetic field.

One of the very best illustrations of magnetic induction is the following: Take a strip of tin plate, so called (which is really sheet iron plated with tin), having dimensions something like 1 inch by 12. If this strip has not been magnetized, either end of it will attract the north pole of the needle in a magnetoscope. Next hold the strip with one end down and bend it to and fro sharply between your thumb and forefinger at various points throughout its length. On examining it with the magnetoscope it is observed that the end which has been held down in the earth's field now repels the north pole of the needle. The bending apparently gives freedom to the particles of iron, and they probably arrange themselves in a new way under the influence of the earth's field. If now the other end of the strip be held down during the bending process, the poles of the strip will be reversed. Explain this by a diagram.

To magnetize a piece of iron, therefore, we need not touch it with a magnet; we need only bring it into the neighborhood of a magnet, *i.e.* into a magnetic field.

This process is called magnetic induction, more properly magnetization by induction.

It is here all-important to note that the original magnet loses none of its own magnetism by inducing magnetism in another body.

If we assume that a magnet is made up of small particles, or molecules, and that each particle is a magnet, then the phenomenon of magnetic induction is easily explained as follows: A piece of iron is made up of molecular magnets which are arranged in no particular order, arranged, in fact, in the utmost disorder, as indicated in Fig. 211. But when this iron is brought into a magnetic field, — when, for instance, it is brought near a magnet, — the north poles of these little magnets will each tend to point in a definite direction; and the south poles each in an opposite direction. The result will be that the little magnets will be brought more or less into line, producing a north pole at one end of the iron and a south pole at the other, as indicated in Fig. 212. The iron is then said to be magnetized by induction.

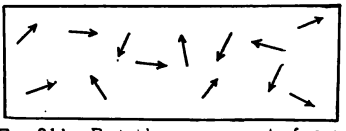


FIG. 211. Probable arrangement of particles in a piece of ordinary iron.

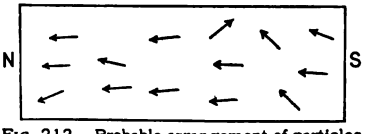


FIG. 212. Probable arrangement of particles in a piece of magnetized iron.

The Electro-magnet

225. One of the most remarkable and fundamental facts in electrical science is that every electric current is surrounded by a magnetic field; or, in other words, every electric current carries with it a magnetic field. Accordingly, a piece of iron brought into the neighborhood of an electric current will in general become magnetized by induction, just as it would if brought near a permanent magnet. **Such an arrangement of an electric circuit and iron is**

called an electro-magnet. As we shall see in a later chapter, the electro-magnet is really the fundamental feature in the telegraph, telephone, dynamo, motor, and transformer.

226. Iron has great conductivity for lines of magnetic force. If a piece of iron be placed in a magnetic field, the lines of force in that part of the field will, in general, change their direction in such a way as to pass largely through the iron. These lines of force exhibit no such preference for brass, zinc, copper, and other non-magnetic metals.

Summary of Leading Facts concerning Magnetism

227. 1. A magnet, free to rotate, sets itself in a definite direction.

2. The direction of the magnetic meridian varies from one point to another on the earth's surface.

3. In the northern hemisphere, the north end of a compass needle dips down; in the southern hemisphere, the south end dips down.

4. Like magnetic poles repel each other; unlike attract.

5. The earth itself is a great magnet.

6. The earth's action on a magnet is a couple.

7. Magnetic quality disappears at red heat.

8. Magnetization is a molecular property.

9. Bodies may be magnetized by induction.

Iron has great conductivity for lines of magnetic force.

References on Magnetism

Ency. Brit. — Art. "Compass." Should be read by every one.

CHRYSTAL — Art. "Magnetism," *Ency. Brit.*

EWING — *Magnetization of Iron and Other Metals.* Undoubtedly the best treatise on the subject in the English language.

THOMPSON, S. P. — *Elementary Lessons in Electricity and Magnetism* (Macmillan Co.). An excellent book for information on this subject.

JACKSON, D. C., and JACKSON, J. P. — *An Elementary Book on Electricity and Magnetism* (Macmillan Co.). Practical, simple, popular, and interesting.

CHAPTER VIII

ELECTROSTATICS

228. If a sheet of dry writing paper which is held against the wall be given a few quick flaps with a piece of flannel, the paper will adhere to the wall quite strongly. The rubber handle of a fountain pen, when rubbed over the sleeve of one's woolen coat, acquires the property of attracting small bits of paper or wood: glass rubbed on silk behaves in the same way.

The substance earliest found to have this property was a fossil resin (ordinary amber) found on the shores of Greece.*

These facts have been known ever since the time of Thales, who lived some six hundred years before the beginning of the Christian era; and, for two thousand years following Thales, these simple facts constituted practically the whole of electric science, excepting some knowledge of lightning and of the electric eel, which no one then imagined to have any connection with the amber phenomenon.

Bodies that have acquired this property of attracting small bits of paper are said to be electrified, or to have an electric charge.

First Law of Electrostatics

229. In more recent times, however, it has been found that *any* two different substances when rubbed together, or even when brought into contact, become electrified.

* The Greeks called this resin *electron* (ἤλεκτρον). And hence later Dr. Gilbert called this phenomenon *electric*, and this gave us still later our word *electricity*.

But before we can examine the evidence for this statement it will be necessary for us to devise some means by which we can detect the presence of an electric charge. One method is that indicated above, viz. the attraction of bits of thread or straw; but this is not a very sensitive test: much smaller charges can be detected by the gold-leaf **electroscope,** an instrument which we shall now describe.

The Electroscope and Proof Plane

230. This device is essentially a metal rod supported on glass, and carrying two leaves of aluminum or gold foil suspended from its lower end, as shown in Fig. 213.

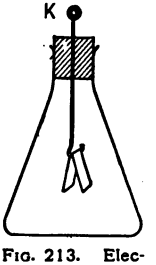


Fig. 213. Electroscope made of Erlenmeyer flask and aluminum foil.

If, now, any electrified body be brought into contact with the knob *K*, or, as we shall see presently, even into the near neighborhood of *K*, the leaves of gold foil will diverge. If the body is not electrified, the leaves will not diverge.

We have thus a simple and fairly delicate means for detecting electric charges. The theory of the electroscope we shall consider presently.

Try touching the tip of a camel's-hair brush on the table top and then to the electroscope. Rub the brush gently over your hair and again bring it to the electroscope. **Since electrification is produced by such slight means, we are led to think that any two different substances brought into contact become electrified.** This is possibly the most fundamental fact of electrostatics, and may be called the **First Law of Electrostatics.**

But we sometimes wish to examine the electrification of a body which is too large to carry to the electroscope,

or of a body which we do not wish to move. In such a case, it is most convenient to wax a penny to the end of a glass or rubber rod. (See Fig. 214.) We may use the

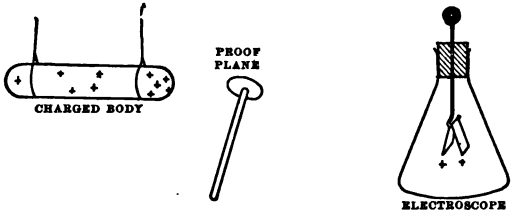


FIG. 214. The use of a proof plane.

glass rod as a handle, and then by touching the penny alternately to the body and to the electroscope, we may determine whether or not the body is charged. Such a device is called a **proof plane.**

CONDUCTORS AND NON-CONDUCTORS

231. A second great discovery in electrostatics was made by Stephen Gray in London (?–1736). He found that when a glass rod is electrified by rubbing, the electrification "leaks off" if the rod is held in the hand, or supported by cotton thread, or by metal, or by a moistened string of any kind. By connecting an electrified glass rod to a distant point by means of a wet string, he was able to transfer the electrification through a distance of many meters.

He found also that if a body containing an electric charge were supported on glass or rubber, or suspended by a dry silk thread, the charge disappeared very slowly. **Those substances which lead off the charge quickly are called conductors; those which prevent the charge from escaping are called non-conductors, or insulators.** A conductor supported upon a non-conductor is said to be

insulated. The following list of substances is arranged in the order of their ability to conduct electricity:

Substances	Relative Conducting Power
Silver	1,000,000,000
Mercury	16,000,000
Gas carbon	400,000
Sulphuric acid, dilute	160
Gutta-percha	0.000,000,000,004

Two Kinds of Electrification

232. A *third* great discovery was made in electrostatics by a French soldier and electrician, Dufay (1698–1739). He found that if a light, *electrified* body, say a pith ball, be suspended by a silk thread, other electrified bodies behave always in one of two ways when they are brought near the suspended body. Some charged bodies will attract the pith ball, and some will repel it. The electrification which appears on glass when rubbed with silk behaves in one way; the electrification which appears on gutta-percha (or a rod of sealing wax) when rubbed with flannel behaves in the opposite way. Dufay called these two kinds of electrification *vitreous* and *resinous*, respectively; but the following names suggested by Benjamin Franklin (1706–1790) are in some respects better, and have been adopted by the whole world:

Definitions. — 1. **That kind of electrification which makes its appearance on glass when rubbed with silk is called positive.**

2. **That kind of electrification which appears on sealing wax or gutta-percha when rubbed with flannel is called negative.**

It will be observed that this definition is a purely arbitrary one. The fact of nature is that there are two

kinds of electrification, but which one we shall call positive and which one negative is a matter of choice. The scientific world has, however, agreed upon the definition given above.

Figure 215 may aid the student to bear in mind not

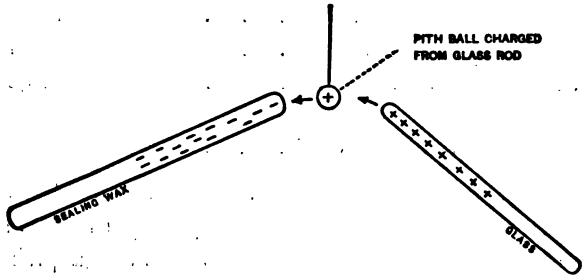


FIG. 215. Differences in behavior of the two electrifications.

only this convention, but also Dufay's evidence for thinking there *are* two kinds of electricity.

It is important here to observe that the pith ball is always repelled by the glass rod when it has been charged from the glass rod, while it is always repelled by the sealing wax if it has been charged from the sealing wax; in other words, **like electrifications always repel each other.**

We now see why it is that the gold leaves of the electroscope always diverge when charged, for whichever of the two electrifications is given to the electroscope the leaves will each be charged in the same way, and will, therefore, repel each other. It is very important to observe that the electroscope gives the same indication, *i.e.* a divergence, for a positive charge as for a negative one. How we can determine when we are dealing with a positively charged body and when with a negatively charged body is explained in the following section.

Experimental Distinction between Positive and Negative Charges

233. First charge the electroscope with a known kind of electrification, say positive. Then if the positive charge on the electroscope be increased, the divergence of the leaves will increase; if, however, we give to the electroscope a negative charge, the divergence of the leaves will diminish.

If we know the original charge on the leaves to be positive, we shall have then no difficulty in saying just what the sign of any other charge is; for if the unknown charge increases the divergence, it must be positive, but if it diminishes the divergence, it must be negative.

SECOND LAW OF ELECTROSTATICS

234. If we assume now that two different substances when brought into contact become electrified, and that bodies having charges of the same sign repel each other, while bodies having charges of opposite signs attract, the next inquiry is, *How* do these bodies attract (or repel) each other? This question, it will be observed, is, for electrical charges, exactly the same one which Newton asked concerning the behavior of pieces of ordinary matter.

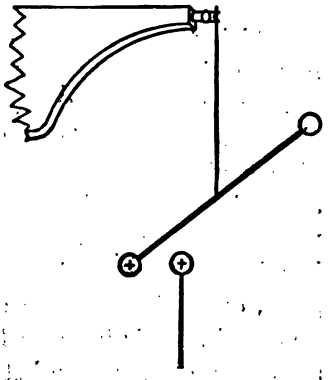


FIG. 216.

The French physicist, Coulomb (1736–1806), answered the question for electricity by a series of experiments which are difficult to repeat, though very simple in idea. He showed that the experimental facts are as follows:

(1) The repulsive (or attractive) **force between two quantities of electrification** (Fig. 216) **is directly proportional to the product of the two quantities.** Equal quantities of electricity can always be obtained by exactly duplicating any process which produces electrification.

(2) **The force** which one charged particle (or a body acting as a particle) exerts upon another **varies inversely as the square of the distance separating the bodies.**

(3) In addition to the above two facts discovered by Coulomb, it was shown by Cavendish and Faraday that **the force** which one charged particle exerts upon another **depends** also **upon the material of the non-conductor which separates them.** This non-conducting material is generally called the **medium, or the dielectric.**

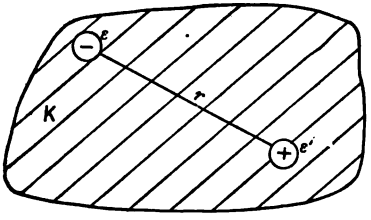


FIG. 217.

The essential thing here is to understand that the force which one charged particle exerts upon another depends upon the medium in which the particles are situated, as well as upon the distance between the particles.

In this respect, then, the attraction of one electric charge for another is quite different from the attraction of one piece of ordinary matter for another, because the attraction which the earth, for instance, exerts upon a piece of quartz, say, is not appreciably affected by inserting between the earth and the quartz a slab of sulphur, glass, or other substance; while if glass, mica, or other non-conductor is inserted between two charged bodies, the attraction or repulsion between these two bodies is diminished.

The Electric Field

235. When a rubber rod receives a charge, there is a region about the rod throughout which this charge is able to exert attractions or repulsions upon other charged bodies. **The space about a charged body over which it is able to exert force is called its electric field.** The force thus exerted upon a charged body is measured in dynes just as gravitational force is measured, and it produces the same effects upon bodies as gravitational force. An electrical force of 13 dynes acting upon a charged body will produce the same acceleration in the body as any other force of 13 dynes.

A Second Method of Producing an Electric Charge. Electrostatic Induction

236. For charging bodies there are other methods besides friction. One of the most interesting of these is the following, which appears to have been discovered by Stephen Gray.* He found that **one may charge a body, provided this body be a conductor, by simply bringing it into the field of another charged body.**

Let A (Fig. 218) be a charged body. A tin can on a block of paraffin, or a hollow brass sphere supported on a glass stem, will answer well. Let B indicate an insulated conductor which is not charged. A block of wood covered with lead foil and supported on a glass or rubber stem will answer well. Bring B into the neighborhood of A — say within three or four centimeters of it — and examine B with the proof plane and electroscope (p. 259) as follows:

1. Touch the proof plane to the charged body, A. Carry this charge to the electroscope. You will get a distinct separation of the gold leaves.

* Poggendorff, "Geschichte der Physik," p. 839.

2. Touch the proof plane to the end of *B* which is nearest *A*, and carry the charge to the electroscope while the leaves still diverge. The divergence of the leaves diminishes, showing that the charge on this part of *B* is opposite to that on *A*.

Repeat the process until the leaves have completely collapsed and have again diverged. They are now charged with the same kind of electrification that is on the near end of *B*.

3. Touch the proof plane to the end of *B* that is most remote from *A*. The leaves begin to collapse again, showing that the two ends of *B* have charges of different sign.

This experiment should always be repeated until the evidence is perfectly clear for thinking that when an uncharged body is brought into the field of a charged body, the uncharged body becomes electrified exactly as indicated in Fig. 218, above. **The charges on *B* are said to be induced, and the phenomenon is called electrostatic induction.**

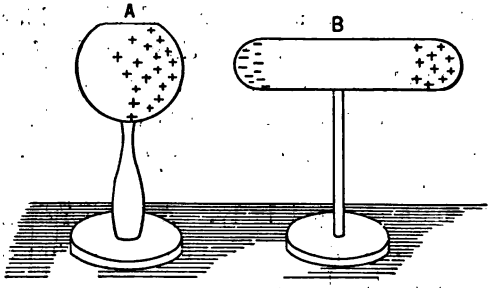


FIG. 218. The phenomenon of electrostatic induction.

At this point it is all-important to note that the induced charge on *B* does not at all diminish the charge on *A*. It is produced by means of the charge on *A*, but not at the expense of the charge on *A*.

If the charged body *A* is now removed from the vicinity of the originally uncharged body *B*, the latter no longer shows any signs of being electrified. **Apparently any**

charged body is able by induction to sort out from an uncharged body equal and opposite charges, attracting those charges that are unlike its own to the side nearest itself and repelling those that are like (Fig. 218).

The unlike charge that is attracted **is called a bound charge; the like charge that is repelled is called a free charge**; for reasons made evident by the following experiment.

While A is near B, connect B with the earth by any conductor (Fig. 219). The like charge that is repelled travels along the conductor and distributes itself over the surface of the earth. In other words, it is free to leave the vicinity of the charged body, A. If the connection between B and the earth is now broken and A is afterwards removed to a distance, it is found that the unlike charge induced by A still remains upon B; or, in other words, the unlike charge on B is not free to leave the vicinity of A and so is called a **bound** charge. This method of induction, therefore, allows us to obtain an unlike charge equal to the original without decreasing the original charge.

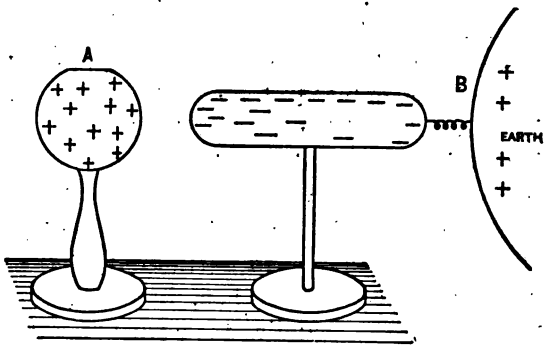


FIG. 219. Charging by induction.

The Electrophorus

237. An excellent illustration of charging by induction is found in the electrophorus, an instrument invented by

the Italian physicist, Volta, for the purpose of producing electric charges quickly and easily. It consists of a plate of hard rubber, rosin, or some other substance which can be readily electrified by friction, and of a metal disk, *C*, which is provided with an insulating handle, *H*. (See Fig. 220.) This metal disk with its handle is called the *carrier*. The hard rubber plate, *P*, generally rests upon a sheet of metal *S*, which is called the *sole plate*. Suppose the hard rubber plate to have been negatively electrified by rubbing with a catskin. Then when the carrier is made to approach the plate of ebonite, the induced charges will be distributed as shown in Fig. 220. When the carrier is placed *upon* the plate, it will be supported by slight elevations, here and there, so that it will not receive a charge from the plate. If now the finger is touched to the carrier, the negative charge will run off to earth and the positive charge will remain **bound**. When, however, the carrier is removed by the handle to a distance from the plate, this bound charge becomes **free** and available for charging other bodies. Evidently this process may be repeated as many times as one likes.*

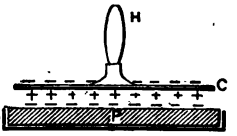


FIG. 220. Electrophorus of Volta.

Every student may easily make an electrophorus for himself by pouring a tin pie plate full of sealing wax, or a mixture of beeswax and rosin, and using another and somewhat smaller pie pan for a carrier.

* Machines have been perfected by the joint labors of Holtz, Toepler, and Wimshurst by means of which one can start with an indefinitely small charge on a body, *A*, and produce an indefinitely large charge on *B*. These machines are based on the simple principles which we have just been studying. The student who desires to study their action will find them described in S. P. Thompson's "Electricity and Magnetism," pp. 55–68.

The Condenser. The Leyden Jar

238. It is found by experiment that, if two conductors such as two sheets of tin foil are separated by a sheet of glass and connected one to a source of positive electricity and the other to a source of negative, a very much larger charge can be imparted than if either conductor were charged alone. Such an arrangement of **two conductors separated by a non-conductor is called a condenser.**

Fig. 221. Leyden jar.

The Leyden jar (Fig. 221) is a condenser usually made of a glass jar, with tin-foil coatings extending about two thirds the distance from the bottom both inside and out. The top is wood or other non-conductor through which passes a brass rod. The upper end of the rod is usually a knob and at the lower end is attached a chain to make contact with the inner coating of tin foil.

To charge the jar the outer coating is grounded and charges from an electrophorus, or electrical machine, are brought to the knob. On connecting the outer coating with the knob the opposite charges on the tin foil flow together, frequently giving a strong spark.

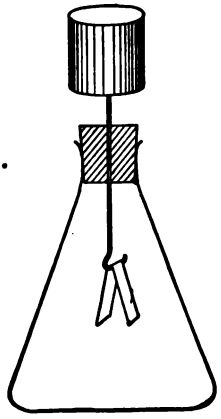


Fig. 222. Equality of positive and negative charges.

Third Law of Electrostatics

239. Let us now modify the electroscope, described on page 259, either by fitting it with a hollow metal vessel, as indicated in Fig. 222, or by connecting it, by use of a

light wire, to a hollow insulated vessel, as diagrammed in Fig. 223.

Any charged body introduced into this can will affect the gold leaves even more than if it were brought up near to the electroscope itself; for, indeed, the can is now a part of the electroscope. And, what is most remarkable, it matters not in what part of the can the charged body is placed, the effect on the gold leaves is just the same.

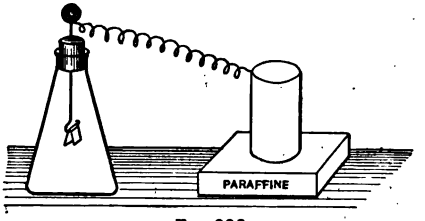


FIG. 223.

We are now prepared to try the following fundamental experiment. Either line the tin can with flannel or introduce into it a roll of flannel in such a way that you can rub on the flannel one end of a rod of hard rubber. Observe then that

1. No effect is produced on the electroscope so long as the rubber rod and the flannel are both left in the can.

2. But if either one be removed, the gold leaves diverge, showing that both flannel and rubber rod are charged.

3. That the charge on the rubber is opposite in sign to that on the flannel.

The same series of experiments may be tried with a piece of silk and a glass rod.

In each case we have two oppositely charged bodies such that their combined effect on the electroscope is zero. The same phenomenon is observed whatever two bodies are rubbed together. The inference is that whenever we produce by friction a certain electric charge, we always produce at the same time an equal charge of opposite sign. This is also true in the case of induction. If we induce

two opposite charges on a conductor and then remove the inducing charge, we find that the induced charges will just neutralize each other: all evidence of electrification disappears, showing that the two induced charges are equal and opposite. This general truth, expressed as follows, may be called the *Third Law of Electrostatics.*

When electrification is produced by friction, by induction, or by any other means, the positive and negative charges so produced are always equal.

Fourth Law of Electrostatics

240. It is very interesting to examine, by means of a proof plane and electroscope, the various parts of the surface of an irregularly shaped conductor, such as that represented in Fig. 224.

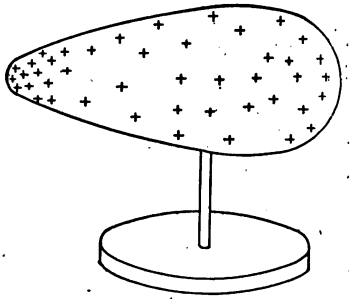


Fig. 224. Electric charge accumulates near sharp end of body.

The proof plane after being brought once into contact with a flat portion of the body, does not produce nearly so great a divergence of the gold leaves as when touched once to either end of the body. The sharper the end (*i.e.* the greater the curvature of the surface), the greater the charge which the proof plane carries away.

We see from experiments of this kind that the electrification is not uniformly distributed over a conductor. The sharper any end or point, the more the charge tends to collect there. When we examine the inside and outside of a tin cup in the same manner, we find that practically all of the charge resides on the outside.

In general, it has been found that the manner in which electrification distributes itself upon a conductor depends only upon the shape of the body and upon the presence of surrounding charges.

To describe *each* of these various distributions would be an infinite task; but we may always be certain that **the charge on the outside of any closed conductor is distributed in such a way that it will produce no electric field — no electric force — inside the closed conductor.**

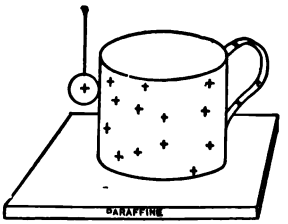


FIG. 225. Charge resides on outside of conductor.

Even in the case of a conductor such as a tin pail which is not quite closed, the electrification is very nearly all on the outside, and the intensity of the electric field inside is very small.

Accordingly, if we wish to protect any instrument from electrical disturbances, we have only to place it inside an electrical conductor which is closed or nearly closed.

Take a small electroscope and place it inside a roll of wire screen, as shown in Fig. 226. If the meshes of the screen are reasonably fine, there is scarcely any electrical disturbance you can produce outside the screen which will affect the sensitive gold leaves inside.

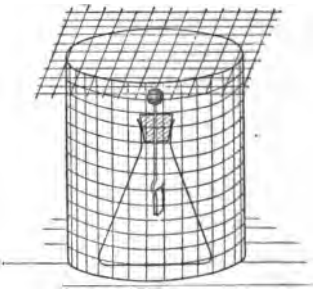


FIG. 226. Electroscope protected by a network of wires.

This great fact that any electrical disturbance outside a closed conductor has no effect on the region inside the closed conductor was thoroughly established

by Faraday.* We may call it the *Fourth Law of Electrostatics.*

Lightning Protectors

241. With this principle in mind, Maxwell suggested that the best method for protecting a house from lightning is to place it inside a network of conductors (Fig. 227), which shall be as nearly as possible equivalent to a closed conductor. Benjamin Franklin, in 1760, first suggested the use of single metallic rods, connected with the earth, as a means of protection against lightning.

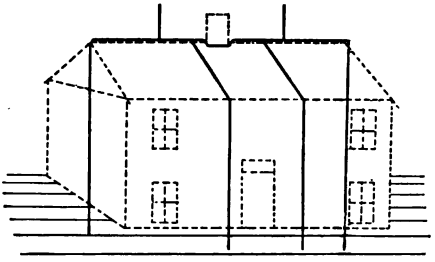


FIG. 227. Maxwell's method of protection against lightning.

SUMMARY OF FACTS ON ELECTROSTATICS

242. 1. Any two different substances become electrified by contact. (*First Law.*)

2. In a general way, bodies may be divided into two classes — *conductors* and *non-conductors.* — GRAY.

3. There are two kinds of electrification — *positive* and *negative.* — DUFAY.

4. Charged particles (or charged bodies which act as charged particles) attract or repel each other with a force which (*a*) varies inversely as the square of the distance, (*b*) varies with the medium, (*c*) varies directly as the products of the two charges. (*Second Law.*) — COULOMB.

* *Michael Faraday* (1791–1869), a man whose work in electricity outranks that of perhaps any other investigator, in a century distinguished above all others for its achievements in physical science.

5. Bodies may be charged by *induction.*

6. When electrification is produced by friction, by induction, or by any other means, the positive and negative charges so produced are always equal. (*Third Law.*)

7. A closed conductor shields the region inside of it from the influence of any electrical charges on the outside of it. (*Fourth Law.*)—FARADAY.

PROBLEMS

1. Explain how a gold leaf electroscope might be used to tell whether the electrification of a body is positive or negative.

2. A gold leaf electroscope is charged. I hold near it a metal plate connected with the earth or simply spread my hand over its knob. The divergence of the leaves at once diminishes. I remove the plate or hand and the divergence increases to its original value. Explain by a diagram.

3. A tree is a fairly good conductor of electricity. A cloud charged with positive electrification floats above the top of a growing tree. Will the tree become charged? If so, how? Make diagram. Is it more likely to be struck by lightning than a spot in the open?

4. A pane of window glass has a sheet of tin foil pasted on each side of it. Why is it possible to give it a larger charge if one of the coatings is "grounded"? Explain by diagram.

References on Electrostatics

S. P. THOMPSON—*Elementary Lessons in Electricity and Magnetism* (Macmillan Co.), pp. 628. Price, $1.40. First chapter especially recommended for the beginner.

M. FARADAY—*Experimental Researches*, 3 vols. The first volume is recommended as the best introduction to electrostatics to be found in the English or in any other language.

OLIVER LODGE—*Modern Views of Electricity* (Macmillan Co.), pp. 422. Price, $1.75. Splendid account of the philosophy of the subject; advanced theories are presented in a lucid manner. Should be read only by those who are quite familiar with the experimental facts.

RIGHI—*Physical Phenomena.* Trans. by A. Trowbridge (Macmillan Co.), pp. 178. Price, $1.10. Remarkably clear and scholarly account of recent work in radio-activity, electron theory, etc.

JACKSON AND JACKSON—*Electricity and Magnetism* (Macmillan Co.); pp. 482. Price, $1.40. Recommended for the beginner.

CHAPTER IX

ELECTRIC CURRENTS

243. If a charged conductor, such as any one of those we have just been studying, be discharged through a wire, it is observed that the wire and the region about it for a short time acquire new properties.

Suppose we choose for our charged body the inside coating of a Leyden jar. On winding a copper wire about a sewing needle, as shown in Fig. 228, no effect is produced on the needle; but if we take the copper wire and (handling it by means of a glass rod) connect one end to the inner coating and the other to the outer, we shall find that when the discharge occurs the needle becomes magnetized; that is to say, this little coil of copper wire has suddenly acquired the ability to magnetize a piece of steel. After the spark has occurred and the two conductors are discharged, it will be observed that the wire no longer has the ability to magnetize steel. **A wire which possesses this ability is said to have an electric current flowing through it.** An electric current produces a magnetic field about the conductor conveying it.

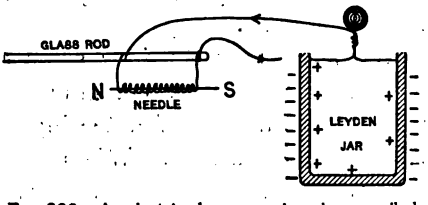


FIG. 228. An electric charge passing along a coiled wire magnetizes a sewing needle.

As we shall presently see, there are many other ways in which electric currents may be produced in wires.

Method of Studying Electric Currents

244. It will be remembered that in taking up the study of magnetism, we examined a number of magnetic phenomena, and employed a compass needle, or magnetoscope, as it is sometimes called, to detect the presence of a magnet. It is thus not difficult to say whether or not any particular piece of steel is a magnet; but it is very difficult to say just what *magnetism* is.

It is so with electrification; we were able to detect its presence very readily by means of the gold-leaf electroscope; though no one has yet been able to find out what *electricity* is. What copper or iron is no one knows, although very much else is known about copper and about iron. Electricity is, then, just as mysterious as copper; but no more so.

We shall now consider some of the phenomena connected with electric currents; but we shall first need some ready method of detecting the presence of an electric current in a wire. When we have once obtained this, we shall be in a position to study some of those remarkable properties of electric currents which have proved so useful during the nineteenth century.

This study we shall take up under two heads; namely:

I. The Production of Electric Currents.
II. Some Effects of Electric Currents.

The Magnetic Field about a Conductor

245. In order that he may clearly grasp the idea that a magnetic field surrounds a wire conveying an electric current, each student should see or perform the following experiment:

Pass a wire through a hole in the center of a piece of cardboard or glass plate (Fig. 229). Support the plate

horizontally and send a fairly large current (say 10 amperes) through the wire. On sprinkling iron filings over the plate they are observed to arrange themselves in circular lines. Jarring the plate while the current is flowing makes the circular form of the magnetic lines more distinct; but, if the plate is jarred while the current is not flowing, the circles are broken. The direction of the lines of force may be determined by placing a compass needle at various points on the plate.

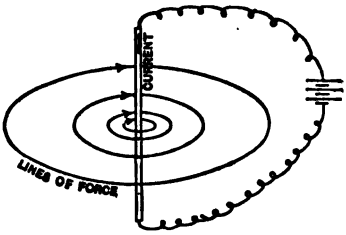


FIG. 229. Showing circular lines of force due to a current.

The direction of the lines of force is used to define the direction of the current as follows (Fig. 230) : **if the wire is grasped in the right hand so that the fingers point in the**

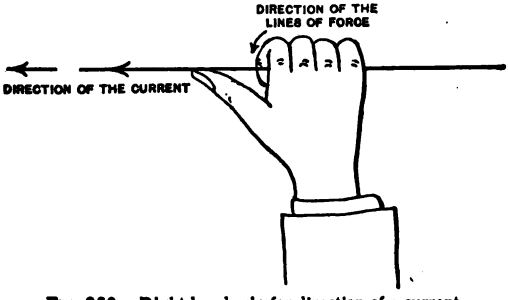


FIG. 230. Right-hand rule for direction of a current.

direction (sense) of the lines of force, the thumb outstretched along the wire points in the direction (sense) of the current.

As an experimental fact, it is then observed that **the direction of the lines of force about a wire carrying a current is related to the direction of the current as the direction of rotation is related to the forward travel of a right-handed screw.** This is called the **right-handed screw relation.**

If the current is passed through a wire bent into a loop, as in Fig. 231, lines of force pass through the loop in a direction perpendicular to its plane; and if the current is

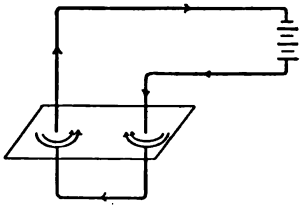


FIG. 231. Lines of force about a single loop.

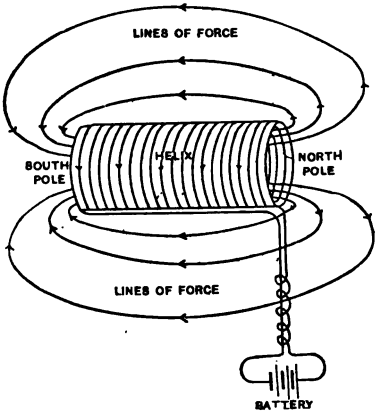


FIG. 232. Lines of force about a helix.

passed through a pile of loops, *i.e.* a coil of wire, the lines of force (Fig. 232) pass along the axis of the coil, making one end a north pole and the other a south. Such a coil is called a **helix**, and behaves like a magnet.

The Galvanometer

246. As we have already seen and shall see many times again, every wire conveying an electric current is surrounded by a magnetic field, which is just like that surrounding a magnet.

This magnetic field is such that, if the wire be placed parallel to a compass needle, the needle will be deflected. Such an instrument is one very commonly used to detect the presence of an electric current, and is called a **galvanometer.** Almost immediately after a current ceases to flow in the wire, the magnetic field about the wire ceases

to exist, and the wire no longer affects the compass needle. But practically as soon as the current begins to flow, **the needle tends to set itself at right angles to the direction of the current.**

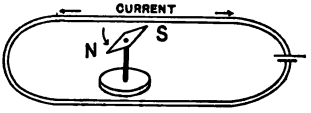


FIG. 233. A galvanoscope in which the magnet is movable, while the wire conveying the current is fixed.

This is the fundamental discovery made by the Danish physicist, Oersted, in 1820; and the first galvanometer ever employed was the wire and compass needle shown by Oersted to his students in the year mentioned.

If we consider the instrument merely as an indicator of currents — and not as a current meter — then we may call it a **galvanoscope.** And the first galvanoscope was probably a freshly prepared frog's leg, which twitched whenever a current passed along the nerve and muscles of the hind leg (Fig. 234). This discovery was made by Galvani (1737–1798), an Italian physiologist; and it is after him, accordingly, that the galvanometer is named.

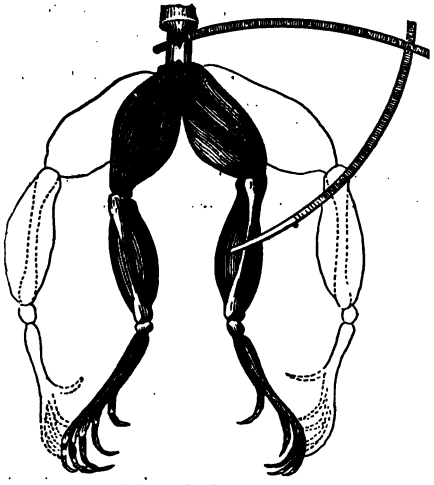


FIG. 234. The first galvanoscope.

It is sometimes more convenient to fix the magnet and allow the coil to move (Fig. 235) than to hold the wire fixed and allow the needle to move as did Oersted. We have already seen that a magnet is surrounded by lines of force, leaving the north pole and running into the south.

When a wire carrying a current is wound into a flat coil, one face of the coil becomes a north pole and the other a south. If the axis of the coil is in a north and south direction (Fig. 235), the north face of the coil will be attracted by the south pole of the magnet and repelled by the north pole. The movable coil then turns, thus indicating the passage of the current.

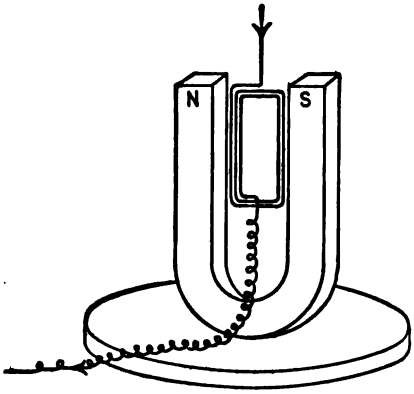


FIG. 235. A galvanoscope in which a part of the wire conveying the current is movable; while the magnet is fixed.

The form of instrument in which the magnet is fixed and the circuit moves is generally called, after its inventor, a **D'Arsonval Galvanometer.**

In general, **if a movable coil of wire carrying a current is placed in a magnetic field, it tends to move so as to include as many lines of force as possible.**

I. THE PRODUCTION OF ELECTRIC CURRENTS

247. With this preliminary statement we proceed to consider the two methods of producing currents which are at present most important, namely:

(*a*) The method of Galvani and Volta — the so-called voltaic cell.

(*b*) The method of Faraday — cutting lines of magnetic force — electro-magnetic induction.

(a) The Voltaic Cell

248. The simple and well-established facts of the voltaic cell are as follows:

At the very close of the eighteenth century (1799), it was discovered by the Italian physicist Volta that all conductors of electricity can be divided into two classes. This division is based upon the following experiments:

If we make a closed circuit out of several different metals, *i.e.* if we make an endless chain in which each link is composed of a different simple substance, such as zinc, copper, or iron, we see that no electric current is produced. **All substances which, when joined together at constant temperature in any order, as in Fig. 236, produce no current, are called conductors of the first class.**

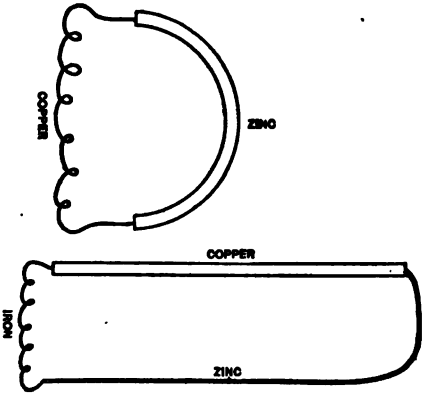


Fig. 236. Circuits made up entirely of conductors of the first class.

Volta found, however, that if into a circuit such as the above he introduced one link composed of a *compound* substance, such as brine (a solution of table salt) or dilute sulphuric acid or copper sulphate, he then always obtained an electric current. Conductors of this kind always undergo chemical decomposition when introduced into a circuit containing two different metals and always yield a current.

Conductors of this kind, which Volta called conductors of the second class, always undergo chemical decomposition when introduced into a circuit containing two different

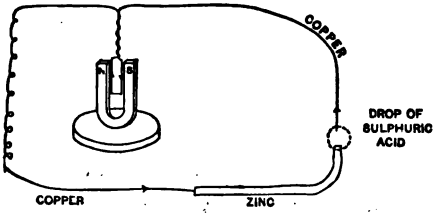


FIG. 237. Combination of conductors of first and second class producing a current.

metals and always yield a current. The modern name for a conductor of the second class is **electrolyte,** *i.e.* any substance which is decomposed when a current passes through it. Conductors of the first class practically include only carbon and the various metals.

Definition of a Voltaic Cell

249. It has been found by experiment that no two conductors when joined together will produce a current so long as they are at the same temperature. We may use any of the three combinations indicated in Fig. 238, but no current will thus be obtained.

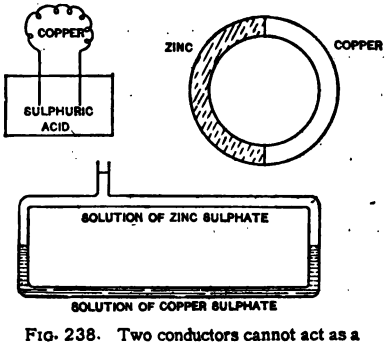


FIG. 238. Two conductors cannot act as a voltaic cell.

But the following combination, suggested by Volta and named after him the *voltaic cell*, will always give a current:

The voltaic cell is defined as three or more conductors in series, each conductor being made of a different substance and not all belonging to the same class.

A Typical Voltaic Cell

250. When a strip of pure zinc is immersed in dilute sulphuric acid, bubbles of hydrogen gas begin to collect

on the surface of the zinc and, at the same time, some of the zinc combines with the acid to form zinc sulphate. When a strip of copper is similarly treated, no such action is observed. If both strips without touching one another are immersed in the acid and connected externally by a wire (Fig. 239), a current of electricity flows through the wire from the copper to the zinc. It is also observed that, though the hydrogen bubbles are now given off at the surface of the copper plate, the chemical action takes place between the zinc and the sulphuric acid and that the current flows as long as the circuit remains unbroken or until either the acid or the zinc is used up.

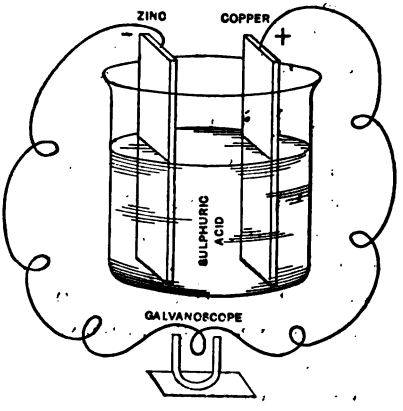


FIG. 239. A voltaic cell.

A circuit which contains two conductors such as copper and zinc, and one conductor such as sulphuric acid which is decomposed when the current passes, is a voltaic cell.

Some Definitions

251. Conductors. Substances like zinc, copper, carbon, and sulphuric acid which readily allow the passage of the current of electricity are called **conductors.**

Electrolyte. A conductor like sulphuric acid which is decomposed as the current passes is called an **electrolyte.**

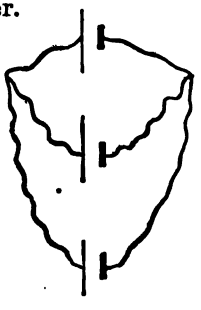
Closed circuit. A series of conductors joined end to end so as to form an endless chain is said to be a **closed circuit.**

It may or may not have an electric current passing through it.

Open circuit. When a closed circuit is cut or interrupted at any point, it is then called an **open circuit.**

Battery. A number of voltaic cells connected together is called a **battery.**

Series. Cells which are connected so that the copper of one cell is joined to the zinc of the next, and so on, are said to be joined up **in series**, and may be represented thus . The short thick line represents the zinc plate and the long thin line the copper.



Parallel, or **multiple.** When all the zincs of a number of cells are connected together and all the coppers are likewise connected together, the cells are said to be **in parallel** or **in multiple** and may be represented thus —

Poles. If there be a voltaic cell in an open circuit, the two ends thus left free are called the **poles** of the cell. The pole which is connected with the copper plate, or with the plate whose electrical behavior is like that of copper toward zinc, is called the **positive pole.** That pole which is connected with the zinc plate, or with a plate whose electrical behavior is like that of zinc toward copper, is called the **negative pole.**

Electrodes. We have already seen that at least one member in the chain of conductors which makes up a circuit of a voltaic cell must be an electrolyte, practically always a solution. The two conductors which connect with this liquid are called **electrodes.**

That electrode by which the current enters the electrolyte is called the **anode**; and that electrode by which the

current leaves the electrolyte is called the **cathode.** In the typical cell which we have just studied, the zinc plate is the anode and the copper plate the cathode.

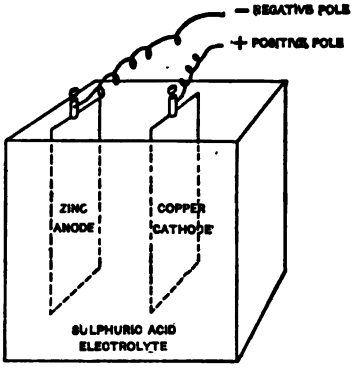


FIG. 240. Various parts of a voltaic cell.

Electrical pressure. When a current flows from one part of an electrical circuit to another, a difference of electrical pressure is said to exist between these two parts. The electrical pressure is that which moves or tends to move electricity from one place to another.

The Direction of the Current

252. A simple experiment gives a reason for regarding the current as flowing from the copper to the zinc instead of in the reverse sense.

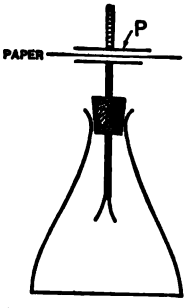


FIG. 241. An electroscope with plate top.

An electroscope is fitted with a plate top (Fig. 241), on this is laid a sheet of dry paper, and on the paper is set another plate, P, with insulated handle. The wire connected to the copper plate of a battery of voltaic cells is touched to the lower plate of the electroscope at the same time the wire connected to the zinc is touched to the upper plate. On removing the plate, P, and the paper, the electroscope is found to have a positive charge. If, however, the zinc terminal of the battery has been touched to the lower plate and the copper to the upper, the electro-

scope receives a negative charge. Hence, the copper of a voltaic cell has a positive (+) and the zinc a negative (−) charge, and we may say that **the flow of current through the external circuit is a transfer of charges from the positive to the negative plate.**

Inside the cell the electrodes in contact with the electrolyte set up a difference of electrical pressure resulting in a flow of current which is maintained by chemical action. Hence **the voltaic cell is merely a chemical device for maintaining a difference of electrical pressure.** We shall see later that the dynamo is a mechanical device for producing the same result.

Three Practical Forms of the Voltaic Cell

253. 1. The gravity cell. This cell is composed of a copper plate immersed in a saturated solution of copper sulphate, and a zinc plate immersed in a solution of zinc sulphate, generally arranged as indicated in Fig. 242.

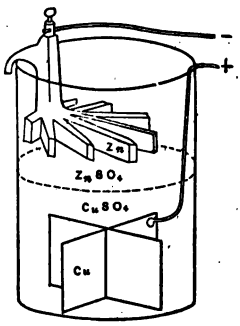


FIG. 242. A gravity cell.

The copper sulphate is somewhat denser than zinc sulphate, and hence remains in the lower half of the jar, while the zinc sulphate, when carefully poured in, floats on top. Hence the name *gravity* cell. The cell must not be left on open circuit because the two solutions will mix by diffusion and the zinc and copper sulphate will be wasted. When the circuit is closed and the electric current begins to flow, we observe that the zinc plate grows smaller, that the copper sulphate grows less and less, while the zinc sulphate increases and the copper electrode becomes covered with a coating of fresh metallic copper.

The metallic zinc has disappeared; the metallic copper has increased, at the expense of the copper sulphate. Accordingly, if such a battery is to furnish a current continuously, it must now and then be supplied with a new zinc plate and with a fresh solution of copper sulphate. The source of energy is here the oxidation of the zinc, which is oxidized — "burned," if you like — to form zinc sulphate.

This cell is well adapted to work in which small currents are needed very frequently or for a long while at a time.

2. **The Leclanché cell.** In this cell (Fig. 243) the two conductors of the first class are zinc and carbon; the electrolyte is a solution of ammonium chloride, sometimes called *sal ammoniac*. With the carbon is mixed some dioxide of manganese, and both are held in a porous jar or small canvas bag. The current easily passes through the pores of the jar or the meshes of the bag. The carbon plate here evidently corresponds to the copper plate of the gravity cell. Here again the zinc is used up whenever the poles are joined and the current flows. The sal ammoniac also diminishes. Accordingly, from time to time, the zinc and sal ammoniac must be renewed.

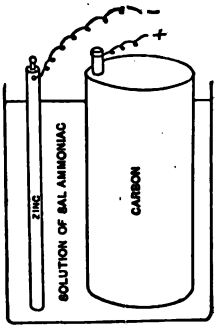


FIG. 243. A Leclanché cell.

The student has probably already asked himself why the oxide of manganese is mixed with the carbon. The answer is found in the fact that, if carbon alone is used, the carbon becomes covered with a layer of hydrogen gas soon after the circuit is closed; because the ammonium chloride (NH_4Cl) breaks up, giving off ammonia (NH_3) and depositing free hydrogen (H) at the carbon electrode.

The chlorine unites with the zinc. This coating of hydrogen causes the carbon plate to cease to act as a

carbon plate and makes it act more like a second plate of zinc. Two zinc plates in sal ammoniac would yield no current. **This is called polarization, which may be defined as a tendency to send a current in a direction opposite to that furnished by the cell.** The effectiveness of a cell is therefore always diminished when polarization occurs. The manganese dioxide uses up the hydrogen and keeps the carbon plate clean. The purpose of the manganese dioxide is then to prevent polarization. It does this by furnishing oxygen which unites with the hydrogen to form water; and once in union with oxygen the hydrogen is harmless.

The Leclanché cell is excellently adapted to furnishing a current for a small time, or for intermittent use, as in the case of a door bell or call bell. It will remain on open circuit with very little waste. As in the gravity cell the source of energy is the "burning" of the zinc, which is here "burned" to form zinc chloride.

The so-called "dry cells" which are now in such common use are, of course, not really dry. They are simply Leclanché cells in which the liquid electrolyte is held in some absorbent material so that it shall not spill during transportation.

3. **The storage cell.** In this cell the electrodes are of lead and lead peroxide, respectively; the electrolyte is dilute sulphuric acid. The lead plate is the negative one; the lead oxide is the positive one.

In Fig. 244 the positive and negative groups of plates are shown as they appear when separated from each other. In Fig. 245 they are seen assembled in the glass jar which contains the electrolyte — the dilute sulphuric acid. When the circuit is closed, the lead peroxide plate gives up a part of its oxygen, while the lead plate, the negative electrode, becomes oxidized, until finally the two elec-

trodes become very much alike, and the current, therefore, becomes less and less, other things being the same. The battery is now said to be discharged. For, as we have already seen (p. 281), two electrodes which are alike never give a current when immersed in any one electrolyte.

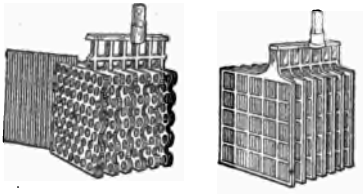


FIG. 244. Showing the separate plates of a storage cell.

But in order to put the cell again in good working shape, it is necessary only to pass an electric current through the cell in a direction opposite to that obtained when the cell was in use. This process is known as **charging** the cell. The charging simply restores the oxygen to the positive plate, and removes the oxygen from the negative plate, leaving it spongy, metallic lead.

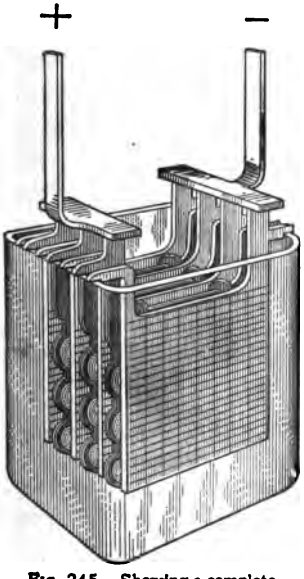


FIG. 245. Showing a complete storage cell.

In a cell of this type, in which the electrodes do not dissolve in the electrolyte, the source of energy is evidently the source from which the charging current is obtained — whatever that may be.

The following two features distinguish the storage cell from most other voltaic cells:

(*a*) The fact that the plates are prepared, *i.e.* made chemically different, by electrical means.

(*b*) The fact that they are capable of yielding currents

which are enormous compared with those from other forms of batteries.

The Commutator

254. In most of our work with batteries, whether in the class room or in the laboratory, we shall need to change the sense of the current in some part of the circuit. This can be done in many ways; but the device represented in Fig. 246 is typical of them all. The block, *ABCD*, is made of wood, and has four holes bored about halfway through it. These holes, placed one at each corner of a square, act as cups to hold mercury. Into the sides of the block are inserted four wires, *E*, *F*, *G*, *H*. Each of these wires connects with the mercury in the cup nearest it. The poles of the battery are joined to two diametrically opposite cups, say *B* and *D*; the wires from the rest of the circuit, say from a galvanometer, are joined to the remaining two cups, *A* and *C*.

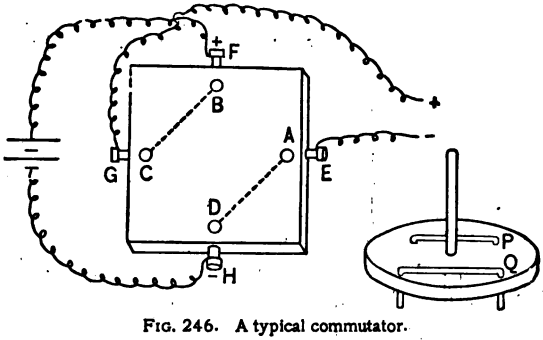


Fig. 246. A typical commutator.

For closing the circuit, two short, thick copper conductors, *P* and *Q*, are mounted on a movable block, as shown in the figure. These connectors, *P* and *Q*, may be placed in the mercury cups in such a way as to connect *B* with *A*, and hence *C* with *D*; or they may be placed so as to connect *B* with *C*, and hence *A* with *D*.

In the one case, the current flows from *E* to *G* through the galvanometer; in the other case, from *G* to *E* through the galvanometer. To change the sense of the current in the galvanometer circuit, we have, therefore, only to lift the top block and rotate it through 90° in either direction.

An instrument which may thus be used to reverse the sense of a current in part of a circuit is called a commutator. To "break the current" we have merely to lift off the top block. **Any device which is thus used to interrupt a current is spoken of as a key or switch.**

The *commutator* of a dynamo or motor is a mechanical device for rapidly changing the connections of the machine in such a way that the current always flows in one sense.

(*b*) The Induction of Electric Currents

255. Induction is the second of the two important methods mentioned above (§ 247) for producing electric currents.

We have already seen (p. 249) that the region about a magnet is endowed with properties totally different from other portions of space. This magnetic field, as it is called, does not appear to the naked eye to be different from any other portion of space; but we know that it is different, for a magnet brought into such a field behaves in a manner peculiar to this magnetic field. As has been pointed out, Faraday described such a magnetic field by conceiving it to be filled with lines of force — purely imaginary lines — drawn, at each point in the field, in the same direction in which the axis of a magnet would set itself if placed at that point. The earth itself being a great magnet, the whole of the space inhabited by the human race is a magnetic field. The lines of force which describe a magnetic field can be resolved into components (p. 41). We accordingly speak of the vertical component and the horizontal component of the earth's field, as represented in Fig. 247, where T represents in direction, sense, and amount the total magnetic force of the earth on any magnet pole; V represents in direction, sense, and amount the vertical component of T on the same pole;

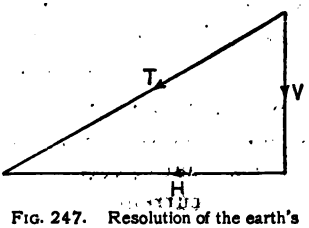


Fig. 247. Resolution of the earth's magnetic field into two components.

and H the horizontal component of T on the same pole.

We are now in a position to consider the simple method by which Faraday, in the autumn of 1831, first succeeded in producing currents without the use of the voltaic cell and without the use of heat.

If we consider any closed circuit (say a piece of copper wire bent into the form of a circle or square) placed in a magnetic field, we may think of it as containing or inclosing a certain number of lines of force, just as a fence might inclose a group of trees.

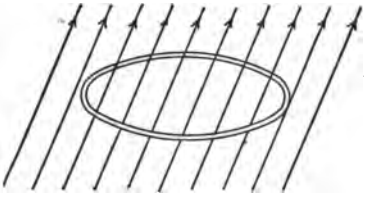


FIG. 248. Magnetic lines of force inclosed by an electric circuit.

In Fig. 248 six lines of force are represented as being inclosed by the wire circle. If by any means the magnetic field were made twice as strong, we should represent this change by drawing the lines of force twice as close together, so that now the circle would inclose twelve instead of six lines. Or, if the copper circle were carried parallel to itself to another region where the magnetic field were not so strong, then we should describe this change by drawing *fewer* lines of force through the circle.

Faraday's discovery was simply this. **When, by any means whatever, the total number of lines of force passing through any circuit is changed, an electric current is produced in that circuit.**

Such a current is called an induced current.

VARIOUS ILLUSTRATIONS OF INDUCED CURRENTS

256. *Experiment* 1. Connect the terminals of a D'Arsonval galvanometer with a copper wire, so that with the suspended coil a closed circuit is formed. Take a single

loop in this circuit, and thrust through it one pole of a permanent magnet. Such a magnet carries its magnetic field (*i.e.* its lines of force) about with it. And when these invisible lines are thrust through the loop, we observe that for the instant a current passes in the galvanometer. (See Fig. 249.) Now remove these lines by removing the magnet; a current passes, but now in the opposite sense. **Observe that in either case the current lasts only so long as the motion of the magnet continues, or only as long as the number of the lines of force is changing.**

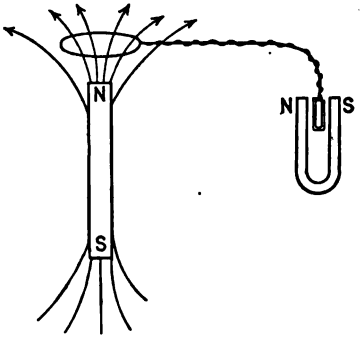


FIG. 249. An electric current produced by thrusting lines of force through a circuit.

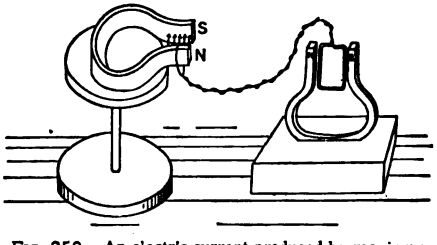


FIG. 250. An electric current produced by moving a coil in a magnetic field.

Experiment 2. Now support the magnet on some convenient stand, and slip the loop of wire over one end of the magnet. (See Fig. 250.) A current is again observed. If we slip the coil off, the sense of the current is reversed.

If the loop of wire be slipped on and off quickly, a current will be produced which flows first in one sense, then in the other.

(i) A current which changes its sense in a regular and continuous manner is called an *alternating* current.

(ii) But if the current during certain intervals of time ceases entirely, it is said to be *intermittent*.

(iii) Those which flow continuously in one sense are called *direct* currents.

From these two experiments, it is evident that it makes no difference whether it is the loop of wire or the lines of force that are moved. **The relative motion of the wire and the magnetic field changes the number of lines of force inclosed, and hence produces the current.**

Experiment 3. Since we live on the surface of a large magnet, it becomes interesting to see how we must move a closed circuit in the earth's magnetic field in order to produce an electric current.

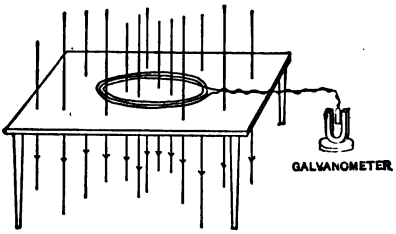


FIG. 251. An electric current produced by capsizing a coil of wire in the earth's magnetic field.

If a large loop of wire be made in one part of a closed circuit containing a galvanometer, we have only to hold this loop alternately in an east-and-west and in a north-and-south vertical plane. When the plane of the loop is vertical and north and south, no magnetic lines from the earth pass through it; they pass along each side of it. But if the coil be suddenly rotated about a vertical axis so as to lie in an east-and-west plane, the earth's lines of force will thread through it, and we observe in the circuit an induced current. If we rotate the coil of wire back into a north-and-south plane, the current is in the opposite sense. If in either of the preceding experiments we leave the total length of wire unchanged and take two turns of wire instead of one, the deflection of the galvanometer is observed to be twice as great as before. Therefore the

magnitude of an induced current depends on the rate at which the lines of force are cut.

Experiment 4. But there are still other ways in which we may alter the number of lines of force passing through a circuit. We have seen that a wire conveying a current is surrounded by a magnetic field. If we bring a loop of our galvanometer circuit near such a battery current, we shall find that the magnetic lines from the battery current will in general thread through the galvanometer circuit, and will thus produce an induced current. In Fig. 252 lines of force marked 3, 4, 5, and 6 each pass through the galvanometer circuit as well as the battery circuit.

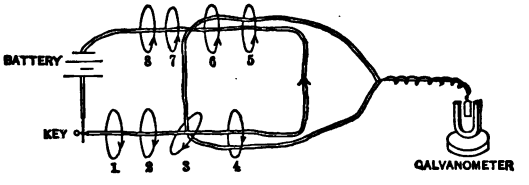


FIG. 252. An electric current produced in a loop of wire by making or breaking a battery circuit in the neighborhood of the loop.

When the motion ceases, the current ceases. In this circuit it is found that **the sense of the induced current is always such as to oppose the motion which produced it.** This statement is known as **Lenz's Law.**

If instead of bringing the loop and the battery current near together we simply place the circuits side by side and suddenly open or close the battery circuit, we observe in each case an induced current in the galvanometer circuit. To interrupt or break the battery current is equivalent to carrying it off to a very great distance. To close the battery circuit is electrically equivalent to bringing it suddenly into the neighborhood of the galvanometer circuit.

Primary and Secondary Circuit

257. The two circuits of which we have just been speaking are so frequently used in physics that special names have been given to them. The circuit **whose current furnishes the magnetic lines of force is called the primary circuit; the one in which the current is induced is called the secondary circuit.**

The facts of the case may be still more generally stated by saying that **any variation of the primary current will,** in general, **induce a current in the secondary.** For as the amount of current in any circuit varies, the number of lines of force about that circuit also varies.

It has been found that when the primary circuit is wrapped about a core of iron it furnishes an enormously greater number of magnetic lines of force than would be furnished if the circuit contained only, say, wood or air. This is the electro-magnet, which was invented and so named by Sturgeon in 1822, and which has proved to be an all-important factor in modern electrical machinery.

The Transformer

258. When both primary and secondary circuits are wrapped about an iron ring or core, as indicated in Fig. 253, the device is called a **transformer.** Whenever the current in the primary circuit, *P*, is made to vary, an induced current appears in the secondary circuit, *S*. Conversely, if a current be passed through the circuit *S* and made to vary, a current will be induced

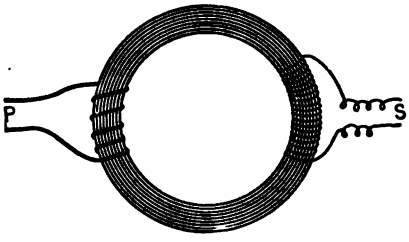


Fig. 253. A transformer.

in the circuit *P*. An instrument of this kind is in wide use for lighting houses by means of alternating currents. The primary coil consists, in this case (Fig. 254), of many turns of fine wire, while the secondary coil consists of a few turns of heavy wire. The result is that a small

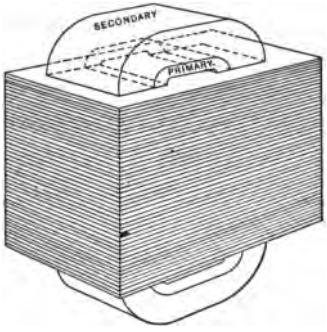


FIG. 254. Transformer cores are built up of iron plates in the manner here indicated.

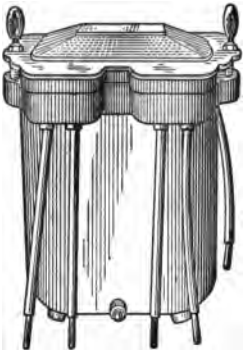


FIG. 255. Illustrating the commercial transformer.

alternating current of high pressure passed through the primary induces a large current of low pressure in the secondary. The exterior form which the transformer takes when used in lighting houses is seen in Fig. 255. This illustration shows merely the iron box which contains the core and coils indicated in Fig. 254.

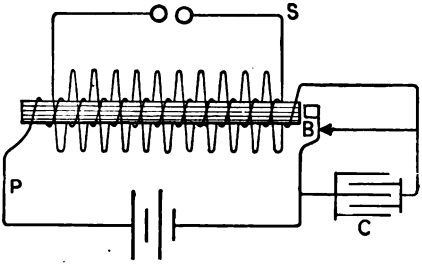


FIG. 256. Diagram of induction coil.

A particular kind of transformer — called an induction coil — is one in which both primary and secondary are wound, one on top of the other, on a cylindrical bundle of iron wires (Fig. 256). The current in the primary is rapidly made and broken by the

attraction and release of the hammer, *B*, thus inducing a current of such high electrical pressure in the secondary that a spark will jump between the terminals of the secondary. This instrument has a wide variety of uses in automobiles, telephone exchanges, wireless telegraphy, the laboratory, and in medical practice.

Sometimes a single coil of wire is wrapped on a bundle of iron wires, so that when the current is interrupted in this coil the lines of force inclosed disappear so rapidly that they produce a powerful induced current in the coil itself, thus causing a large spark at break. These are called *make-and-break spark coils* and are largely used for igniting the explosive mixture in gas engines.

259. *Experiment* 5. **The dynamo experiment.** A coil containing a large number of turns of wire is connected to a delicate galvanometer. This is the coil marked *secondary* in Fig. 257. Another coil, of a small number

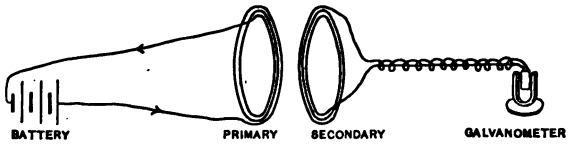


FIG. 257. The dynamo experiment.

of turns, marked *primary*, is connected to a battery and is set near the secondary. If the secondary is suddenly turned 90° into a horizontal position, a current flows through the galvanometer. Let us call the sense of this current +. If now the coil is rotated another quarter turn into a vertical position, the galvanometer shows a current in the opposite, or −, sense. Turning the coil through a third quarter, a − current is again produced; but, on the last quarter turn into the original position, the current is again +. The results of the experiment are summarized in the following table:

Rotation about a Horizontal Axis from the Vertical Position	Current
0° to 90°	+
90° to 180°	−
180° to 270°	−
270° to 360°	+

Evidently, if the rotation had been continuous, through half of each turn we would get a current in one sense and through the other half in the opposite sense. This is precisely the effect produced in the dynamo described in the succeeding paragraph.

The Dynamo

260. The dynamo is a machine consisting of a strong electro-magnet which we may consider the primary and a movable coil which we may consider the secondary. This movable coil is rotated by some mechanical means, such as an engine, so that the number of lines of force passing through the coil is continually altered.

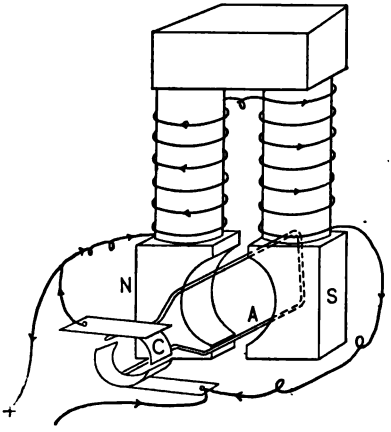


Fig. 258. The dynamo.

The movable coil *A* (Fig. 258), which is rotated in the magnetic field, is called the **armature**: the electro-magnet *NS*, which constitutes almost the entire weight of the modern dynamo and which furnishes the magnetic field, is called the **field magnet**. The current produced by the armature

is, in general, an **alternating current**, *i.e.* one which flows first in one sense, then in the other, changing, perhaps, a hundred times a second. But sometimes this armature is fitted with a device, *C*, which changes the connections of the armature coils with the using circuit whenever the sense of the current in these coils changes. The result is that the current in the using circuit always flows in *one* sense, although the current in any armature coil is alternating. This device is essentially a **commutator**, such as we have already studied (p. 290), only here the commutator is automatic. Such a machine is called a **direct current generator**. Sometimes the entire current from the armature is passed around the iron core of the field magnet. A generator of this type is shown in Fig. 259, and is said to be **series-wound**. These machines were formerly much used for arc lights. Another type of

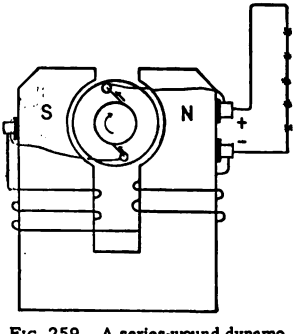


FIG. 259. A series-wound dynamo.

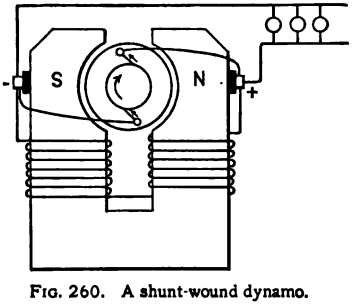


FIG. 260. A shunt-wound dynamo.

dynamo and that most frequently employed is one in which only a part of the armature current is diverted to the field magnets. These dynamos are said to be **shunt-wound**, and will be understood from an examination of Fig. 260. The quantitative discussion of the dynamo must be left for more advanced study.

The Bell Telephone

261. The ordinary telephone shown in Fig. 261 is an excellent illustration of the manner in which currents are induced in a circuit when the number of lines of force passing through that circuit is changed.

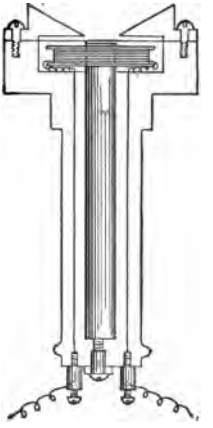


Fig. 261. A Bell receiver.

This form of telephone (which is now used only as an instrument for receiving messages, but which was formerly used also to transmit messages), consists merely of a thin iron diaphragm, underneath which is supported a permanent magnet, carrying on the end nearest the diaphragm a helix of fine wire. (See Fig. 261.) The lines of force which spread out from the upper end of the magnet tend to pass along the iron disk and to keep within it as much as possible. (See § 226.) Connect the two ends of the helix to the two terminals of a moderately sensitive galvanometer. Observe that when one thrusts the disk by means of his finger or lead pencil a little nearer to the end of the magnet, a current shows itself on the galvanometer; and when the diaphragm is released and springs back into its position of equilibrium, a current in the opposite sense is set up. In the same way, when the disk is set in motion by the air waves due to the human voice, the total number of lines of force passing through the helix is changed. This is the **Bell transmitter.**

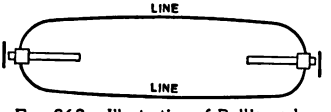


Fig. 262. Illustration of Bell's early telephone system.

The induced currents thus sent over the line are received by a similar instrument at the other end (Fig. 262).

The modern transmitter, devised by Berliner, is a **microphone.***

II. Some Effects of Electric Currents

A. Chemical Effects. Electrolysis

262. Every one has heard of plating spoons with silver, and bicycles and skates with nickel, even if he has never seen the process. All of these are instances of the general fact, largely established by Sir Humphry Davy, that when a current is passed through a solution, that solution is, in general, broken up into two other substances.

In the first year of the nineteenth century, the same year in which Volta described the Voltaic cell, it was found by Carlisle and Nicholson that a current passed through water will break the water up into hydrogen and oxygen. Hydrogen is given off at the cathode and oxygen at the anode.

Experiment 1. In order to pass a current through water it is necessary to dissolve some salt, or acid, in it to make the liquid a good conductor. The most convenient thing is to add 10 or 15 per cent of strong sulphuric acid.

Take a Wolff's bottle half full of the dilute acid; through two of the holes in the bottle insert two strips of sheet platinum soldered to copper wires, as indicated in Fig. 263: connect these to a battery of several cells joined up in series. A storage battery of five or six cells is most convenient. Gas will be given off from each electrode, and may be collected for examination by attaching one end of a rubber tube to the remaining neck of the bottle, and immersing the other end of the tube under a soap-solution. The escaping gas will thus blow a series of soap-bubbles which

* The student will find an account of telephone transmitters in Jackson and Jackson's "Electricity and Magnetism," pp. 354–356.

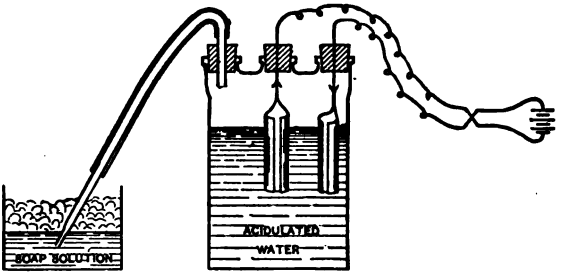


FIG. 263. The decomposition of water by an electric current.

may be exploded by a lighted taper (*Care!!*); and it has been shown that the gases are evolved in just the right proportion to explode, without leaving behind any oxygen or any hydrogen, *i.e.* the proportions of hydrogen and oxygen present are chemically equivalent.

Experiment 2. By using a U-tube with electrodes, one sealed into each branch, the gases can be collected before they have a chance to mix. If the arms of the tube are graduated, one can measure directly the volume of hydrogen set free, and also the volume of oxygen. Accurate measurements of this kind show that when the gases are collected under equal pressures the volume of hydrogen is exactly double that of oxygen. But chemists tell us that these

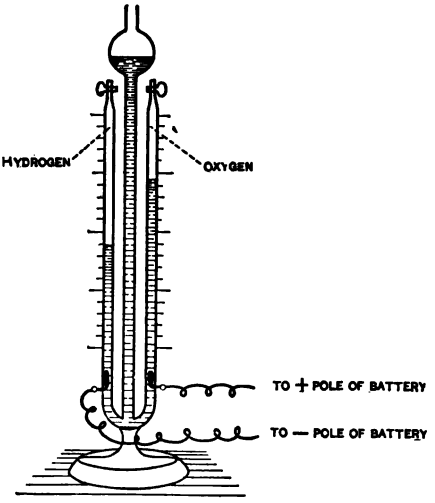


FIG. 264. Electrolysis of water in which the hydrogen and oxygen are collected in separate vessels.

are exactly the volumes of hydrogen and oxygen that unite to form water.

Experiment 3. Instead of acidulated water, we may use a solution of lead acetate as an electrolyte. The phenomenon is most conveniently seen or projected when the solution is placed in a flat vessel such as is represented in Fig. 265.

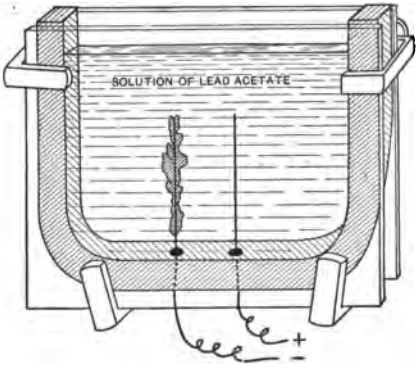


FIG. 265. Electrolysis of lead acetate.

If we use a pair of platinum wires as electrodes, it will be seen that the lead acetate, which is a clear transparent solution, is broken up so as to yield dark metallic lead at the negative pole of the battery, *i.e.* at the same pole where the hydrogen appeared in the preceding experiment. In general, hydrogen and all the metals behave as if they rode through the electrolyte on the current, *i.e.* they collect at the same pole at which the current leaves the electrolyte.

Experiment 4. If we immerse two copper plates in a solution of copper sulphate and pass a current from one plate to the other, the mass of the plate by which the

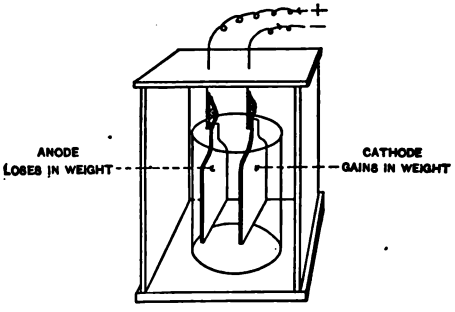


FIG. 266. The copper voltameter.

current enters will diminish, while the mass of the plate by which the current leaves will grow larger; but the copper solution remains practically unchanged. By accurately weighing the plates before and after the passage of the current, and noting the duration of the current, one can determine just how much copper any given current will deposit in one second of time. **Such a cell as this fitted with electrodes and electrolyte is called a voltameter.** The copper thus deposited is very pure, and this process is now largely employed for refining copper.

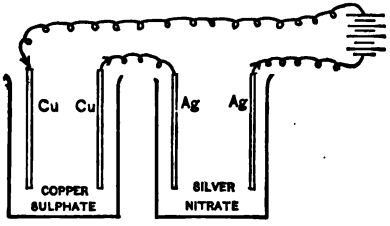


FIG. 267. The current produces equivalent decomposition in cells containing different electrolytes joined up in series.

Experiment 5. When several voltameters are connected in series after the manner shown in Fig. 267, the same current will pass through each of them. The weights of the metals deposited, while not equal, are chemically equivalent.

FARADAY'S LAWS OF ELECTROLYSIS

263. Faraday made a very careful study of many phenomena such as we have just been considering, and summarized his results as follows:

First Law

The amount (*i.e.* the mass) of an electrolyte which is decomposed by an electric current is proportional to the current flowing and to the time during which it flows.

Second Law

When an electrolyte, or a series of electrolytes, is decomposed by an electric current, the components into which it is

separated are always chemically equivalent, *i.e.* they are set free in just such amounts as may recombine and form a chemical compound without any left-over material.

Unit of Current. The Ampere

264. Since the amount of metal deposited in one second is directly proportional to the current passing, it is evident that we may use the amount of metal deposited in one second as a measure of the current. Accordingly the International Congress of Electricians, which met at London in 1908, agreed to define the **unit current as one which when passed through a solution of nitrate of silver deposits silver at the rate of 0.00111800 gram per second.** This unit is called an **ampere.**

Just why a current of this particular value is chosen as unity, and the process of reasoning which connects the ampere with the centimeter, gram, and second, are matters which must be left for more advanced study.

PROBLEMS

1. How much silver will be deposited by a current of 300 amperes in 8 hr.?

2. A current is measured by a silver voltameter. The cathode plate increases in weight 0.04 gm. in 1 hr. What is the current strength?

3. The electrochemical equivalent of aluminum is 0.0000936. How much aluminum can be obtained by a current of 3000 amperes in 10 hr.?

4. The United States furnishes about 275,000 tons of copper annually. How long would it take to refine this electrically if 10,000 amperes of current were at our command? A current of 1 ampere deposits 0.000329 gm. of copper per second. 1 lb. = about 453 gm.

5. How many grams of zinc will be used by a gravity battery which furnishes 0.003 ampere of current for 30 days? The electrochemical equivalent of zinc is 0.000338 gm. per second.

B. Heating Effects of Electric Currents

265. In these days every one is familiar with the fact that when an electric current is passed through a carbon filament, the temperature of the filament may be increased to such a point that the carbon becomes white-hot.

Trolley cars are heated by electric currents passing through coils of wire. Electrically heated flatirons and chafing dishes are frequently used. Iron chains are now made by welding together the two halves of each link by means of an electric current. Even the rails in some railway tracks are welded together by the same process.

Between the years 1840 and 1843 Dr. Joule made a very careful study of the heat produced by an electric current. He found that *for any one piece of wire* the amount of heat produced by any current depends upon two things only:

1. Upon the amount of current flowing, *i.e.* upon the number of amperes.

2. Upon the time during which it flows.

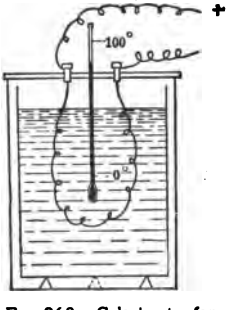


Fig. 268. Calorimeter for measuring the rate at which heat is produced by an electric current.

By use of a calorimeter containing a coil of wire, it was shown that when the current in the coil of wire was *doubled*, the heat was developed *four* times as fast as before; and when the current was *trebled*, the heat was imparted to the water *nine* times as rapidly as before.

But if any particular current were allowed to flow first for 5 minutes, then for 10 minutes, it was found that the amounts of heat (calories) developed in these two intervals were in

the ratio of 1 : 2. And Joule found that in general he could describe the results of his experiments as follows:

Joule's Laws of Heating

1. In any given conductor, the heat developed by an electric current in any given time varies directly as the *square* of the current.

2. In any given conductor, the heat developed by any given current is directly proportional to the time during which the current flows.

3. In any conductor the heat produced by a given current flowing for a given time is directly proportional to the resistance of the conductor.

Electrical Resistance

266. Hitherto, we have been considering the effect of a current upon a single conductor. Suppose, however, we take a series of different conductors and join them end to end, and pass one current through the entire chain. Will the heating effect be the same on each link of the chain? An experimental answer to this question may be obtained as follows:

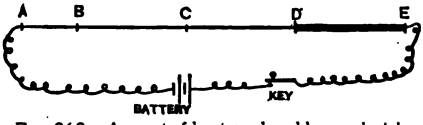


FIG. 269. Amount of heat produced by an electric current depends upon length, diameter, and material of wire.

Join together four wires as represented in Fig. 269, where

AB is 10 cm. of iron wire;	0.25 mm. diam.
BC is 20 cm. of iron wire;	0.25 mm. diam.
CD is 20 cm. of copper wire;	0.25 mm. diam.
DE is 20 cm. of iron wire;	1.00 mm. diam.

Through this series pass current sufficient to heat the part *AB* red hot. This will require about 4 amperes. Observe, then,

(1) that the part BC (which is exactly similar to AB except in length) is just as hot as AB, and consequently is giving off twice as much heat as AB;

(2) that the copper wire CD (which is exactly like BC except in chemical composition) is very much cooler than BC;

(3) that DE (which is exactly like BC except that its area of cross section is larger) is much cooler than BC.

From experiments of this kind, in which, however, all the quantities involved were accurately measured, Joule concluded that the amount of heat which a given current develops in any conductor in a given time depends upon three factors:

(1) *The length* of the conductor, the amount of heat being directly proportional to the length;

(2) *the chemical composition*, *i.e.* the kind of material of which the conductor is made; thus we have seen that the iron wire becomes much hotter than copper wire of the same size; in like manner a carbon filament becomes much hotter than an iron wire of the same size, other things being equal;

(3) *the area of cross section*, the amount of heat being inversely as this area. For this reason the insurance companies require wire of not less than a certain area of cross section to be used in leading a current to incandescent lamps. If the wire be too small, there is danger of its getting hot and setting fire to the building.

But for any given conductor these three constants, *length*, *chemical composition*, and *cross section* may all be grouped into a single constant which is called the **electrical resistance** of the conductor and which we may denote by R.

The resistance of a wire, or any other conductor, is simply a quantity which determines how much heat a given current will produce in it in a given time.

Returning now to the series of conductors represented in Fig. 269, it is evident that the conductor BC has twice the resistance of the conductor AB; for the same current generates twice as much heat in BC as in AB.

The Unit of Resistance. The Ohm

267. A conductor is said to have **unit resistance** when its resistance is equal to that offered by **a column of mercury of uniform cross section, at 0° C., when its length is 106.3 cm. and its mass is 14.4521 gm.** This unit is called an **ohm.**

When a wire is said to have a resistance of three ohms, it is meant that a given current will generate heat in that wire three times as rapidly as in the mercury column above described.

The advanced student will find the connection between the *ohm*, the *centimeter*, and the *second* a most interesting chapter in electrical science.

Joule's Law in Terms of Algebra

268. Let us denote by

R the resistance of any conductor, measured in ohms;

C, the current flowing in this conductor, measured in amperes.

Then the amount of heat, H, generated in this conductor during t seconds of time will be given by the following equation:

$$H = \frac{C^2Rt}{4.2} \text{ calories}$$

This may be considered as the "defining equation" for R, since we have already learned how to measure each of the other three quantities involved.

In like manner, if we denote by

l, the length of any conductor, measured in centimeters;

s, the area of cross section, measured in square centimeters;

k, a constant depending upon the chemical constitution of the conductor, then we may write for the resistance of the conductor,

$$R = \frac{kl}{s}$$

This is to be considered as the defining equation for k, which is called the **resistivity** of the wire; for each of the other three quantities appearing in the equation have already been defined.

The Incandescent Lamp

269. We may, then, increase the heat either by increasing the current or by increasing the resistance. Heat is developed wherever resistance of any kind is located. It is for this reason that long slender carbon filaments of high resistance are used in incandescent lamps. The heat is located almost entirely in the filament, scarcely at all in the wires which lead the current to the filament.

Note also that it is the heat, not the electric current, which makes the filament luminous. If the carbon filament were raised to the same temperature by any other means, it would be just as bright.

Figures 270 and 271 indicate the manner in which

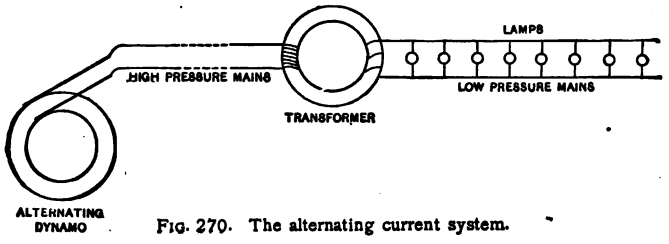


FIG. 270. The alternating current system.

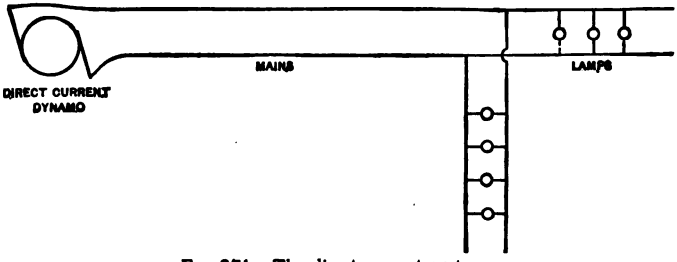


FIG. 271. The direct current system.

incandescent lamps are usually connected in the alternating and direct current systems respectively.

PROBLEMS

1. How much heat will be generated in 5 hr. by a current of 10 amperes flowing through a resistance of 40 ohms?

2. Two incandescent lamps are alike in every respect, except that one carries twice as much current as the other. Compare the amounts of heat generated in these lamps in equal times.

3. The resistance of an incandescent lamp is 110 ohms. How much heat will be generated in 1 hr. by a current of $\frac{1}{2}$ ampere?

4. A stereopticon rheostat which carries a current of 25 amperes has a resistance of 3 ohms. How much heat will be generated in 40 min.?

5. An arc light has an effective resistance of 5 ohms and requires 9.6 amperes to operate it. How much heat will be given off on burning the lamp for 10 hr.?

C. Magnetic Effects of an Electric Current

270. The student who has employed a galvanometer to detect electric currents already knows that a wire conveying an electric current is surrounded by a magnetic field.

But it is not enough to know merely that the region about a current is a magnetic field; we want to know what kind of a field it is, at least what are its direction and sense at any point. One of the easiest methods of

making a map of such a field is to carry a small magnet about and note the direction which the magnet (*i.e.* the axis of the magnet) assumes at each point.

In the case of an electric current, this examination is easily made by passing a wire, say two or three meters long, through a hole drilled in the center of a small glass or cardboard plate, or even a wooden block. The plate may be supported by a clamp, as shown in Fig. 272.

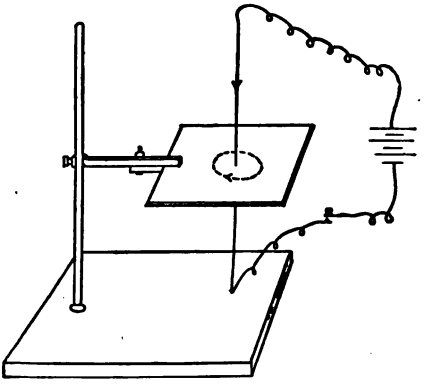


FIG. 272. Mapping a magnetic field about a wire conveying a current.

If, on this plate, we place a ring of *very small* compass needles, we find that whenever a current is passed through the wire the magnets all *tend* to *set* themselves tangent to a circle about the wire. In this experiment a good strong current, say from 10 to 50 amperes, is needed, in order to minimize the disturbance due to the earth's field.

If we reverse the sense of the current, it is seen that the little magnets tend to set themselves tangent to the same circle as before, but *now* in the opposite sense.

The Electro-magnet

271. If a rod of iron be placed inside a helix, such as that shown in Fig. 273, the lines of force passing through the coil are enormously increased in number. Such a device is called an **electro-magnet.** The behavior of the electro-magnet is well illustrated in the ordinary telegraph sounder. Here the operator at a distant station

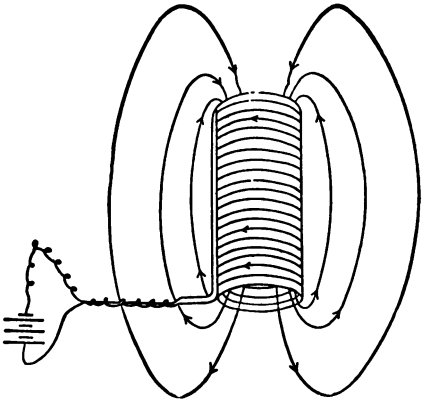


FIG. 273. Lines of force about a helix.

closes the circuit; the electro-magnet furnishes a strong magnetic field, attracting to its poles a small piece of iron, *A* (Fig. 274), called an **armature.** This armature is pivoted at *B*, and hence can rotate slightly about an axis through *B*. When the circuit is closed, the armature hits a stop which gives the sharp metallic click familiar to every one who has ever been in a telegraph office. When the

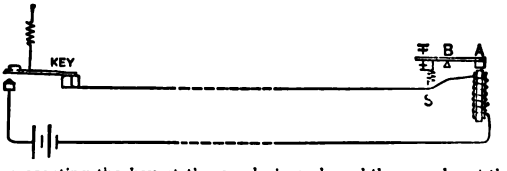


FIG. 274. Representing the key at the sender's end, and the sounder at the receiver's end, of a telegraph line.

circuit is broken, the armature is pulled back against another stop by means of a spiral spring, *S*.

The details of the **sounder** are shown in Fig. 275.

The electric bell, which makes and breaks the circuit automatically, is very much like the tele-

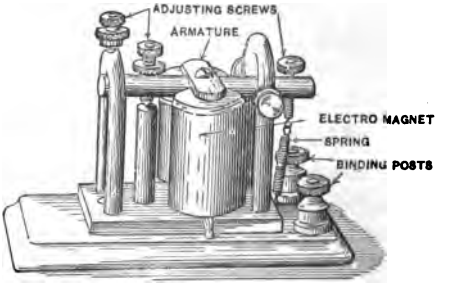


FIG 275. A sounder.

graph sounder, only here, when the circuit is first closed by pushing the button at *K* (Fig. 276), the current runs through the armature; but the iron core of the magnet immediately becomes magnetized; the armature is pulled away from the metallic stop on which it rested, and the current is thus interrupted; but as soon as the current is broken the armature falls back by means of a spring into its initial position, and the circuit is thus again closed.

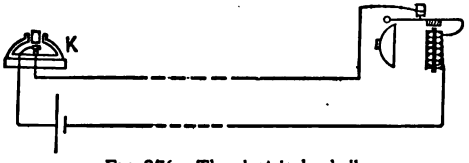


FIG. 276. The electric doorbell.

In this manner the armature is kept in continual vibration; and may, therefore, be employed as a bell clapper, the bell continuing to ring so long as the push button is held down at *K*.

D. Behavior of a Movable Circuit in a Magnetic Field

272. Up to this point we have been studying some of the properties of an electric current considered by itself.

We have found

(1) that if the conductor is an electrolyte it is decomposed by the current;

(2) that the conductor is heated by the current;

(3) that the current produces a magnetic field about the conductor.

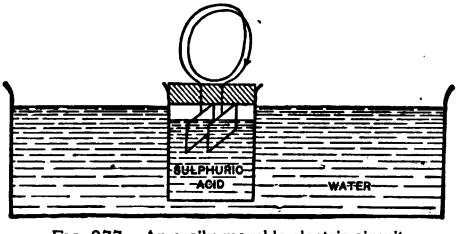


FIG. 277. An easily movable electric circuit.

We are now ready to determine by experiment how an electric circuit behaves when placed in another magnetic field, *i.e.* in a field

other than its own. To answer this question we shall need a circuit which is free to move while a current is flowing in it. We can easily accomplish this by floating a small voltaic cell in a large dish of water. A small coil of copper wire, consisting of half a dozen turns, with zinc and copper plates soldered to its respective terminals, can be fitted, through a cork, into a small and rather deep glass beaker, as shown in Fig. 277.

As soon as the glass is half filled with dilute sulphuric acid and floated on water we have at our disposal a current in a circuit which is free to move. If the floating cell is inclined to be unstable, a small piece of sheet lead laid in the bottom will remedy the difficulty.

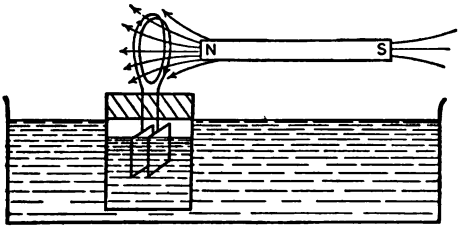


FIG. 278. Movable coil *rotates* so as to include lines passing through in the right-handed sense.

Two phenomena are observed when a strong bar magnet is brought near this floating coil, viz.:

(1) If the N pole of the magnet be extended toward the coil, the circuit will always rotate so as to present that face in which the current is flowing in a clockwise sense, *i.e.* the south face of the coil is attracted by the north pole of the magnet.

(2) But this motion of rotation is not the whole of the phenomenon. For if the magnet be held in a fixed position, end on *in front of the coil*, it is observed that the coil is *translated* as a whole, moving so as to include more lines of force.

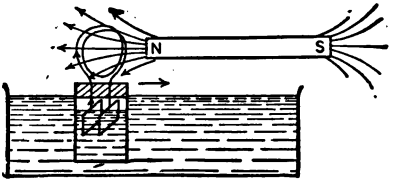


FIG. 279. Movable circuit is *translated* so as to include as many lines of force as possible.

And, in general, it has been proved that a circuit in which an electric current is flowing always sets itself so as to include as many magnetic lines of force as possible.

The D'Arsonval Galvanometer

273. A still simpler illustration of this behavior of a current in a magnetic field is the following.

Mount a Weston magnet upon a block of wood, as indicated in Fig. 280. Between the poles suspend a coil of half a dozen turns of No. 20 copper wire, using for the suspension a spiral of, say, No. 30 wire. Connect the coil by means of another spiral below to a single cell and key. If the plane of the copper coil be set parallel to the magnetic lines of force as indicated, the coil will rotate whenever the circuit is closed; because by rotating it can set itself so as to include a much larger number of lines of force.* This device in the form shown in Fig. 280 is called a D'Arsonval Galvanometer.

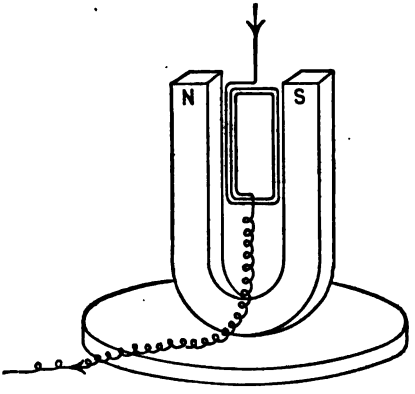


FIG. 280. A galvanoscope in which a part of the wire conveying the current is movable: while the magnet is fixed.

It is essentially the instrument which Lord Kelvin devised in 1867 for receiving messages over the Atlantic

* This experiment (which can be prepared in ten minutes) should be thoroughly examined, repeated, and mastered by every student. Almost any D'Arsonval galvanometer will serve in place of the apparatus described.

cable. The suspended coil is connected with the copper wire inside the cable, and whenever a current is sent over the line, the suspended coil turns itself so as to include more lines of force. When the current ceases the coil returns to its position of equilibrium. The angular deflection of the coil is indicated by the motion of a spot of light: and the operator reads the message by the motion of this spot. In later instruments the coil is also provided with a fountain pen so that it records its own deflections on a moving strip of paper.

The Electric Motor

274. The electric motor is also a suspended coil of this kind. Here the coil is generally wrapped about an iron core on a horizontal steel shaft which is suspended in bearings so that the coil rotates with very little friction. This coil is called the **armature** and is placed between the poles of an electro-magnet so as to be in a strong magnetic field. If a current is now passed through any one coil of wire on the armature coil, the coil will tend to rotate so as to include more lines of force; but when it has once set itself in a position where it includes as many lines as possible, the commutator (simply a split ring fastened on the shaft and insulated from it) changes the sense of the current in the coil, so that now the wire tends to rotate 180° farther so as to include more lines of force. Each turn of wire on the armature behaves in the same way, so that

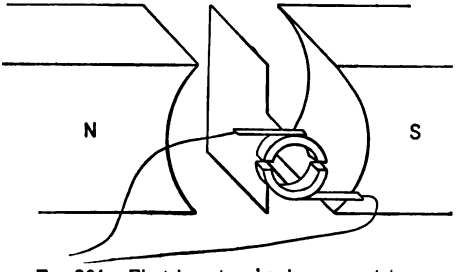


FIG. 281. Electric motor showing commutator.

there is a continual moment of force acting upon the armature. By means of a belt and pulley a large part of the energy furnished to the motor may be transmitted to other machines and there employed to do work.

In regard to its structure and circuits (Fig. 282), the motor is almost identical with the shunt-wound dynamo. Its external appearance is often quite different from that of a dynamo, for the motor is generally smaller and has also to be protected against various unfavorable working conditions. Thus a motor underneath a trolley car or a motor vehicle must be protected against mud and slush.

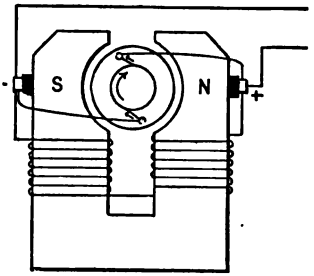


FIG. 282. A shunt-wound motor.

Another important difference between the dynamo and the motor is the following:

When an armature is rotating in a magnetic field there is generated in that armature an electrical pressure. In the case of the motor this electrical pressure — generally called the **back electromotive force** — is such as to oppose the current which is feeding and driving the motor. The result is that the motor takes less current as its speed increases. When the armature is standing still, as at starting, there is, of course, no back electromotive force. Most motors are therefore provided with a **starting box**, *i.e.* with a set of wire coils which will reduce the electrical pressure at the armature, until the armature acquires sufficient speed to accomplish this by its own back electromotive force. This starting box may be seen on the front platform of any electric car.

Electrical Pressures in Series

275. A voltaic cell or a dynamo may be looked at from several different points of view. A battery may be considered as an instrument in which chemical energy is transformed into heat or some other useful form of energy. In like manner, a dynamo may be viewed as a machine for transforming the energy delivered by a steam engine into heat, or light, or some form of mechanical energy.

But we may also consider a battery as a pump which exerts a pressure tending to drive electricity (whatever that may mean) through a circuit. In fact, a battery may be said to exert an **electrical pressure** along a circuit. For instance, if a gravity cell and a lead storage cell be placed in series in the same circuit, with their negative poles together, as indicated in Fig. 283, it is found that the current in the circuit *always* flows from the positive to the negative pole *of the storage cell*, *i.e.* the current flows in the same sense in which it would flow if the storage cell alone were in the circuit. Considered as a force pump for electricity, the storage cell is always able to exert more electrical pressure than the gravity cell. When *two* gravity cells are joined up against one storage cell, as in Fig. 284, it is observed that the resultant pressure is al-

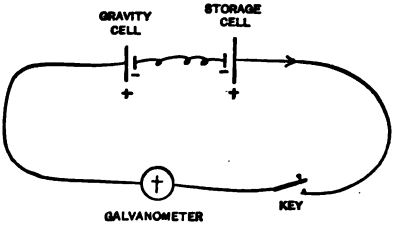


FIG. 283. Electrical pressure of gravity cell compared with that of storage cell.

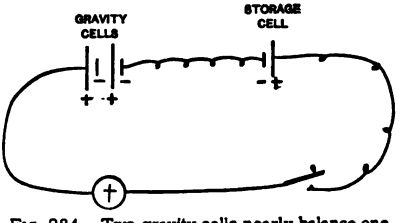


FIG. 284. Two gravity cells nearly balance one storage cell.

most zero, for the current becomes almost zero. The electrical pressure of one storage cell is then nearly equal to that of two gravity cells.

But if we join *three* gravity cells in series, each acting in the same sense, and oppose them to one storage cell, as indicated in Fig. 285, the current will then flow in a sense opposite to that in which the storage battery alone would send it. The electrical pressure of one lead storage cell is then less than that of three gravity cells. This is indicated by the directions of the arrows in Figs. 283 and 285.

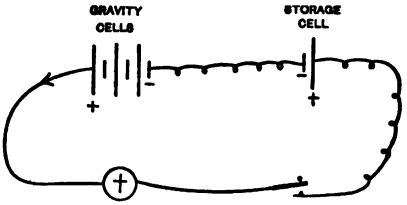


FIG. 285. Three gravity cells more than balance one storage cell.

By similar experiments it is easily shown that the electrical pressure of any three cells joined end to end, with positive poles all one way, is three times as great as that of either cell alone. In general, **when a number of sources of electrical pressure are in series, the resultant electrical pressure is their algebraic sum.**

OHM'S LAW

276. In 1827 the three following facts were announced by Ohm:

1. **In any circuit in which the resistance is kept constant,** the current is doubled whenever we double the electrical pressure, trebled when we treble the pressure, etc.; and, in general, **the current is directly proportional to the electrical pressure.**

2. **When in any circuit the electrical pressure remains the same, the current will vary inversely as we vary the resistance of the circuit.**

3. **When in any circuit the current is to remain constant, the electrical pressure must be varied directly as the resistance varies.**

These three facts constitute **Ohm's Law.**

Unit of Electrical Pressure. The Volt

The quantity which we have hitherto denoted by **electrical pressure is generally called electromotive force** and is usually abbreviated to **E.M.F.** It has been agreed by the scientific world to take as unity **that electromotive force which will send one ampere of current through a resistance of one ohm,** and to name it **the volt** in honor of the Italian physicist, Alessandro Volta (1745–1827).

In other words, it has been agreed to call E.M.F. (E) unity when current (C) and resistance (R) are unity.

Hence, **Ohm's Law may be written,**

$$\textbf{Current in amperes} = \frac{\textbf{E.M.F. in volts}}{\textbf{Resistance in ohms}}$$

$$\text{or, briefly, } C = \frac{E}{R}$$

The E.M.F. of a gravity cell is about one volt; that of a Leclanché cell, 1.5 volts; that of a lead storage cell, about 2.4 volts.

A more particular statement of Ohm's Law is now possible, namely: **In any electrical circuit the total E.M.F. in volts, divided by the total resistance in ohms, is numerically equal to the current in amperes.** But if, instead of considering the entire circuit, we consider only a portion of it, such as that between the points a and b (Fig. 286), then it is found that Ohm's Law holds in the same way

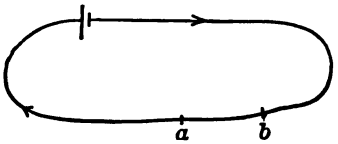


FIG. 286. Ohm's Law applied to a portion of a circuit.

for this portion of the circuit as for the whole circuit, *always provided there is between a and b nothing of the nature of a battery or dynamo.*

Ohm's Law for this portion of the circuit would then read as follows: *The difference in electrical pressure, in volts, between the points a and b, divided by the resistance, in ohms, between the points a and b, is numerically equal to the current, in amperes, flowing in that part of the circuit.*

Some Illustrations of Ohm's Law

1. *Shunt Circuits*

277. When any two points in a circuit, such as A and B in Fig. 287, are connected by two conductors, such as r_1 and r_2, one of these conductors is said to be a **shunt** on the other. In a case of this kind the question arises, What is the total resistance between the points A and B? This problem is easily solved by applying Ohm's Law to each of the conductors, r_1 and r_2. Let us denote by C the total current; by c_1, the current in r_1; and c_2, the current in r_2. Now since neither the point A nor the point B becomes charged, it must follow that as much electricity arrives at A as leaves it. Hence,

total current between A and B

$$= \text{current through } r_1 + \text{current through } r_2,$$

or

$$C = c_1 + c_2$$

But by Ohm's law, $c_1 = \frac{E}{r_1}$ and $c_2 = \frac{E}{r_2}$, where E is the electrical pressure between A and B. Adding c_1 and c_2 we have, therefore,

$$C = E\left(\frac{1}{r_1} + \frac{1}{r_2}\right) = E\left(\frac{1}{R}\right)$$

where R is the total resistance between A and B.

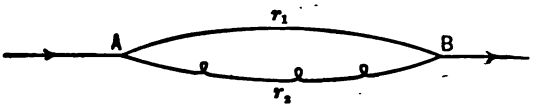


FIG. 287. A shunt circuit: r_1 is a shunt on r_2; and *vice versa*.

Hence it follows that **when two conductors are placed in parallel, the reciprocal of the total resistance is equal to the sum of the reciprocals of the resistances of the separate conductors.**

2. *The Incandescent Lamp Circuit*

278. Thus in an incandescent lamp circuit, where the electrical pressure between the mains is kept constant, say 110 volts, the total current is the sum of the currents in the individual lamps. The resistance of a single 16-candle-power lamp is about 220 ohms.

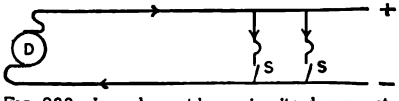


FIG. 288. Incandescent lamp circuit: dynamo at *D*; switch at *S*.

Hence, if only *one* lamp be lighted, the resistance between the mains will be 220 ohms, the total current in the mains will be $\frac{110 \text{ volts}}{220 \text{ ohms}}$, *i.e.* half an ampere.

But, if *two* lamps be turned on, the resistance between the mains will be the reciprocal of $\frac{1}{220} + \frac{1}{220}$ *i.e.* 110 ohms; and hence the total current in the mains will be $\frac{110 \text{ volts}}{110 \text{ ohms}}$ *i.e.* one ampere. In like manner, we may obtain the resistance due to any number of lamps.

3. *Division of Current*

279. Where two shunt circuits have unequal resistances, it is clear that the larger current will flow along the con-

ductor of less resistance. And from Ohm's law it follows (Fig. 287) that

$$E = c_1 r_1 = c_2 r_2 \text{ or } \frac{c_1}{c_2} = \frac{r_2}{r_1}$$

That is, **in shunt circuits the current divides in inverse ratio to the resistances.**

4. *The Voltmeter*

280. From what we have already learned concerning shunt circuits, it is clear that if two points, A and B, on a conductor are shunted by a very high resistance, this shunt will not carry sufficient current to affect appreciably the electrical pressure between A and B.

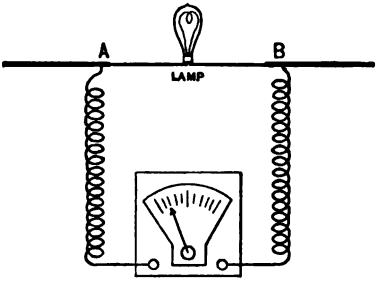


FIG. 289. A voltmeter used to measure the pressure between the terminals of an incandescent lamp.

Now a voltmeter consists, in nearly every case, of a high resistance D'Arsonval galvanometer. When the terminals of such a galvanometer are connected to the terminals of, say, an incandescent lamp, the very slight current which passes through the galvanometer will be proportional to the electro-motive force between the terminals of the lamp. Such an instrument is therefore called a **voltmeter.** The voltage between the terminals of a motor or storage battery would be measured in the same way.

5. *The Ammeter*

281. Suppose, however, that we wish to measure the amount of current passing through an incandescent lamp. Here we must evidently pass the entire current through

the instrument, else we shall not get a measure of it. At the same time our measuring instrument must not appreciably diminish the current which we desire to measure. In order to accomplish this end, we select a short, thick piece of metal having an extremely small, but *constant*, resistance.

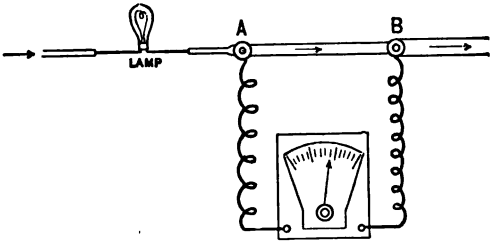


FIG. 290. An ammeter, consisting of a low resistance, AB, placed in series with main circuit, and a voltmeter shunted on AB.

This resistance (AB of Fig. 290), is placed in series with the lamp circuit. Then about AB is shunted a voltmeter, which of course measures the electro-motive force between A and B.

But since the resistance, R, between A and B is constant, it follows that the E.M.F. between A and B will depend only upon the current strength, *i.e. the reading of the galvanometer will depend only upon the current strength.* Such an instrument, consisting of low resistance, AB, and a voltmeter as a shunt, is known as an **ammeter.**

In portable ammeters and voltmeters, as well as those used in power stations, the moving coil is generally supported on an axis which works between jeweled bearings, and is not suspended by a wire as in most of the laboratory galvanometers.

PROBLEMS ON OHM'S LAW

1. A current of 1 ampere and an electro-motive force of 50 volts is required to feed an incandescent lamp. What is the resistance of the lamp in ohms?

2. What is the total resistance of two wires, R_1 and R_2, when joined *in series* as indicated in Fig. 291?

3. A Leclanché cell is used to ring a doorbell. The resistance of the wire in the bell is 2 ohms, the resistance of the line is ½ an ohm, and the resistance of the cell is 1 ohm (Fig. 292). The E.M.F. of the cell is 1.5 volts. What current will be produced when the circuit is closed?

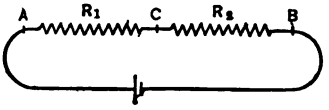


FIG. 291.

4. The E.M.F. of a single gravity cell is 1.1 volts; the resistance of each cell is 3 ohms. If three of these cells be joined up in series, as indicated in Fig. 293, and the poles of the battery be connected by a resistance of 1 ohm, what current will flow in the circuit?

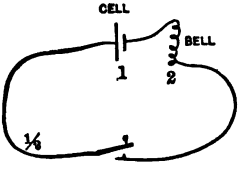


FIG. 292.

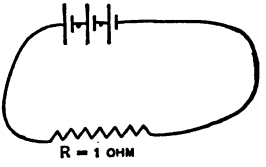


FIG. 293.

5. A telegraph sounder has a resistance of 70 ohms, and it requires a current of 0.2 ampere to work it. Will 20 gravity cells such as described in the preceding example be sufficient? Will 40 cells suffice? Calculate the current given by each battery.

6. A gravity cell of 1.1 volts is joined up in opposition to a storage cell of 2.1 volts, *i.e.* their positive poles are connected. What is the resultant E.M.F. of the two cells?

7. Two wires, R_1 and R_2, alike in all respects, and having each a resistance of 20 ohms, are joined in *parallel* as shown in Fig. 294. What is the sum of their resistances, *i.e.* what is the resistance of that part of the circuit which lies between the points A and B?

8. Three gravity cells, each having a resistance of 3 ohms and an E.M.F. of 1.1 volts, are joined "in parallel" (as indicated in Fig. 295), and then connected to a coil of wire having a resistance of 8 ohms. Find the current in the coil. How much current passes through each cell?

9. If it takes 100 volts to generate a current of 20 amperes through a resistance of 5 ohms, what electro-motive force will be necessary to generate one fourth the current through a resistance of 20 ohms?

10. A dynamo gives an E.M.F. of 110 volts. Ten lamps each of 220 ohms resistance are joined in parallel, or multiple arc, to the terminals

of the dynamo. What is the current? What would be the current if the lamps were joined in series?

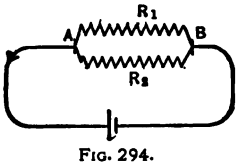


FIG. 294.

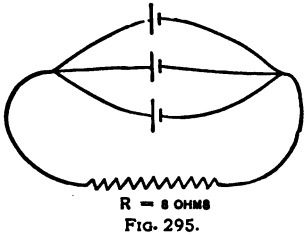


FIG. 295.

11. Two parallel branches of a divided circuit have resistances of 1 and 3 ohms respectively. How will a total current of 48 amperes divide itself between these two branches?

12. What is the combined resistance of a wire of 10 ohms shunted with a wire of 20 ohms?

Unit of Power. The Watt

282. Power has already been defined as the rate of doing work. The unit of electrical power is **the watt,** which **is defined as the rate of work represented by a current of one ampere flowing under an electro-motive force of one volt,** or

$$\textbf{watts} = \textbf{volts} \times \textbf{amperes}.$$

Since the watt is inconveniently small for rating dynamos, **the kilowatt = 1000 watts** is most frequently used. Attention has already been called to the fact that

$$1 \text{ horse power} = 746 \text{ watts}.$$

PROBLEMS

1. A current of 100 amperes flows through a circuit under an E.M.F. of 220 volts. What is the power in the circuit?

2. A generator in a street car power house can give 2750 amperes at 575 volts. What is its power in kilowatts? What at least must be the horse power of the engines that drive it?

3. A 16-candle-power incandescent lamp requires $\frac{1}{2}$ ampere at 110 volts. How many watts per candle power does the lamp consume?

4. An electrically heated flatiron takes $\frac{1}{2}$ ampere of current at 110 volts. How many watts does it consume? How much heat is generated per minute?

5. An inclosed arc lamp requires 4 amperes at 50 volts. What power does it consume? If its candle power is 2000, what is the wattage per candle power?

Summary of Facts concerning Electric Currents

283. Electric currents are generally produced in one of the three following methods, viz.:

1. By use of the voltaic cell.

2. By induction. Dynamo currents and telephone currents are of this kind.

3. Directly from heat; method not discussed in this volume.

The most important properties of electric currents are the following:

1. They decompose all chemical compounds which are conductors. Faraday's Laws of Electrolysis.

2. They heat conductors through which they pass. Joule's Law of Heating.

$$H = \frac{C^2Rt}{4.2}$$

Electrical resistance may be defined as that property of a conductor which determines how much heat a given current will produce in it in a given time.

3. A conductor conveying a current is surrounded by a magnetic field.

4. A conductor conveying a current tends to set itself so as to include the greatest possible number of magnetic lines of force. This is the principle of the electric motor, and of galvanometers in which the coil is free to move.

Definitions of Practical Electrical Units

Unit of Current

One **ampere**: defined arbitrarily as that current which deposits silver at the rate of 0.00111800 gram per second.

Unit of Resistance

One **ohm**: defined arbitrarily as that resistance which is offered by a column of mercury of uniform cross section at 0° C., when its length is 106.3 centimeters and its mass is 14.4521 grams.

Unit of Electro-motive Force

One **volt**: defined as that electrical pressure which will produce one ampere of current in a resistance of one ohm.

Unit of Power

One **watt**: defined as power from one ampere under an electrical pressure of one volt.

References on Electric Currents

S. P. Thompson — *Elementary Lessons in Electricity and Magnetism* (Macmillan), pp. 628. Price, $1.40. Chapter III especially recommended for the beginner.

Jackson and Jackson — *Electricity and Magnetism* (Macmillan), pp. 482. Price, $1.40.

Cajori, Florian — *History of Physics* (Macmillan), pp. 330. Price, $1.60.

M. Faraday — *Experimental Researches*, 3 vols.

G. Chrystal — Art. "Electricity" in *Ency. Brit.*

Oliver Lodge — *Modern Views of Electricity* (Macmillan), pp. 422. Price, $1.75. Advanced theories are presented in a lucid manner.

Arts. "Telephone" and "Telegraph" in *Ency. Brit.*

K. Edgcumbe — *Electrical Engineer's Pocketbook* (Whittaker.)

CHAPTER X

LIGHT

As under the head of Sound we considered those phenomena which produce the sensation of hearing, so now, under the head of Light, we proceed to consider those phenomena which produce the sensation of sight; and, following the method used in previous chapters of this volume, we begin with a study of some of the fundamental phenomena of light, and lastly consider what is often called the "nature of light."

Definitions

284. When we are looking at the various objects in a room, it is not difficult, in general, to select those which we might still be able to see in case no light were admitted to the room from the outside. If the room were completely shut off from any outside light, the chairs and tables in it would became quite invisible, while a burning match or a red-hot coal in the fireplace, a lighted gas jet, or an incandescent lamp would each illuminate the room, and at the same time be seen by its own light. Bodies of this latter class are said to be **self-luminous**, while bodies of the former class, including the ordinary furniture of a room, are said to be **non-luminous**. They shine by borrowed or reflected light. Is the moon luminous or non-luminous? The sun? Luminous bodies are generally, but not always, hot. A phosphorescent clock face is easily seen at night, but can hardly be called a hot body. The student who recalls his

daily duties will find that most of his work is performed by the aid of borrowed, or diffused, light. For instance, you are now reading this page by light which is reflected from the white paper to the eye.

Some substances, like water, air, and thin layers of gelatine, permit us to see distinctly other objects through them. Substances of this kind, including many kinds of glass and some crystals, are said to be **transparent.** There are other substances, however, such as iron, wood, and ink, through which bodies cannot ordinarily be seen. Such substances are said to be **opaque.** But every one will recall certain other substances, such as milk in thin layers, thin tissue paper, lightly ground glass, even ink in thin layers, and thin shavings of wood, through each of which objects are faintly or indistinctly seen. These bodies form an intermediate class between transparent and opaque bodies. They are called **translucent.**

If bodies are taken in layers of proper thickness, it may be easily shown that there is no sharp line between the most opaque and the most transparent substances. Metals, such as gold and silver, which are very dense and ordinarily very opaque, become translucent, and even transparent, when taken in sufficiently thin layers, as may be easily shown by depositing a thin film of silver on glass; while, on the contrary, what we call "perfectly clear" water, when taken in sufficiently thick layers, permits almost no light to pass through it. Geographers who have studied the bottom of the sea have found that those parts of the ocean which lie at a depth of several kilometers are in almost complete darkness.

We now proceed to consider some of the fundamental and general properties of light, including the light which is emitted from luminous bodies and the light which is reflected from non-luminous bodies.

Rectilinear Propagation of Light. Light moves in Straight Lines

285. If a man steps behind a tree, it is very well known that he becomes partly or entirely invisible to one looking toward the tree. The reason of this is that the light proceeding from any part of the man behind the tree cannot pass through the opaque tree, cannot take a curved path around the tree, and therefore cannot reach the eye of the observer. Evidence of this kind shows us that, if light

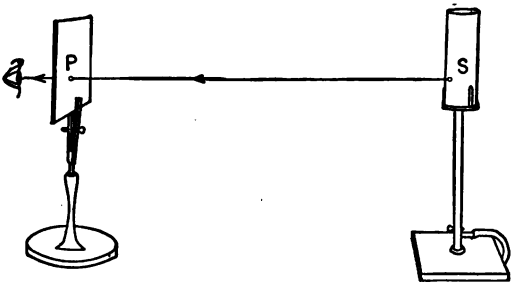


Fig. 296. Light travels in straight lines.

does not travel in straight lines, it travels in lines which are very nearly straight.

In order to test this matter more accurately in the laboratory, we may proceed as follows: As a source of light, use a lamp, either gas or oil, provided with a tin chimney in which has been drilled, at a point on a level with the flame, a hole from one to two millimeters in diameter. This round hole, near the flame, may be considered as a luminous point, and not only for this, but for many other purposes, in the study of optics is a most excellent source of light. At some distance, say one meter, in front of this point-source clamp a sheet of cardboard with a pinhole in it. The pinhole, *P* (Fig. 296), and the source of light, *S*, are two points which serve to define

a straight line. We might think of this line represented by a thread stretched tightly between these two points. Now place your eye behind the pinhole, and look toward the hole in the opaque lamp-chimney. You will find that any opaque object placed on this straight line, *PS*, will render the luminous point invisible. **This means that light from the luminous point can reach the pinhole in the card only along the straight line joining these two points.** This principle is illustrated also by the fact that an opaque object casts a shadow which is similar to the object itself.

But from the following it will be seen that this state-

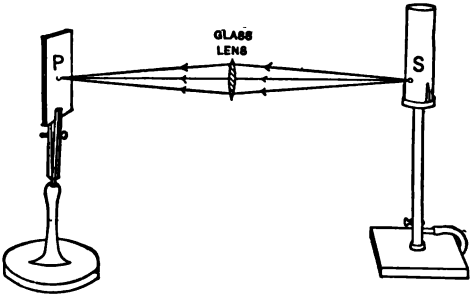


FIG. 297. If a medium is not homogeneous, light does not travel through it in straight lines.

ment is not quite true as it stands. Suppose that we place in this straight line an ordinary spectacle lens, as represented in Fig. 297, so that an *image* of the luminous point is produced upon the pinhole on the cardboard. To do this we have placed the spectacle lens so that the straight line, *SP*, passes approximately through the center of the lens. Let us now interpose a coin, somewhat smaller than the lens, immediately in front of the lens so that the straight line passes through the center of the coin. We observe that this coin does not prevent all the light from the luminous point reaching the pinhole, the only differ-

ence being that it is not quite so bright as before. The path of the light from the source to the pinhole is no longer a straight line. All the light which forms the image now passes around the edge of the coin. But the reason of this is very evident, for we have now interposed a piece of glass in the path of the light, and, indeed, it has been found that, **in general, when light has to pass through materials of different kinds it does not travel in the same straight line through them all.**

If now we remove the spectacle lens and allow the coin to remain, in Fig. 298, we shall find that the latter pre-

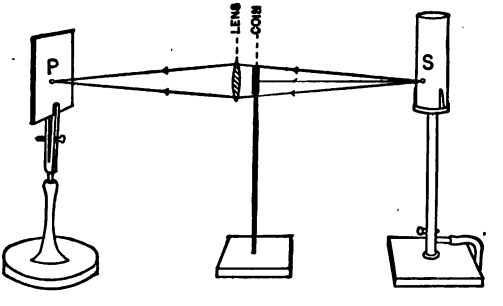


FIG. 298.

vents any light from the source reaching the hole in the cardboard. The entire path of the light is here through air alone.

Definition. **A body,** such as the air in a room, or a piece of glass, **every part of which is like every other part is said to be homogeneous.**

Accordingly, we may describe the behavior of light by saying that in any homogeneous medium light travels in straight lines.

The line indicating the direction in which the light travels at any point is called a ray of light.

Illumination. The Intensity of a Source of Light

286. At any point on a surface **the amount of light which falls on unit area is called the illumination** at that point. Because light travels in straight lines, the light coming from a small source, *S* (Fig. 299), and covering an area of one square centimeter at a distance of one meter will cover an area of four square centimeters at a distance of two meters. In other words, it follows from the straight line propagation of light that **the illumination produced by a point source at any distance varies inversely as the square of the distance from the source.** Ordinarily the same fact may be regarded as true for a source of light that is not a point.

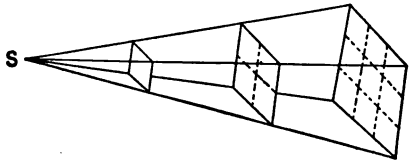


Fig. 299. Illumination varies inversely as the square of the distance.

If two sources of light produce the same illumination on screens placed at the same distance from each source, then these sources are said to have the same **intensity.**

If a screen is equally illuminated by two different sources of light, then the intensities of these sources are proportional to the squares of their distances from the screen.

The Photometer

287. The simplest method of comparing the intensities of two sources of light is, perhaps, that devised by the chemist Bunsen. Here the sources of light, candle, gas flame, incandescent lamp, etc., are mounted one at each end of a graduated wooden bar. Between the sources is a slider which carries a small white paper screen with a translucent grease spot near the center. If this screen be

viewed from the side on which it is more brightly illuminated the grease spot will appear darker than the remainder of the screen. But if it is viewed from the side on which the illumination is less, the grease spot will appear brighter than the rest of the paper. This is evident from the fact that the greased portion transmits light more easily than the rest of the paper. Evidently the illuminations are equal when the grease spot appears to be of the same brightness as the rest of the card. But when the illuminations are equal, the intensities of the sources are in the ratio of the squares of the distances from the slider to the sources. Such an instrument is called a **photometer.**

Shadows

288. If any opaque object be held in front of a screen which is illuminated only by a point-source, there appears on the screen a certain area which is not illuminated at all. This dark area and the space between it and the opaque

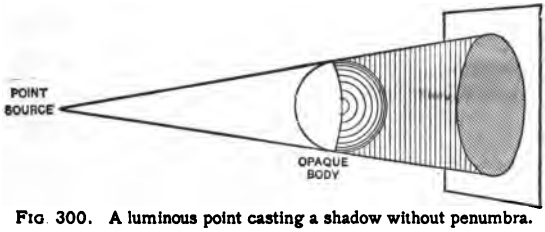


Fig. 300. A luminous point casting a shadow without penumbra.

body are called the **shadow** of the opaque object. The fact that this dark area is geometrically similar to the outline of the opaque object is a consequence of the fact that light moves in straight lines in a homogeneous medium. The fact that shadows produced by opaque objects are similar to the objects may also be considered as experimental evidence that light *does* travel in straight lines.

Hitherto we have considered shadows of objects illuminated by point-sources only. Suppose now that the source is no longer a luminous point, but a luminous body, made up of an infinitely great number of luminous points. In this case it is observed that the shadow consists of two parts, one a central dark portion, which is illuminated by no point on the bright body. This part of the shadow is called the **umbra.** Immediately surrounding the umbra is another region which is illuminated by some, but not all, portions of the luminous body. This region is called the

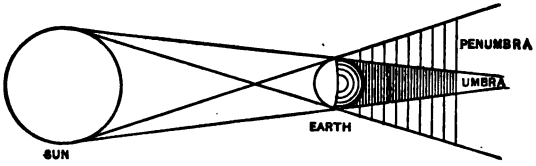


FIG. 301. A luminous body casting a shadow with penumbra.

penumbra. The region immediately outside the penumbra is illuminated by all portions of the luminous body. (See Fig. 301.)

Ordinary arc lights, although they give one the impression of being "great balls of fire," produce shadows which are remarkably sharp; that is, they give almost no penumbra. We infer, therefore, that they must be very nearly point-sources. That this is the fact, we can easily observe by looking at an arc light through several deeply colored glasses.

IRRADIATION

289. Any very bright object, such as the sun, or the filament of an incandescent lamp, always *appears* to be larger than it really is. Physiologists explain this by saying that when the eye produces an image of any bright object upon the retina, those portions of the retina bordering upon the image are also affected. Hence the retina has

the impression of an image a little larger than the real image, and hence we imagine the object to be a little larger than the real object. This phenomenon is called **irradiation.** Look through a piece of dark red glass at one of the small holes in the tin lamp chimney. Note that its apparent size is decreased as the brightness of the object lessens.

Diffraction

290. In another part of the chimney, cut a slot about 30 millimeters long and 1 or 2 millimeters in breadth.

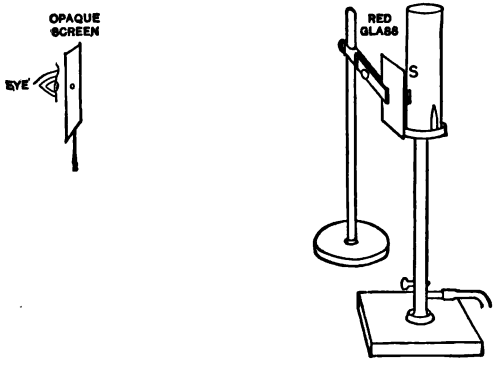


Fig. 302. A luminous line appears wider when seen through a small aperture.

Suppose that this slot, *S* (Fig. 302), is placed immediately in front of the flame. As we look at it with the naked eye, the slot appears to have a certain width. But now examine it looking through a small pinhole in a visiting card held very close in front of the eye. **The slit, *S*, appears now very much diminished in brightness, but very much wider.***

* A piece of red glass placed in front of the slit will prevent irradiation, and make the phenomenon clearer. This phenomenon is still better seen if we substitute for the visiting card a single straight cut made on the opaque film of an ordinary photographic plate.

The ray PH (Fig. 303) prolonged backward gives us one edge of the apparent slit; the ray QK prolonged backward gives us the other. The small cone of rays, SHK, is spread out into a much larger cone, $PHKQ$, so that the slit S appears to have a width CD. The effect then of a small opening on a bundle of rays passing through it is to spread out the rays. **When a cone of light rays enters a small opening and is spread out into a cone of larger angle, this phenomenon is called diffraction.**

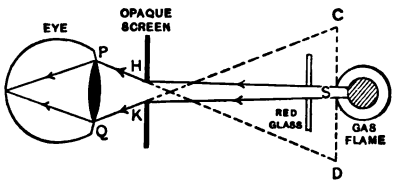


FIG. 303. Horizontal section through preceding figure.

This is not unlike what happens when water waves pass through an opening in a breakwater. If BAC (Fig. 304) is a wall separating the quiet water from the open sea, the crests of the waves, before they enter the opening at A, are straight; when they strike the wall BC, only a small portion of the wave enters at A. And it is observed that this small portion of the wave front does not proceed in one straight line, but spreads out so that the successive crests are nearly circular lines, as shown in Fig. 304. This spreading apart of the wave-normals in water is likewise called diffraction.

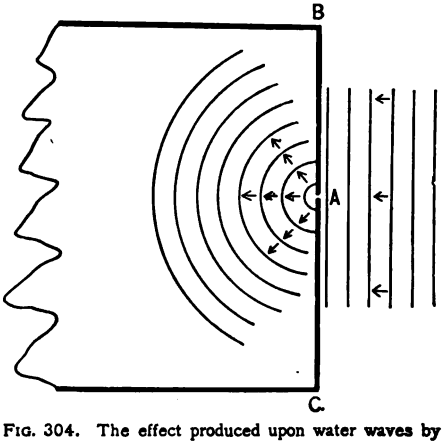


FIG. 304. The effect produced upon water waves by a small opening in a breakwater.

The line which is normal to the crest or front of a wave *at any point* indicates the direction in which that wave is travelling *at that point.* This normal is analogous to a **ray** of light which is a line indicating the direction in which the light travels at any point.

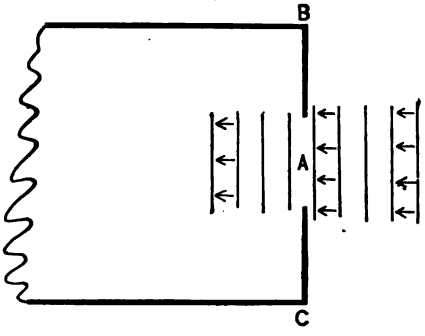


FIG. 305. *A* is a large opening in a breakwater. Waves do *not* pass through a small opening in this manner.

If, in the case of the breakwater, the opening at *A* be large, the wave-crests proceed through it and remain straight, as shown in Fig. 305; so also it is found, if we use a large opening in the visiting card (Fig. 302), the light rays proceed straight through without change of direction, *i.e.* the apparent size of the opening in the lamp chimney is not appreciably changed by viewing it through a large hole in the card.

From the preceding experiments, it is evident that **light travels in straight lines**

(1) **When the medium is homogeneous**; and

(2) **When the rays are not compelled to pass through any very small openings.**

Light falling upon a Body

291. We have just been considering the behavior of light when it has a free path in which to travel. Suppose, however, a beam of light strikes upon some body lying in its path.

If the body has a very smooth surface, it will generally happen that a large part of the light which falls upon it

from any direction is sent off in some other direction. This is called **reflection.** If the body is a rough one, such as ground glass or a piece of newspaper, the light which falls upon it is scattered in all directions. Such light is said to be **diffused**; and the process is called **diffuse reflection.**

But some of the light which is incident upon a body will penetrate the body; this we know because objects can be seen through all substances if only the body is made thin enough. The rays which pass through a body are said to be **transmitted.**

If, however, the body is so thick as to be opaque, then those rays which enter the body never leave it. Such rays are said to be **absorbed**; and the process is called **absorption.**

Reflection

292. A very simple and direct manner of studying the reflection of light is the following. A cross is made of two pieces of pine wood mortised together, as indicated in Fig. 306. One piece is slotted so as to receive a strip of plate glass, *G*.

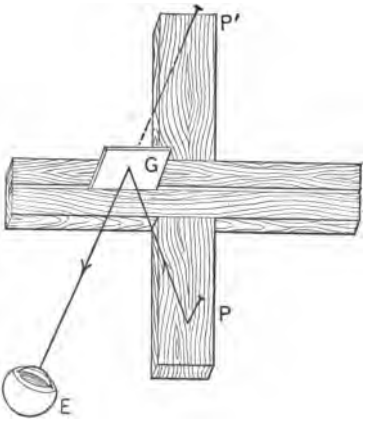


Fig. 306. A convenient device for studying the laws of reflection.

If a pin be placed at *P*, this strip of glass will act as a mirror. To an eye placed somewhere in front of the glass, say at *E*, the image of the pin will be seen at *P'*. We can now easily place a *second* pin at the point *P'*, so that it coincides with the *image* of the *first* pin. The second pin

at P' should be viewed over the top of the glass strip. The glass strip may well be ground or greased on the rear surface so that the reflection from the first surface only is seen.

The first pin now locates the position of an object; while the second pin locates the position of its image, produced by a plane mirror. Concerning this image it is important to note the following five facts:

1. On observing this image from various points of view, it is seen that the image does not change position as the eye changes position. The image has a fixed position in space quite independent of the observer.

2. If, however, either the object or the reflecting surface be moved, the image also moves.

3. If now we draw a straight line from P to P' (Fig. 307), we find by use of a square that the reflecting surface is at right angles to this line.

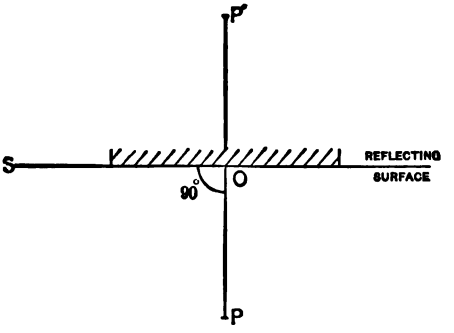


FIG. 307. $P'O = PO$.

4. Not only so, but on measuring the perpendicular distance, $P'O$, from the image to the reflecting surface, and also the perpendicular distance, PO, from the object to the reflecting surface, it is found that these distances are equal.

5. If O (Fig. 308) is the point where the ray from P meets the reflecting surface and the normal ON is drawn, the angles EON and NOP are equal.

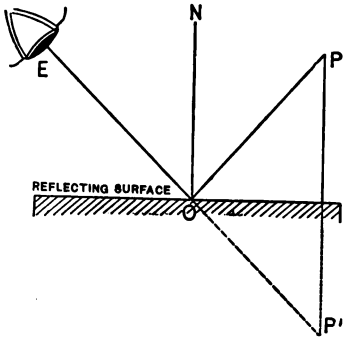


FIG. 308. Illustrating the law of reflection at plane surfaces.

Definitions. **The angle between the normal to the surface and the incident ray is called the angle of incidence. The angle between the normal and the reflected ray is called the angle of reflection.**

But this is exactly the manner in which we have found water waves behaving. When a wave coming from any direction strikes a solid pier at a definite angle of incidence (Fig. 309),

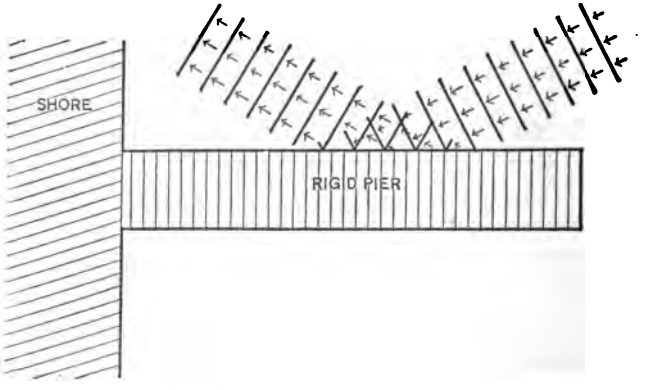


FIG. 309. Reflection of water waves.

the reflected wave leaves the pier at the same angle on the other side of the normal.

Laws of Plane Reflection

293. These facts may be summarized as follows:

(i) **The image of a point in a plane mirror lies on the perpendicular let fall from the point to the mirror, and lies as far behind the mirror as the point lies in front of it.**

(ii) **At each point of the reflecting surface, the angle of reflection is equal to the angle of incidence.**

(iii) **The reflected ray, the normal, and the incident ray all lie in the same plane.**

PROBLEMS

1. A man sees his reflection in a plane mirror 20 ft. away. How far away from him does his image appear to be?

2. If the luminous point, P, is placed at a very great distance from the mirror, the rays of light falling upon the mirror will be practically parallel. Show that in this case the reflected rays will also be parallel to each other.

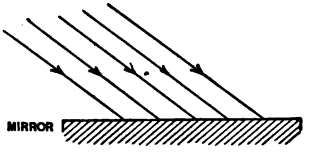


Fig. 310. Reflection of parallel rays at a plane surface.

3. An object lying in front of a plane mirror is moved 2 cm. farther away from the mirror. How much does this change the distance between the image and the object?

4. What is the difference between the phenomena of reflection from white blotting paper and from a piece of window glass?

5. Why are the reflections of trees as seen in the surface of a pond always inverted?

6. Draw a diagram of a clock face as seen in a plane mirror.

7. A standard candle and an electric lamp of 16 candle power are placed 4 ft. apart. How can one find by experiment a point on the direct line between the two at which the illuminations from the two sources are equal? How far from the candle would this point be?

8. In a Bunsen's photometer a standard 16-candle-power lamp gives the same illumination when 20 cm. from the screen as is given by another lamp when 15 cm. from the screen. What is the candle power of the second lamp?

9. Which gives the better illumination on the page of a book, a 10-candle-power student lamp 2 ft. from the page, or a 100-candle-power incandescent gas burner 6 ft. away?

Velocity of Light

294. A single flash of lightning lasting only a small fraction of a second will illuminate the entire landscape in such a way that the most distant parts of the landscape are seen at the same instant as the nearer parts. It is evident that if light did not travel at an enormous speed this would not be the case. If, for instance, light traveled with the same speed as sound, it would be a full second after the flash of lightning before we could see points as far away as 1200 feet; and four seconds later we could see points a mile away. It was such evidence as this, perhaps, that led the ancients to think that light traveled with an infinite speed. But Galileo was not satisfied with a rough estimate of the speed of light. As we have already seen, he had a profound faith in accurate experiment; accordingly he proposed to *measure* the velocity of light, as follows:

Two observers, each provided with dark lanterns, were to be stationed on two distant hilltops at night. When the first observer, at *A* (Fig. 311), sent a flash of light

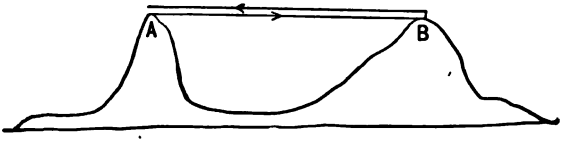


Fig. 311. Galileo's plan for measuring the speed of light.

from his lantern, the second observer, at *B*, was to make a similar signal as soon as the first flash was perceived. The interval between sending the first signal and receiving the answer might be supposed to be the time taken by the light in making one round trip between the hills. This experiment was tried by the Florentine Academy.

The conclusion was that the time required was too short to be appreciable.

We now know, however, that the time lost by the second observer in perceiving the first flash and in sending a return signal would occupy more time than is required for light to make a half dozen trips around the world.

295. But the following most ingenious method was discovered and tried by Ole Roemer during the years 1675–1676 at the Paris Observatory.

Jupiter has eight moons, one of which is larger and brighter than any of the others, and is called the "first

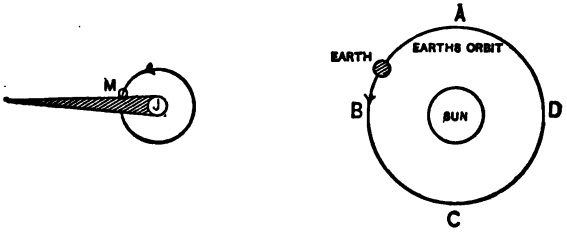


FIG. 312. Roemer's method for measuring the speed of light.

satellite." Every time this moon, *M*, revolves about Jupiter, *J*, it is eclipsed to an observer on the earth, *i.e.* it passes into the shadow of Jupiter, as indicated in Fig. 312.

Roemer measured the period of this first moon very carefully, *i.e.* determined very exactly the *average* interval of time between two of its successive eclipses. Knowing this, of course he could predict the time of future eclipses. This he did; but on comparing the predicted times with the observed times, he found that whenever the earth was in that half of its orbit next to Jupiter, *ABC* (Fig. 312), these eclipses occurred each a little earlier than the predicted time. But when the earth was in that half of its

orbit away from Jupiter, *i.e.* in the half marked *ADC*, the eclipses were each a little tardy in their appearance. And in particular when the earth was at the point *B*, nearest to Jupiter, the eclipse occurred about 8 minutes earlier than the predicted time; while at *D*, a point most remote from Jupiter, the eclipse took place more than 8 minutes late. From this Roemer concluded that 8 minutes and 18 seconds is the time required for light to travel from the point *B*, or the point *D*, to the center of the earth's orbit, *i.e.* from the sun to the earth. This distance is 154 million kilometers. Hence, for the velocity of light he obtained approximately $\frac{154 \times 10^9 \text{ m.}}{498 \text{ sec.}} = 309$ million meters per second.

This, of course, is the rate at which light travels in a vacuum; for the region between the earth and sun is probably a very perfect vacuum. But experience shows that there is very little difference between the speed in air and in vacuum.

Modern methods for determining the velocity of light are better than that of Roemer. The best determination is that of Professor Michelson, who finds that in a vacuum the velocity of light is 299,853,000 meters per second. For all practical purposes we may use the more easily remembered figure, 300 million meters per second. Reduce this speed to miles per second, and enter the result on the margin of this page.

Speed of Light in Media other than Air or Vacuum

296. It will be observed that any determination of the velocity of light must include the measurement of a distance and an interval of time.

In order to measure the velocity of light in water, or any other kind of ordinary matter, it is evident that we

must employ a much smaller distance than that used by Roemer. But if we employ a small distance, the time interval becomes so minute that its measurement is a matter of the utmost difficulty. This difficulty was first overcome by the French physicist, Foucault, who, in 1850, proved, beyond all doubt, that light travels at a much slower rate in water than in air or in vacuum. And indeed it has been shown since then that **light travels more slowly in all kinds of matter than it does in a vacuum.**

Refraction at a Plane Surface

297. We are now, for the first time, prepared to consider what is one of the most important phenomena in the whole subject of optics.

Indians in spearing fish show themselves very familiar with the fact that the fish is not exactly where he appears to be in the water, but is always a little lower down. Boys in wading a creek generally learn that they have to roll their trousers higher than at first they thought necessary; for the creek is always deeper than it appears to be. A lead pencil placed in a tumbler of water and looked down upon from one side appears to be sharply bent at the point where the pencil enters the surface of the water.

One explanation of these three, and of all similar, phenomena is easily obtained by an experiment which is described by Lord Bacon. He places a small piece of metal, say a coin, in the bottom of an opaque bowl; and then gives his eye such a position, E, that he just *cannot* see any part of the coin, P (Fig. 313). If now water be carefully poured into the bowl while neither the position of the eye nor that of the coin is changed, it will be seen that the coin is lifted into view, as indicated in

Fig. 314. The change in direction of the ray, *PS*, when it leaves the water and passes into air, is called **refraction.**

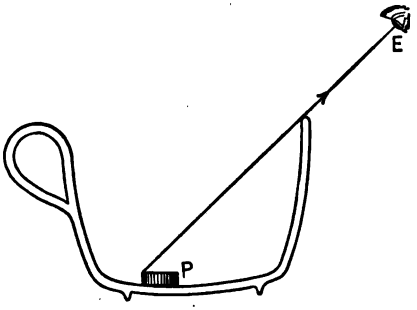


FIG. 313.

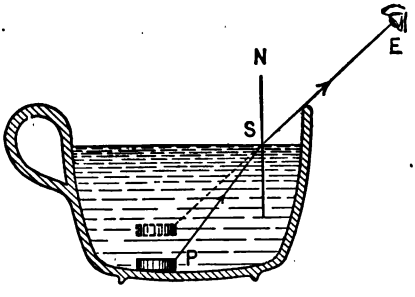


FIG. 314. Coin is lifted into view by the water.

In general, it is found that **the ray emerging from water to air is bent away from the normal, *NS*, while a ray which enters the water from the air is always bent toward the normal.**

The same phenomenon happens when a ray enters the air from glass, as may be easily seen by drawing a straight pencil mark on a piece of white paper, and then viewing this line through a thick block of glass, or even a small piece of plate glass. Except when observed straight through the glass, the line seems to be broken at the edge of the block, and appears to shift position always to the other side of the line from which the eye observes it. This can be easily explained by assuming that here, as in the case of the coin in water, the ray of light is bent toward the normal on going from air to glass, or, what is the same thing, is bent away from the normal on going from glass to air.

What happens in the case of water and air, or glass and air, happens in general whenever a ray of light passes from any medium to any other.

Index of Refraction

298. The first man who described all these phenomena in a simple manner was Willebrord Snell (1591–1626), a Dutch mathematician. His description is as follows:

Let AS (Fig. 315) be any bounding surface separating two different media, say water and air. Take any point, S, on this surface and describe a circle about it as a center. Let QS be any ray incident at S; the problem is to find the direction of the refracted ray, SP. Snell discovered that this could always be done as follows; viz. from Q let fall a perpendicular QL on the line NS, which, is drawn normal to the surface at S. Then, if we define PM as the perpendicular distance of P from this same normal, the ratio of QL to PM is a constant for any two media. This constant is called the **index of refraction.**

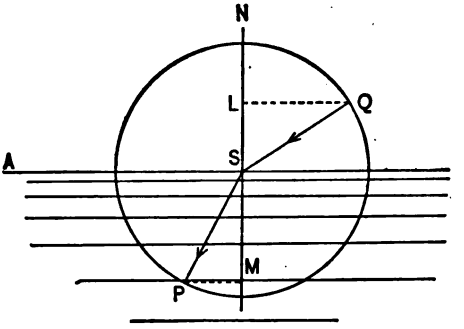


Fig. 315. Snell's description of the refracted ray.

$$\frac{QL}{PM} = \text{constant} = \text{index of refraction}$$

For water and air this constant is about 1.33; for ordinary glass and air its value is about 1.52.

Definitions. **The angle QSL, between the incident ray and the normal, is called the angle of incidence**; it is generally denoted by i. **The angle PSM, between the refracted ray and the normal, is called the angle of refraction**; it is generally denoted by r.

For those who are familiar with trigonometry it will be evident that

$$\text{index of refraction} = \frac{\sin i}{\sin r}$$

Laws of Refraction

299. The facts described above may be summarized as follows: —

(i) **For any two media the ratio $\frac{QL}{PM}$ (Fig. 315) is a constant and is called the index of refraction.**

(ii) **On entering any medium of higher refractive index the ray of light is bent toward the normal. On entering a medium of lower refractive index the ray is bent away from the normal.**

(iii) **The incident ray, normal, and refracted ray lie in the same plane.**

PROBLEMS

1. Draw a diagram representing the path of a ray of light through a piece of plate glass.

2. Why does a straight stick in water appear bent at the surface of the water?

3. Why does still water always appear shallower than it really is?

4. Why is it necessary to "shoot under" a fish?

5. A ray of light strikes a piece of glass making an angle of incidence of 45°. The index of refraction from air to glass is $\frac{3}{2}$. Construct the path of the ray into the glass.

Total Reflection

300. Any one who takes the pains to examine minutely a freshly fallen snowflake will be impressed with the fact that snow is made up of small but transparent crystals of ice. The question now forces itself upon us: How does transparent ice acquire the brilliant whiteness of snow? The same phenomenon has been observed by any one who has ever broken up a piece of clear ice with a ham-

mer. The moment the ice is broken into small pieces it appears white. The same is true of finely broken glass. To explain this phenomenon, let us consider any point in the interior of a solid piece of glass. Suppose a ray of light (Fig. 316) to leave the point P_1, and pass on through

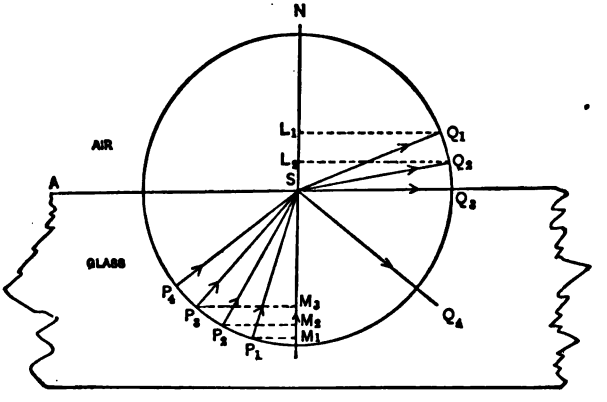


FIG. 316. The phenomenon of total reflection.

the surface AS. The angle of incidence is P_1SM_1. The incident ray has the direction P_1S, but is refracted in the direction SQ_1. Imagine another ray to start from the point P_2: the angle of incidence now becomes P_2SM_1, the angle of refraction increases to, say, Q_2SL_2. As the angle of incidence increases still farther, the refracted ray finally takes the direction SQ_3, that is, the refracted ray just grazes the refracting surface. The perpendicular from Q_1 let fall upon the vertical line NM_1 has now its largest possible value. If we still further increase the angle of incidence to P_4SM_1, **it is observed that refraction does not occur**; but that the ray is reflected back at S into the glass. This phenomenon is called **total reflection**, and occurs whenever the angle of incidence exceeds the angle P_3SM. There is then a certain limiting angle of incidence

for any ray in glass, such that if this angle be exceeded the ray will not escape from the glass, but will be sent back into it. This is called the **critical angle. The critical angle may therefore be defined as that angle of incidence which gives an angle of refraction equal to 90°.** The following table gives the critical angles for a few of the more important substances:

Table of Critical Angles

Diamond	24°
Ruby	36°
Rock Salt	41°
Crown Glass	42°
Turpentine	43°
Water	48°

A test tube containing only air and held in a tumbler of water shows total reflection very nicely. Much of the light attempting to pass from the water through the test tube fails, because the angle of incidence is greater than 42°. A tumbler of water held a little above the level of one's eyes readily shows total reflection from the lower surface of water. (See Fig. 317.)

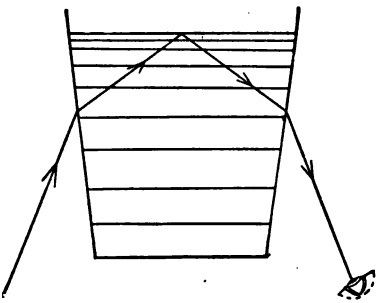


FIG. 317. Total reflection in a tumbler of water.

We now see how snow appears so white. Any stray ray of light which enters a body of snow will, in general, penetrate only a few of the transparent crystals before it will strike some refracting surface at an angle greater than the critical

angle. It will then be totally reflected and will finally find its way out into the air again. A large proportion (not all) of the rays of light which fall on snow will therefore be sent back. But this is all that we mean by a white body, viz. one which reflects a large portion of all the light which falls upon it.

301. From Fig. 318, it will be evident that a right-angled prism, with equal sides, may be used as a mirror for rotating a beam of light through 90°. Such a prism made of crown glass has a critical angle of 42°, while the angle of incidence for light entering a face at right angles is always 45°. Mirrors of this kind therefore require no silvering and are very frequently used. Heavy glass prisms of this form are employed as sidewalk lights, since they have the advantage of reflecting into the cellar that light which would otherwise fall upon the space immediately under the pavement.

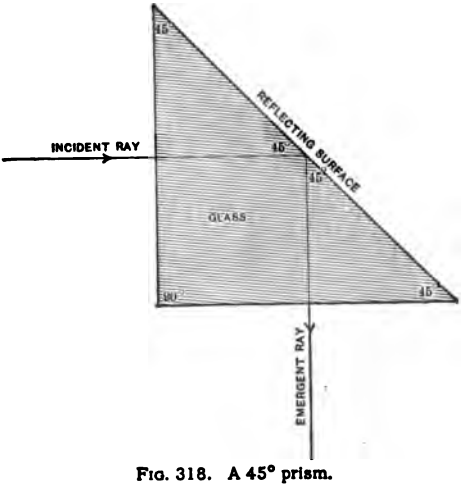


Fig. 318. A 45° prism.

Explanation of Refraction

302. We shall now assume that the student is fairly familiar with the phenomenon of refraction; and proceed to inquire whether it is like any other phenomenon which we have already studied, whether there is any simpler

class of facts to which we can refer it as a special case, whether, in short, we can *explain* the phenomenon of the bending of the rays.

An Illustration of Refraction

Let us suppose that a regiment of soldiers is marching over a country interspersed with plowed fields; and suppose also that the regiment moves with a speed of 3 miles an hour over the smooth ground and 2 miles an hour over the plowed ground. Let AB (Fig. 319) represent the line which separates the smooth ground from the plowed. The arrows indicate the line of march, *i.e.* the arrows are at every point perpendicular to the front of the ranks.

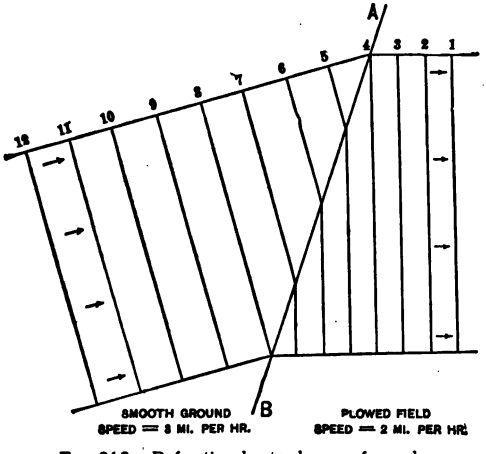


Fig. 319. Refraction due to change of speed.

Consider any rank, say No. 8, which is just entering upon the rough ground. Evidently the right-hand end will be slowed up, for the left end is now traveling one mile an hour faster than the right — the front will now form a slightly bent line, until the whole rank has crossed into the plowed region: and then the rank will again be straight, **but it will be headed in a direction slightly different from that which it previously had.** It will, in fact, be **refracted.** Not only so, but the line of march will be refracted toward the normal to the boundary line BA.

We have already seen (p. 349) that light travels more slowly in glass than in air. The refraction of rays of light is therefore strictly analogous to the refraction of the line of march, each depending upon a change of speed in passing from one medium to another.

We have already (p. 150) found by experiment that the speed of water waves depends upon the depth of the water in which they travel — the deeper the water the faster the wave travels. If now we consider a series of waves, whose crests are represented by the straight lines in Fig. 320, it is evident that when these waves cross the boundary line *AB*, they will be deflected just as the soldiers are, and just as the rays of light are. This analogy between waves and light is essentially due to Huygens (1629–1695), a brilliant Dutch physicist and mathematician.

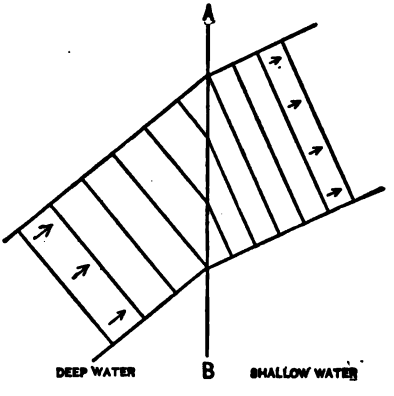


Fig. 320. Refraction of water waves due to change of speed.

Physical Meaning of Index of Refraction

303. From what precedes it will be evident that the amount of bending (refraction) which a ray undergoes in passing from one medium to another depends entirely upon the ratio of the speeds of light in these two media: If the speed is the same in each, there is no change in the direction of the ray. If the speed is less in the second medium, the ray is bent toward the normal.

From Fig. 321 it is evident that, if CA represents the distance which light travels in air, during the time required for it to travel from B to D in glass, we have

$$\frac{\text{speed in air}}{\text{speed in glass}} = \frac{CA}{BD} = \text{refractive index}$$

When, therefore, we say that the refractive index of a piece of glass is 1.52 we mean that light travels through air 1.52 times as fast as through this kind of glass. In like manner we know from the refractive index of water that light travels $1\frac{1}{3}$ times as fast in air as in water.

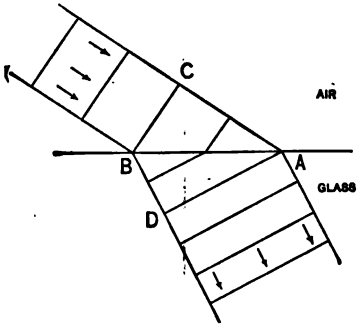


FIG. 321. The physical meaning of refractive index.

We have already seen (p. 162) that energy is, in general, transmitted in either one of two ways, viz. by currents or by waves. By which of these methods does light travel from one point to another? By currents, or by waves?

LIGHT A WAVE MOTION

304. The first decisive answer to this question was given by Dr. Thomas Young (1773–1829), a London physician. He argued that **if light is a wave motion, we ought to be able to add two trains of light waves together and produce darkness,** just as (see p. 159) we add together two trains of waves in strings and thereby produce rest. If, however, light consists of currents of matter (emitted by the luminous body), as Newton had suggested, one cannot obtain darkness by adding two currents together. On the contrary, the two currents would produce *more* disturbance than either one alone, *i.e.* more light than either one alone.

The manner in which Dr. Young added two rays of light is very simple, and will be clear to any student who has examined the phenomenon of diffraction described above, p. 339. We there found that a very small beam of light on passing through a small opening was spread out into a cone of light.

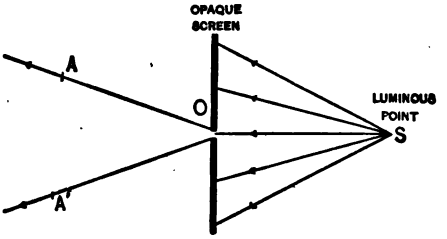


FIG. 322. Diffraction of light through a single opening.

An eye placed anywhere between the points A and A' (Fig. 322) will receive upon its retina, as upon a screen, one single series of disturbances through the opening at O. What is really seen is the image of S produced by the small opening at O, and by the lens in the eye. Remember, the whole of the region included in the angle AOA' is filled with light.

But if *two* apertures be made side by side in the opaque screen, each will furnish a wedge of light, as represented in Fig. 323. These two wedges, AOA' and BOB', will overlap in the region between B and A'. An eye placed anywhere in this region will receive upon its retina two different series of disturbances, one from O, and another from O'.

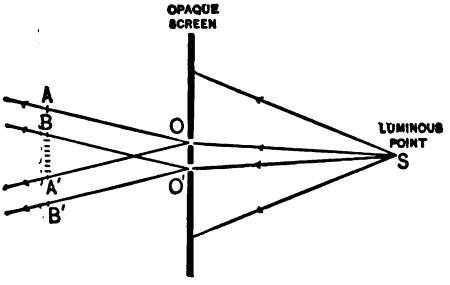


FIG. 323. Two rays of light interfere to produce alternate brightness and darkness.

Under these circumstances one sees in the region between B and A' an image of the opening at S; and the image is filled with an alternation of bright

and dark bands. These bands are symmetrically placed on each side of a bright central band.

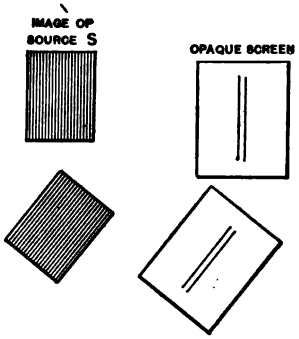


FIG. 324. Showing that the dark bands depend upon the two slits.

It will be observed also that as the opaque screen of cardboard is rotated in its own plane these bright and dark bands also rotate, remaining always parallel to the slits. (See Fig. 324.)

Accordingly (Dr. Young argued) if light consists in a wave motion, we may expect to find that at any point, P (Fig. 325), **where the respective distances of the two openings, PO and PO', differ by one half a wave length, these two trains of waves annul one another and produce darkness**; but at any other point, Q, where the two paths, QO and QO', differ by one whole wave length, or any whole number of wave lengths, there the brightness will be greater than that due to either train alone. And this is exactly what he found to be the fact. **Light from a bright point shining through two small openings in a card behaves like water waves from the sea rushing through two small openings in a breakwater.** Two trains of waves which combine with each other to produce alternately bright and dark bands in this manner are said to **interfere**; and the phenomenon is called **interference**.

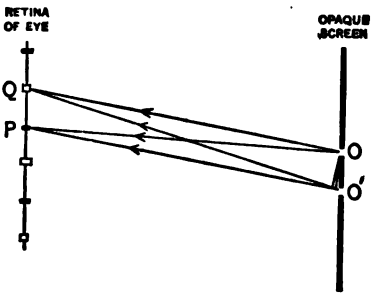


FIG. 325.

Summary of Evidence for Thinking Light a Wave Motion

305. 1. Light is diffracted as are waves when passing through a small opening.

2. Light is reflected as are waves.

3. The speed of light depends upon the medium through which it travels, as does the speed of waves. This change of speed gives rise to refraction in each case.

4. Two rays of light interfere to produce alternate darkness and brightness, just as two trains of sound waves interfere to produce silence, and two trains of waves in a string interfere to produce rest.

Wave Front. Rays

306. With this justification we shall hereafter speak of light as a wave motion. The continuous surface, each point of which is moving in the same phase, we shall call the **wave front.** For instance, a particular wave front would be a surface at each point of which a particle is at the end of its swing in one direction. This wave front may be thought of simply as a surface which is at any point perpendicular to the *ray* of light which passes through that point.

"There really are no rays. The harder one tries to isolate a 'ray' by itself, by letting light go first through a narrow slit or pinhole, and then passing it through a second slit or pinhole, the more do we find it impossible, for then we notice the tendency to spread more than ever. If the word 'ray' is to be retained at all in the science of optics, it must be understood to mean nothing more than the geometrical line along which a piece of the wave front marches."—Sylvanus P. Thompson, "Light, Visible and Invisible," p. 12.

This wave front corresponds to a wave crest as water

waves traverse the surface of the sea. It is important, however, to note that the wave front, in the case of light waves, is always a surface, never a line, as is the crest of a water wave.

Color produced by Dispersion

307. The explanation of the colors of the rainbow was first given by Newton (1643–1727). He showed that when a beam of white light, such as sunlight, falls upon a prism of glass (Fig. 326), this beam is separated into a large number of different colors. The red is refracted least of all and the violet most of all; the remaining colors of the rainbow are interspersed between these two colors in the following order:

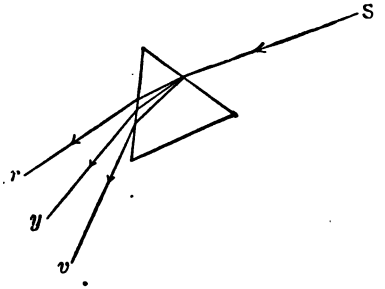


Fig. 326. A prism deviates violet light most and red light least.

Violet
Indigo
Blue
Green
Yellow
Orange
Red

This observation of Newton's is most easily verified by holding a prism immediately in front of the eye and viewing a small naked gas flame or a flame covered with the slotted tin chimney described above, p. 339.

If S (Fig. 327) denotes the source of light, and I, the point of incidence of any ray, the angle between any re-

fracted ray and the line *SI* produced is called the **angle of deviation**, or simply the **deviation**, for that ray.

Since any image is seen in the direction in which the ray enters the eye, it is evident that those rays which are most refracted will give an image on the side away from the source, as shown in Fig. 327. In the case of a glass prism

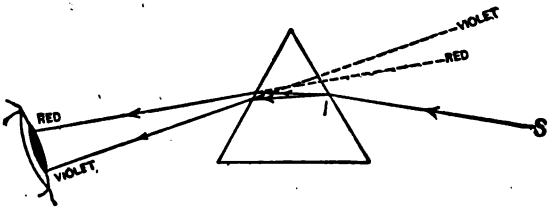


FIG. 327. A source of light viewed through a prism has the red edge of its image on the side next the source.

the blue light is most refracted. Accordingly we assume that in glass blue light travels at a slower speed than red light. In like manner, yellow light travels at a slower speed than red, but at higher speed than blue.

The band of color into which any source of light is thus drawn out by a prism is called a prismatic spectrum.

From the preceding it is evident that the order in which the colors are arranged in this spectrum is simply the order of their speeds in the material of which the prism is made.

COLOR PRODUCED BY ABSORPTION

308. On p. 333 we have considered the distinction between transparent and opaque bodies. It has been found, however, that most bodies are transparent to some colors and opaque to others. Red glass, for instance, is transparent for red light, but absorbs almost all other colors. A large amount of smoke or dust in the atmosphere makes air behave in the same way and produces the so-called

"red sunsets." A solution of indigo is more transparent for blue than for any other color.

Color is sometimes produced by absorption at surfaces. A red brick building, for example, may be illuminated by white sunlight, but the bricks still appear red, because the brick surface has the power to reflect only red rays, other colors being largely absorbed in the surface of the brick.

Application of Preceding Principles

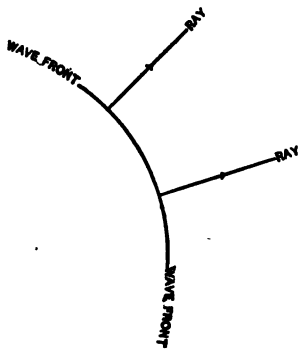


Fig. 328. Rays considered as normals to wave front.

309. Concerning optical instruments, in general, it may be said that they are simply devices for changing the directions of rays of light.

As we have seen from the preceding pages, a ray of light is simply a normal to the wave front. It is evident, therefore, that anything which changes the shape or direction of the wave front will change the direction of the rays. One of the commonest devices used for this purpose is a lens.

Lenses

310. Let S denote a luminous point, sending out rays in many directions. The wave front, wf_1, will be a spherical surface of which S is the center.

Suppose now that we place before any wave front a piece of glass, L, which is bounded by two spherical surfaces, *i.e.* a piece of glass which is thick in the middle and tapers off gradually all around to a thin edge. Since light travels only two thirds as fast in glass as in air, it is evident that the middle portion of the wave front will be much more delayed in passing through the glass than those portions

near the edge; for the thicker the glass the greater the delay. If, therefore, the wave front is convex, on its advancing side, before it enters the glass, it will be *less* convex when it emerges from the glass. Such a piece of glass is called a **converging lens.** The middle portions may, indeed, be delayed so much that a wave front, which is convex before it enters the glass, is concave when it leaves the glass. Such a case is represented in Fig. 329.

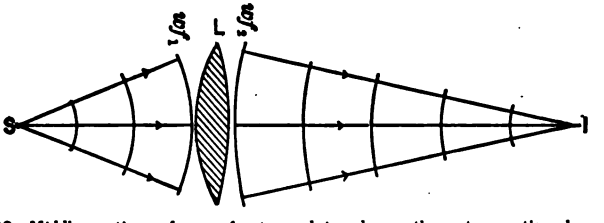


FIG. 329. Middle portions of wave front are delayed over the outer portions by passing through thicker glass.

If, however, we interpose in the path of the light a glass which is thinner in the middle, as in Fig. 330, it is evident that the outer portions of the wave front, wf_1, will be de-

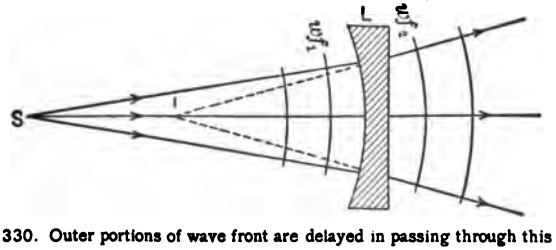


FIG. 330. Outer portions of wave front are delayed in passing through this lens.

layed over the central portions. The result of this will be to make the wave front *more* convex than before it entered the glass. A lens of this kind is called a **diverging lens.**

THE FOCAL LENGTH OF A LENS

311. Let us suppose that the rays of light which fall upon a lens are parallel, or, what is the same thing, let us

suppose that the incident wave front is a plane. The rays of light which come to us from the stars, the sun, the moon, or any very distant object, are nearly parallel. Under these circumstances a converging lens will retard

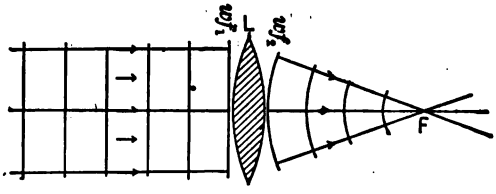


FIG. 331. Focal length of a converging lens.

the middle of the wave front and change it into a concave surface, as indicated in Fig. 331.

Lenses are ground in such a way as to make this emergent wave front very nearly spherical. The center, *F*, of this spherical surface is called the **principal focus** of the lens, or, in other words, **the principal focus is the point where, after refraction, parallel rays meet. The distance from the principal focus to the center of the lens is generally called the focal length** of the lens, and is, in most cases, very approximately equal to the true focal length of the

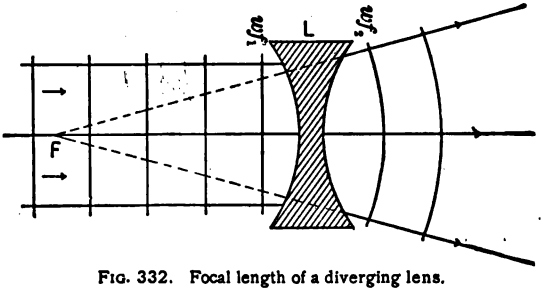


FIG. 332. Focal length of a diverging lens.

lens. The exact definition of focal length is a matter too difficult for introduction here.

If the lens be a diverging one, the same definitions hold,

only in *this* case the effect of the lens is to push ahead the center of the plane wave front, *i.e.* the emergent wave front is a convex spherical surface and has its center of curvature on the side of the lens next the source of light. The focal length of such a lens (Fig. 332) may be said to be negative.

Returning now to the converging lens (Fig. 333), let us place a luminous point at F; the direction of each ray will be exactly reversed, and the wave front which emerges from the lens will be plane. The emergent rays will, therefore, be parallel. This is the usual method of producing parallel rays.

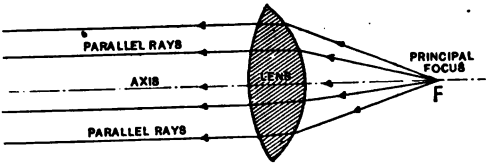


FIG. 333. Rays from the principal focus are parallel on emergence.

POWER OF A LENS

312. The **power** of a lens is defined as **the reciprocal of the focal length.** For, the shorter the focal length of a lens, the greater the change it produces on the direction of rays passing through it; hence its power is said to vary inversely as its focal length.

A lens whose focal length is 1 meter is said to have a power of 1 **dioptric.** A lens whose focal length is $\frac{1}{2}$ meter has a power of 2 dioptrics; and so on. **The power of a lens is simply its ability to change the divergence or convergence of rays incident upon it, and is measured by the reciprocal of its focal length.**

SPHERICAL MIRRORS

313. We next consider the effect of a concave spherical surface on a plane wave front. Imagine the surface to be

polished so that it will reflect the rays which are incident upon it.

Let wf_1 (Fig. 334) be any plane wave front. It is evident that the *outer edges* of the wave front will strike the mirror first and will be started back before the central portion of the wave front has reached the mirror.

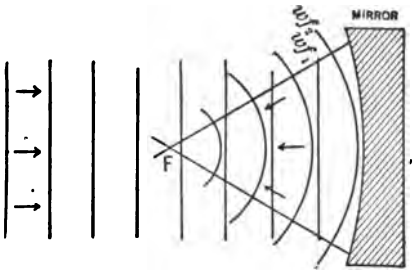


FIG. 334. Concave mirror acts like a converging lens.

The effect of this will be to make the reflected wave surface concave on the advancing side, *i.e.* **a concave mirror acts in the same manner as a converging lens.** Its principal focus is at the center, *F*, of the spherical wave front, just as in the case of a lens.

It can be shown by experiment, as well as by elementary geometry, that **the distance of the focus from the center of the mirror is exactly half the radius of the mirror.**

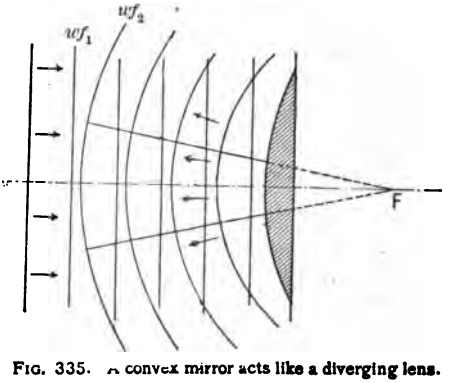


FIG. 335. A convex mirror acts like a diverging lens.

In **a convex spherical mirror** the emergent wave front has a greater convexity than the incident wave front. Hence such a mirror **acts precisely like a diverging lens**; *i.e.* the emergent wave front is more convex than the incident wave front.

The Optical Axis of a Lens

314. The only surfaces which can be ground in lathes with accuracy and ease are spherical surfaces, hence many lenses are made with spherical surfaces, *i.e.* the shape of a lens may be considered as the shape of a figure bounded by two spheres.

The line joining the centers, *A* and *B*, of these two spheres is called the axis of the lens.

If one face of the lens is a plane, we may consider this plane as a portion of a sphere of infinite radius. The center of such a sphere will lie on the normal to the plane surface.

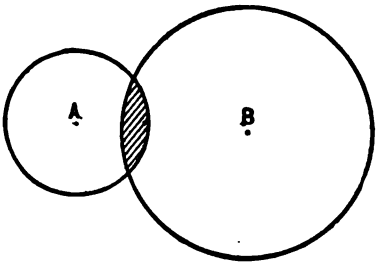


Fig. 336. A converging (convex) lens.

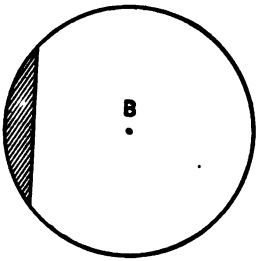


Fig. 337. A converging lens (plano-convex) with one plane surface.

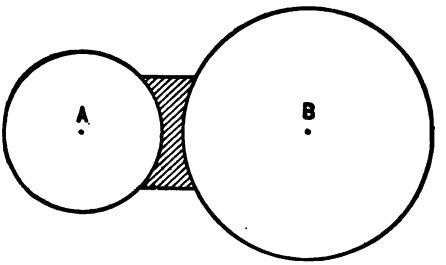


Fig. 338. A diverging (concave) lens.

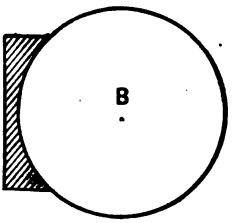


Fig. 339. A diverging lens (plano-concave) with one plane surface.

It will be observed that the axis of a lens is normal to each surface of the lens at the point where it passes through the surface.

The Image produced by a Lens

315. Up to the present we have considered only the wave front produced by one luminous point, *O*. And

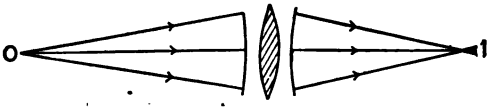


Fig. 340. Image of a point produced by a converging lens.

we have seen that the principal effect of a lens is to change the shape of this wave front so that the rays converge to another point, *I* (Fig. 340). This point *I* is called the **image** of the point *O*. If the lens be a diverging one, the

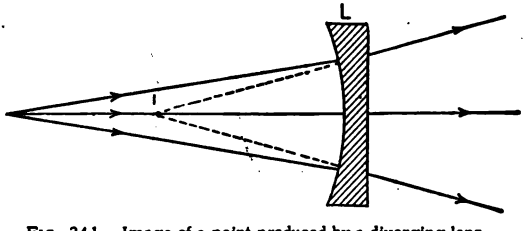


Fig. 341. Image of a point produced by a diverging lens.

effect will be to scatter the rays, so that their prolongations backward converge in a point *I* (Fig. 341). This point *I* is also called the image of *O*.

The Location of an Image

316. To determine the position of this image, *I*, in a simple graphical way, we have only to remember the following general principles:

(1) **A ray falling on a lens in a direction parallel to its axis passes, on emergence, through the principal focus.**

(2) **An incident ray which passes through the principal focus of a lens will emerge in a direction parallel to the axis.**

Thus, if O (Fig. 342) be a luminous point whose image in the lens L is desired, we have only to **consider two rays emitted by the point O, one, OA, parallel to the axis, and the other, OF_1, passing through the principal focus, F_1.** On

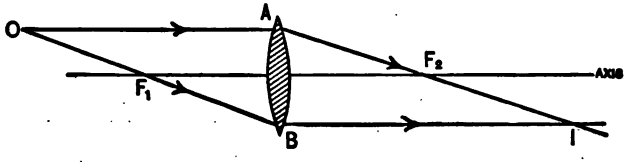


FIG. 342. Graphical determination of the position of an image.

emergence OA must take a direction AF_2, passing through the other principal focus, F_2, while OF_1 on emergence will take a direction, BI, parallel to the axis of the lens. The point of intersection, I, of these two rays is the image of the point O. All other rays emitted by O, and falling on the lens, pass through I.

REAL AND VIRTUAL IMAGES

317. Those images in which the refracted rays actually intersect, as in Fig. 342, **are called real images. Those in which only the prolongations backward intersect, as in** Fig. 341, **are called virtual images.** Is the image produced by a plane mirror real or virtual?

IMAGES OF SURFACES

318. For optical purposes, a body may be considered as a collection of luminous particles, or as a luminous surface.

To find the image of a body which is not a point, we have, therefore, only to find the image of a large number of points. These may be located in succession by the same method which we have employed above for a single point.

Figure 343 shows what kind of an image is given by a converging lens when the body O is placed at a distance

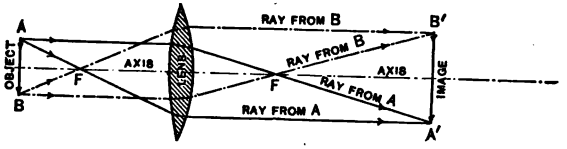


FIG. 343. Illustrating the construction of an image in a convex lens.

from the lens which is greater than the focal length. **The image is inverted and real.** If the lens be diverging, as in Fig. 344, we may trace as many points as we choose by

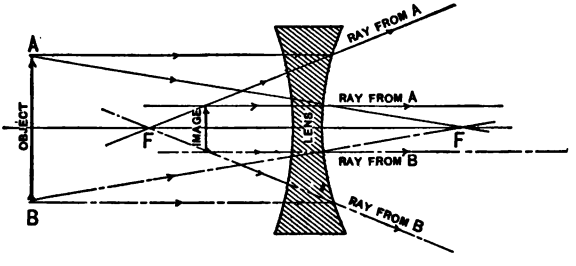


FIG. 344. Illustrating the construction of an image in a concave lens.

the same method. Here we see that **the image is virtual and erect.**

CONJUGATE FOCI.

319. On locating by experiment the position of the image of any object (Fig. 345 shows the construction for

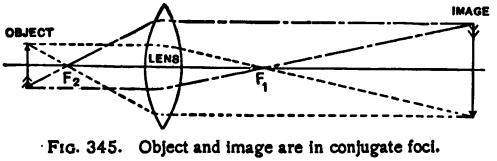


FIG. 345. Object and image are in conjugate foci.

a convex lens), it is found that a simple numerical relation exists between the focal length of the lens and the respec-

tive distances of object and image from the center of the lens. It may be thus expressed:

$$\frac{1}{\text{focal length of lens}} = \frac{1}{\text{image distance}} + \frac{1}{\text{object distance}}$$

or, in symbols, $$\frac{1}{F} = \frac{1}{u} + \frac{1}{v}$$

Image distance (u) and object distance (v) are called conjugate focal distances, and the positions of image and object are the conjugate foci of the lens.

For any real image the relation as stated is always true. For any virtual image (Fig. 344 illustrates the construction for a concave lens) the focal length of the lens may be computed by regarding one of the conjugate focal distances as negative, which one can usually be easily determined by the conditions of the problem.

It is invariably the case that a **real image** may be regarded as formed by the **actual** intersection of rays; *i.e.* a real image is formed in a **possible position.** For a lens it will lie on the side of the lens away from the object, for a mirror on the side toward the object. **Virtual images,** on the other hand, always lie in **impossible** positions; namely, behind a mirror or in front of a lens.

It is also observed that

$$\frac{\text{size of object}}{\text{size of image}} = \frac{\text{object distance}}{\text{image distance}}$$

The same relation is true for spherical mirrors, remembering that the focal length of a mirror is half its radius of curvature, and that a concave mirror acts like a convex lens while a convex mirror acts like a concave lens.

The Image produced by a Convex Lens

320. A very interesting and instructive experiment can be performed as follows:

A screen and candle are set some distance apart and a convex lens is placed between them. If the lens is moved near to the screen, a small, inverted image of the candle may be obtained on the screen. If the lens is placed near to the candle, a very large, inverted image is formed, frequently too large to be received on the screen. By adjusting the distance between candle, screen, and lens, a position may be found where image and object are equal in size but the image is still inverted. When the lens is very near the candle, no image is formed on the screen; but, on viewing the image directly through the lens, it is seen to be larger than the object and erect.

These observations may be thus summarized:

(i) *The image is erect, virtual, and larger than the object when the object is inside the principal focus.*

(ii) *The image is inverted, real, and larger than the object when the object is outside the principal focus and inside twice the focal length.*

(iii) *The image is inverted, real, and the same size as the object when the object is at twice the focal length.*

(iv) *The image is inverted, real, and smaller than the object when the object is outside twice the focal length.*

The Achromatic Lens

321. We have seen (§ 307) that, when white light passes through a prism, it is both deviated and dispersed. The same thing happens in ordinary lenses and the images formed have bands of color around them. Such lenses in a camera or telescope do not give satisfactory results. The difficulty is obviated by the use of an **achromatic lens.**

All kinds of glass do not give the same dispersion nor is their dispersion proportional to their deviation. Therefore a convex lens of crown glass (1 in Fig. 346) which gives large deviation and small dispersion is combined with a concave lens of flint glass (2 in Fig. 346) which gives small deviation and small dispersion in the opposite direction. The dispersions of the two lenses neutralize, leaving a colorless image, while some deviation still remains. Such **a combination of a convex crown glass lens with a concave flint glass lens is called achromatic.** It is possible to correct the dispersion perfectly for *one* color, but not for all, because each color has its own index of refraction.

FIG. 346. The achromatic lens.

PROBLEMS

1. The image of an object placed 20 cm. from a lens is found to be inverted at a distance of 15 cm. from the lens. What is the focal length of the lens?

2. A camera is focused on a house 25 ft. away. The image is 4 in. from the lens. What is the focal length of the lens?

3. The image of a post 12 ft. high and 20 ft. from the camera is 5 in. long. How far from the lens is the image?

4. A concave mirror has a radius of curvature of 10 cm. Find the position and size of the image of a pin 3 cm. high, placed 8 cm. in front of the mirror.

5. Determine the focal length of a lens which gives an image at 40 in. from the lens when the object is erect, 8 in. from the lens, and on the same side as the object.

6. The focal length of a convex lens is 10 cm. Construct the image of a pin 3 cm. high as seen through the lens if the pin is set 20 cm. from the center of the lens. Is it real or virtual, inverted or erect?

7. Construct the image of an arrow placed within the principal focus of a convex lens. What kind of an image is formed?

8. Describe by a diagram the image of a tree as seen through a concave lens.

9. An image of a candle is formed by a concave mirror whose radius of curvature is 20 cm. Draw diagrams to show the kind and appearance of the image (*a*) when the candle is 15 cm. from the mirror, (*b*) when the candle is 5 cm. from the mirror.

10. A man stands before a convex mirror. Describe by diagram the appearance of his image.

OPTICAL INSTRUMENTS

I. *The Pinhole Camera*

322. If light is admitted to a darkened room through a small opening, there will appear on the opposite wall or on a properly placed screen inverted images of outside objects (Fig. 347). The formation of these images is a result of the rectilinear propagation of light.

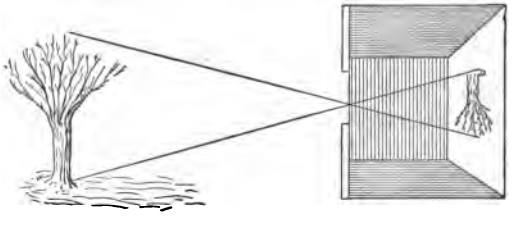


FIG. 347. Images by small openings.

A camera may be constructed on this principle by which fair pictures may be taken. The box has at one end any suitable device for holding a plate or film. At the other, light is admitted through a pinhole. The image is formed precisely as shown in Fig. 347.

II. *The Photographic Camera*

323. A photographic camera, which is perhaps the simplest of all optical instruments, is merely a box fitted at one end with a converging lens and at the other end with a device for holding a plate or film which is sensitive to

light. The sides of this box are usually made of folded leather so that the distance of the plate, *P* (Fig. 348), from the lens, *L*, may be varied at will.

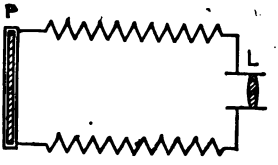


FIG. 348. Section of photographic camera.

This rear end of the camera is furnished also with a ground glass which may be placed in the same position that the sensitive plate is to occupy later.

The *first* step in taking a photograph consists in placing this ground glass in such a position that the lens will produce upon it a sharp image of the object whose photograph is desired. This is called **focusing**.

In the hand camera the proper position of lens for each distance of object is marked on the camera; so that time and trouble of focusing is avoided.

The *second* step is to cover the lens so that no light can enter the camera; then put the sensitive plate in the position formerly occupied by the ground glass and **expose** the plate by again uncovering the lens for the proper length of time.

The *third* step is to **develop** and **fix** the plate. The lights and shadows are now just reversed from what they are in the subject. The plate in its present position is, therefore, called a **negative**.

The *fourth* step, called **printing**, again reverses the lights and shadows and gives the final picture.

Characteristics of a Good Lens

324. A good photographic lens is one:

(*a*) Which has the same focal length for yellow as for violet light; yellow rays being those which are principally used in focusing on the ground glass, violet rays being those which most affect the sensitive plate.

(*b*) Which has the same focal length for the edges as for the center of the lens. When this is so, the whole of the lens can be used, and the photograph taken with a short exposure. Such lenses are said to be **rapid** or **fast.**

(*c*) Which produces a distinct image, not only at the center, but clear out to the edges of the sensitive plate for which it is intended. Such a lens is said to have a **flat field.**

(*d*) Which gives the same magnifying power at the center and at the edge of the plate. If the lens is not carefully corrected so as to produce this result, buildings and geometrical figures will be distorted. A lens so corrected is said to be **rectilinear.**

III. *The Human Eye*

325. The eye, from an optical point of view, is a camera in which the sensitive photographic plate is replaced by a sensitive membrane, called the **retina.** This retina is connected with a nerve, called the **optic nerve,** which conveys the impression to the brain.

In the front part of the eye (Fig. 349) is placed a converging lens, *O*; this is called the **crystalline lens.** This lens projects upon the retina inverted real images of external objects. The distance between this lens and the retina remains practically constant. How, then, can the eye see objects which lie at different distances from the eye? For it is evident from Fig. 345 that the distance of the image from the lens varies with the distance of the object from the lens. The answer to this question lies in the

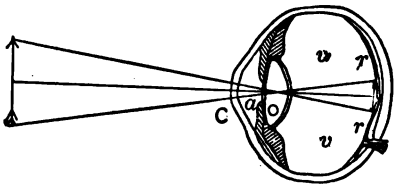


FIG. 349. Longitudinal section of the human eye.

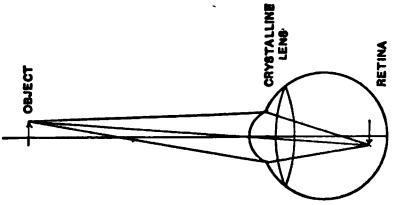
fact that the muscles of the eye are able at will to change the focal length of the crystalline lens. The power of this lens is thus varied automatically to suit the distance of the object. This process is called **accommodation.**

In front of the eye hangs a dark-colored diaphragm, in the center of which is an opening called the *pupil.* This aperture varies with the amount of light falling upon the eye. As the light grows brighter, the pupil contracts; as the light grows less, the pupil enlarges. Examine the pupil of your own eye in a mirror after remaining for a few minutes in a dark room; in a moment after coming into the light the pupil contracts again to its natural size.

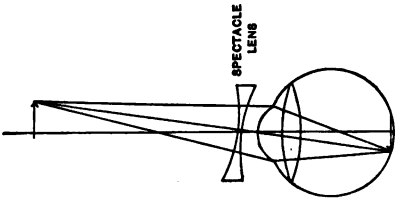
IV. *Spectacles. Reading Glasses*

326. We have seen that the human eye is provided with a mechanism for adjusting itself so as to see distinctly objects which lie at different distances. The normal eye when relaxed is adapted for rays which are parallel, *i.e.* for seeing objects which lie at a very great distance — practically an infinite distance. If one wishes to view a nearer object, he must accommodate his eye.

But it happens that there are some eyes, in which the accommodating mechanism is quite perfect, but which have eyeballs with too



A short sighted eye; uncorrected.



The same eye corrected by a diverging lens.

FIG. 350. The short sighted eye.

long an axis. The lens is too far away from the retina. The image falls short of the retina. Such an eye is said to be **short-sighted**, because it is only when the object is very close that the individual can see it distinctly. See Fig. 350.

When the axis of the eyeball is too short, the image is formed behind the retina. Such an eye can see distinctly, when relaxed, only by means of convergent rays. This defect is called **far-sightedness** and can be counterbalanced, *within limits*, by means of accommodation. But reading at normal distance, say 12 inches, is impossible. What is needed in this case is a converging lens.

Draw a diagram corresponding to Fig. 350, showing the behavior of a long-sighted eye before and after correction with spectacles.

V. *The Simple Microscope*

327. This instrument consists usually of a single converging lens of short focal length. The distinguishing feature of the simple microscope is that the lens is always placed at a distance from the object which is less than the focal length. Now, if the object, *O* (Fig. 351), were

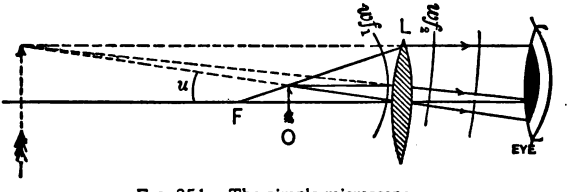


FIG. 351. The simple microscope.

placed exactly at the focus, the power of the lens would be just sufficient to flatten out the curved wave front, wf_1, so that it would emerge as a plane wave front. But if the object be placed between the focus and the lens, as in this case, wf_1 will be too convex for the lens to overcome its

curvature entirely; the emergent wave front, wf_2, will therefore still be convex on its advancing side. The image, I, will, therefore, lie on the same side of the lens as the object, O. The image will, therefore (p. 374), be virtual and erect. Since the image and the object subtend the same angle, u, at the center of the lens, it is evident that the image will be larger than the object in the following ratio:

$$\frac{\text{size of image}}{\text{size of object}} = \frac{\text{distance of image from lens}}{\text{distance of object from lens}}$$

We might, of course, bring an object up close to the eye so that it would subtend an angle as large as that of the image seen through the microscope, but unfortunately the eye cannot accommodate for these short distances. The necessity for the microscope arises then from our limited power of accommodation.

VI. *The Astronomical Telescope*

328. The astronomical telescope is an instrument used for viewing very distant objects. The rays of light which fall upon the telescope are, therefore, generally parallel.

The purpose of the telescope is twofold:

(1) To gather a large amount of light and condense it into a small bundle of rays, so that these can all enter the pupil of the eye and thus make any luminous *point* appear much brighter than it would to the naked eye.

(2) To magnify the angle subtended by two luminous points, and thus make them appear farther apart than when seen by the naked eye.

These purposes are accomplished most simply by two converging lenses, L and E. The lens L is called the object lens, or simply the **objective**; the lens E is called the eye lens, or, more frequently, the **eyepiece**.

The wave front, wf_2, sent out by a star is practically plane when it strikes the objective, as is indicated in Fig. 352.

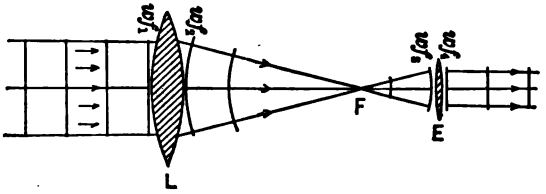


FIG. 352. The astronomical telescope.

The effect of the objective is to turn this plane wave front into a spherical one, wf_2. Let F denote the center of the wave front wf_2. This is called the principal focus of the telescope. The eye lens is so adjusted that its principal focus also lies at F. **Therefore, the rays which emerge from the eye lens are again parallel.**

The effect, therefore, of a large telescope, such as that at the Lick Observatory or at the Yerkes Observatory, is to take a bundle of parallel rays forming a cylinder a yard or so in diameter and condense it into a small cylinder of parallel rays just large enough to enter the pupil of the eye. So that the star (which is practically a luminous point) appears immensely brighter when viewed through a telescope than it does to the naked eye.

If two stars close together, say a double star, be viewed through a telescope, *each* will send a beam of parallel rays

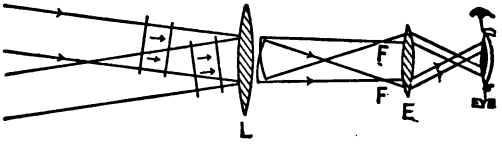


FIG. 353. The telescope as used to magnify an angle.

through the instrument, as indicated in Fig. 353. **But the two parallel beams which emerge from the eye lens**

make with each other a much larger angle than do the two parallel beams which enter the objective. The telescope thus magnifies the angular distance between two stars. It thus enables us to recognize stars as double when to the naked eye they appear single. For we can only distinguish two luminous points as separate when they subtend at the eye an angle which is greater than about two minutes of arc.

VII. *The Opera Glass*

329. The earliest telescopes did not employ a converging eyepiece such as those represented in Figs. 352 and 353; but instead they used a double concave eye lens, the power of which was equal to and opposite in sign to that of the human eye. A diagram of the telescope with which Galileo discovered four satellites of Jupiter is shown in

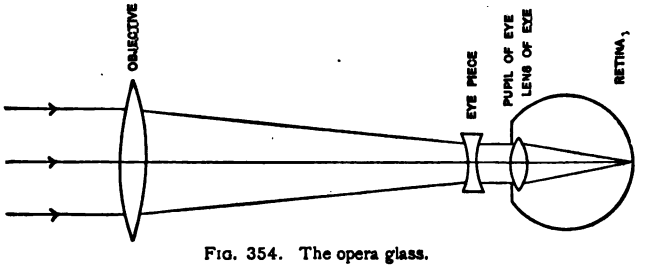


FIG. 354. The opera glass.

Fig. 354. On account of its compactness and because it gives an upright image, this same principle is employed in the modern opera glass.

If we assume what is nearly true, namely, that the eyepiece produces a curvature of the incident wave front, which is exactly equal and opposite that produced to the human eye, then by placing the objective of the opera glass at such a distance that its principal focus lies on the retina, we shall see the object distinctly; and since the

image is much larger than that produced by the naked eye, we shall see the object magnified. In short, the eyepiece and the eye act together, as if they constituted a piece of plane glass.

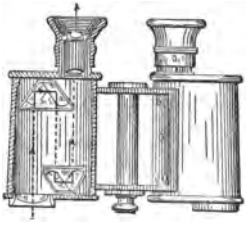


FIG. 355. The prismatic opera and field glass as perfected by Abbe in Germany.

A new type of opera and field glass, called the **binocular** (Fig. 355), has been developed in recent years which gives a much larger field and higher magnification than the ordinary opera glass. Instead of a concave eyepiece a convex lens is used just as in the telescope. In order to make the instrument short and to erect the image the light is totally reflected four times by prisms. The objectives are placed farther apart than in the old form of opera glass. The result is a much larger field of view.

VIII. *The Spectroscope*

330. The instruments we have hitherto studied behave practically in the same manner for all kinds of luminous rays, whether white or green or red or blue. We now take up the consideration of an instrument which enables us to distinguish between each of these various kinds of light. This is accomplished most simply by the use of a prism, which, as we have already seen (p. 362), rotates a blue wave front through a larger angle than it does a red wave front.

Any instrument employed to separate light of different colors or, more generally, light of different wave lengths, is called a **spectroscope.**

The **prism spectroscope** is generally made up of three parts, viz. a prism and two astronomical telescopes. They

are arranged as indicated in Fig. 356, where S is the source of light which is to be examined. F_1 and L_1 are the focus and the objective of one telescope, F_2 and L_2

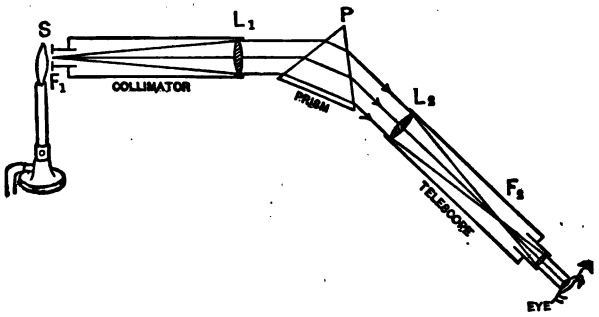


FIG. 356. A prism spectroscope adjusted to view one particular color in the spectrum.

the focus and objective of the other telescope. The prism, P, is placed between them. At F_1 is a narrow slit, which is illuminated by the light to be examined. From each point on this slit will proceed a bundle of rays which will pass through the lens L_1 and emerge as parallel rays.

The purpose of this telescope is to make the rays parallel; it is, therefore, called the **collimator**. These parallel rays fall upon the prism; but when they emerge from the prism, each different color has a different direction, but all the rays of any one color have the same direction.

The telescope L_2, which is called the **view telescope**, is made movable so that it can be turned into the proper direction for observing any desired color. An eye placed in front of the eye lens will there see at F_2 a colored image of the slit at F_1. By rotating the telescope about a vertical axis, one can determine just what colors are present in any source of light. The series of colored images thus seen is called the **visual spectrum** of the source of light, S.

By placing a photographic plate at F_2, these various

images of the slit can be photographed. An instrument of this kind for photographing spectra is called a **spectrograph**.

It is with such an instrument that much of our knowledge of the physical constitution of the sun and stars has been obtained.

Summary of Fundamental Phenomena in Optics

331. 1. Light moves in straight lines

(*a*) when the medium is homogeneous; and

(*b*) when the rays are not compelled to pass through any very small openings.

Hence intensity of illumination varies inversely as square of distance from source.

2. Rays of light when they strike a body are reflected, transmitted, or absorbed. Laws of plane reflection.

3. Speed of light in vacuum is very nearly 300 million meters per second.

4. Speed of light in all kinds of matter is less than in a vacuum.

5. The refraction of light at any surface depends upon the relative speeds of light in the media on each side of this surface.

6. The total reflection of light.

7. Evidence that light consists in a wave motion.

(*a*) Light is diffracted, as are water waves, in passing through a small opening.

(*b*) Light is reflected, as are water waves and sound waves.

(*c*) Speed of light depends upon medium, as speed of water waves depends upon depth of water.

(*d*) The rays of light may interfere and produce darkness.

8. Converging lenses and concave spherical mirrors delay *center* of incident wave front, and hence make wave fronts more concave on the advancing side of the wave front.

9. Diverging lenses and convex spherical mirrors delay *edges* of incident wave front, and hence cause wave front to emerge more convex than it was on entering.

10. Achromatic lenses are made up of two elements such that the dispersion of one just annuls that of the other.

PROBLEMS

1. A man stands 3 ft. in front of a plane mirror. What is the distance between the man and his image?

2. Compute the time which would be required for a ray of light to travel once around the earth at the equator. What distance would sound travel during this interval of time?

3. Explain the argument of Dr. Thomas Young for thinking light a wave motion. Make a diagram, showing, *first*, how light spreads out on passing through a small hole, and, *second*, how Young added two pencils of light together.

4. The focal length of one lens is 25 cm., of another, 120 cm. Compare the *power* of the two lenses.

5. Distinguish clearly between *focus* and *focal length*.

6. The focal length of a converging lens is 20 cm. Make a diagram, to scale, showing how to construct the image of a point which is 2 cm. above the optical axis and 30 cm. distant from the lens.

7. An object 2 mm. in diameter, placed 2 cm. in front of a converging lens, gives an image which is 8 mm. in diameter. How far is the image from the lens?

8. Explain by diagram the optical behavior of the human eye.

9. What is the purpose of the bellows in an ordinary photographic camera?

10. Explain why a cube of glass never shows any prismatic separation of the rays.

11. Distinguish between the *intensity of a source* of light and the *illumination* due to a source of light.

12. What experiment can you suggest to show that the convex lens of ordinary glass is not achromatic?

13. From the diagram here given (Fig. 357) explain the action of the ordinary projection lantern.

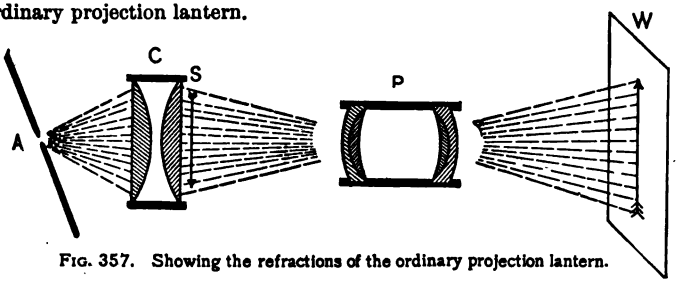


FIG. 357. Showing the refractions of the ordinary projection lantern.

14. What is meant by the statement that the focal length of a given convex lens is 2 ft.? By graphical methods, construct the image of a luminous point placed 6 ft. in front of such a lens and 1 ft. above the optical axis.

15. Show by graphical method that when an object is placed between the focus and the reflecting surface of a concave mirror the image is erect.

References on Light

S. P. THOMPSON — *Light, Visible and Invisible* (Macmillan Co.). Price, $1.50. A course of six lectures addressed to beginners. The language is simple and clear; the important facts are emphasized. The book is one of great value to high school students and to teachers.

LOMMEL — *The Nature of Light* (D. Appleton & Co.). Price, $2.00. Written by an eminent German scholar. Elementary, clear, reliable.

TAIT — *Encyclopædia Britannica*, Art. "Light."

RAYLEIGH — *Encyclopædia Britannica*, Art. "Optics."

CARPENTER — *Encyclopædia Britannica*, Art. "Microscope."

GILL — *Encyclopædia Britannica*, Art. "Telescope."

YOUNG — *The Sun* (Int. Sci. Series). Price, $2.00.

AMES — *Textbook of Physics*, Chapter on "Light" (American Book Co.).

BIDWELL — *Curiosities of Light and Sight* (Swan Sonnenschein & Co., 1899). Five excellent popular lectures upon the eye, color, vision, etc.

HASTINGS — *Light, a Consideration of the more Familiar Phenomena of Optics* (Scribners, 1901).

MICHELSON — *Light Waves and their Uses* (Univ. Chicago Press, 1902).

DERR — *Photography for Students of Physics and Chemistry* (Macmillan, 1906).

EDSER — *Light for Students* (Macmillan).

WRIGHT — *On Light.* Highly recommended for assistance in preparing demonstrations with the lantern.

APPENDIX

QUESTIONS FOR REVIEW

1. In 10 sec. the velocity of a train decreases uniformly from 80 $\frac{\text{ft.}}{\text{sec.}}$ to 60 $\frac{\text{ft.}}{\text{sec.}}$. What is its rate of acceleration? What is the distance traversed during the 10 sec.? (Central High, Pittsburg, Pa.)

2. Define and illustrate motion and acceleration.

A body starting from rest passes over 256 ft. in 4 sec. What is its acceleration? (Wheeling, W. Va.)

3. In the inclined plane experiment, a group of students found that the ball rolled 30 cm. the first second, 120 cm. in the first two seconds, 270 cm. in the first three seconds, and 480 cm. in the first four seconds. *Without using a formula* find the velocity at the end of the third second and the acceleration in centimeters-per-second per second. (Joliet, Ill.)

4. If the acceleration of a ball rolling down an inclined plane is 80 $\frac{\text{cm.}}{\text{sec.}}$ per second, what velocity will it have at the bottom, the plane being 19.6 m. long? (Central High, Scranton, Pa.)

5. Define mass, weight, inertia, density, molecule.

(Barringer High, Newark, N.J.)

6. How high is a balloon from which a body would fall to the earth in 7 sec.? With what velocity would the body strike the earth? What distance would it traverse during the seventh second?

(Birmingham, Ala.)

7. A ball, starting from rest and rolling down a smooth, inclined plane for 3 sec., moves 5 m. during the last second; state (*a*) its acceleration, (*b*) how a change of inclination of the plane would alter the acceleration, (*c*) how a change in the size of the ball would alter the acceleration, (*d*) how a change in the density of the ball would alter the acceleration. (Regents of the State of New York)

8. A stone is dropped from a balloon, taking 40 sec. to hit the ground.
(1) With what velocity in feet per second did it strike the ground?
(2) Would a rifle ball, leaving the gun at a velocity of 1000 ft. per second, hit the balloonist, and why? (Neglect the resistance of the air.)
(East Side, Bay City, Mich.)

9. A boat slides down a shoot-the-chutes, and 10 sec. after starting it strikes the water with a velocity of 120 ft. per second. Find (*a*) the acceleration, (*b*) the length of the incline, (*c*) the distance the boat goes during the last second. (Morris High, New York)

10. A body is thrown downward, with a velocity, at the instant of leaving the hand, of 12 m. per second. After falling freely for 4 sec., what will be its velocity? Take 10 $\frac{\text{m.}}{\text{sec.}^2}$ as the acceleration due to gravity. (Central, Pittsburg)

11. A stone dropped from a bridge is seen to strike the water 3 sec. later. Find (*a*) the height of the bridge, (*b*) the velocity of the stone when it strikes the water. (Barringer, Newark)

12. Define force, work and energy.
What is the force acting on a body whose mass is 1000 gm., when the acceleration is uniform and equal to 5 cm.-per-second per second? Is the force constant, or impulsive? (Bradford, Pa.)

13. A stone falls from an air ship that is 2048 ft. above the earth. With what velocity does the stone strike the ground?
(Girls' High, Brooklyn)

14. A 96-lb. ball is supported by two cords attached to it, one 3 ft. long, the other 4 ft. The other ends of the respective cords are fastened to points on the ceiling 5 ft. apart. By the parallelogram of forces find the tension on each cord. (Montclair, N.J.)

15. A motor boat can travel 12 mi. per hour in still water. If the current in the stream is 3 mi. per hour, how long will it take the boat to travel (*a*) 2 mi. upstream, (*b*) 2 mi. downstream? If kept headed at right angles to the current, how far will the boat travel in 10 min.
(Regents)

16. Give illustrations of "inertia" in the two senses or aspects and give a definition for it. (North High, Columbus)

17. Two forces of 6 and 8 dynes respectively act at right angles to each other on a mass of 2 gm. (*a*) What is the resultant force? (*b*) What is the resultant acceleration? (*c*) What is the momentum at the end of 3 sec.? (Chattle High, Long Branch, N. J.)

18. What is the magnitude of a force which in 20 sec. will give a mass of 80 gm. a velocity of 200 cm. per second? (Normal, Charleston, Ill.)

19. State Newton's second law of motion.

In throwing a cricket ball to reach a distant point, it is usual for the ball to be thrown with an upward inclination and not horizontally. Give a reason for this and explain why the inclination to the horizontal will be less if the ball be thrown with greater speed. (University of Cambridge)

20. Two forces, 20 lb. and 70 lb., act in the same direction and 15 ft. apart. Find the resultant and point of application of resultant.

(Yeatman High, St. Louis)

21. Explain carefully with the aid of diagrams how a boat can sail in directions other than that in which the wind is actually blowing.

(University of Oxford)

22. The brakes of a certain car can stop it in $1\frac{1}{2}$ sec. when it is going at the rate of a mile a minute. What acceleration must they give the car and in what distance can it be stopped? (Boys' High, Brooklyn)

23. What force, acting upon a carriage weighing 1500 lb. for $6\frac{1}{4}$ sec., would give it a velocity of 10 mi. per hour? (Ft. Wayne, Ind.)

24. Why does pounding a carpet free it from dust? Account for the dust driven out at the back as well as at the front of the carpet.

(Girls', Brooklyn)

25. A half-ounce bullet is fired with a velocity of 1400 ft. per second from a gun weighing 7 lb. Find the velocity in feet per second with which the gun begins to recoil, and the mean force in pounds weight that must be exerted to bring it to rest in 4 inches. (University of Oxford)

26. (*a*) What velocity will a 6-dyne force impart to a 12-gm. mass if it acts for 18 sec.?

(*b*) How many ergs of work will have been done by this force after it has moved the body 3 cm.? (East High, Cleveland)

27. A uniform bar 6 ft. long has weights of 22 lb. and 44 lb. respectively, suspended from the extremities. Where must the fulcrum be placed to produce equilibrium? (Neglect the weight of the bar.)

(Spokane, Wash.)

28. A circus performer stands on the middle of a "slack" rope. The parts of the rope make right angles with each other. Calculate to one decimal the tension on each part of the rope, the weight of the performer being 120 lb. (Morris, New York)

29. Two horses of unequal strength must be hitched as a team. The one can pull 360 lb., while the other pulls 90 lb. In the double-tree, 50 in. long, where must the pin be placed to permit an even pull?
(North, Columbus)

30. (*a*) What is the force with which gravity acts upon a 10-gm. mass?
(*b*) What velocity will it give this mass in 10 sec.?
(*c*) When it has acquired a velocity of 20 cm. per second, what is its momentum? (Glenville High, Cleveland)

31. A weight of 100 gm. is suspended from one end of a bar; the combination balances on a fulcrum 6 cm. from the point of suspension of the weight and 30 cm. from the center of gravity of the bar. Find the weight of the bar. (Regents)

32. A 5-gm. mass is moving at the rate of 12 cm. per second. Find its kinetic energy. Define unit in which answer is expressed. (Montclair)

33. A train of 300 tons weight starts from rest and at the end of 5 min. its velocity is 30 mi. per hour. Find its acceleration. What is its kinetic energy at the end of the fifth minute? (Rayen, Youngstown, O.)

34. Devise a scheme for weighing a turkey, using only a stick of uniform density and cross section and 50 cm. long, a cm. scale, some strong cord, and a flatiron known to weigh 6 lb. Be sure your plan provides for eliminating the weight of the stick. Illustrate by working out a numerical problem. (Malden, Mass.)

35. A uniform bar of wood 120 cm. long, weighing 10 gm. per centimeter of length, has a mass of 1000 gm. suspended 10 cm. from one end. From what point in the bar must it be suspended in order to be in equilibrium? (Hyde Park, Chicago)

36. A rifle bullet leaves the muzzle of the rifle in a horizontal direction with a velocity of 800 $\frac{\text{m.}}{\text{sec.}}$. Find the horizontal distance and the vertical distance it will travel in 2 sec. (Central, Pittsburg)

37. Two strings, 6 and 10 ft. long, attached to a horizontal beam and meeting at a point so as to form a right angle, support a 50-lb. weight. What is the pull on the 10-ft. string? (Girls', Brooklyn)

38. A and B carry a fish weighing 54 lb., slung between them from the middle of a 10-ft. oar. One end of the oar rests on A's shoulder, but the other end is pushed 1 ft. beyond B's shoulder. What part of the weight does each carry? (East, Cleveland)

39. Find graphically the resultant of 40 lb. N. E. and 70 lb. W.
(Yeatman, St. Louis)

40. How much can a man weighing 185 lb. lift with a lever of the first class, 10 ft. long, the fulcrum being 2 ft. from the weight? (Lima, O.)

41. (*a*) Give an example of a force acting without producing work.

(*b*) Give an example of a force acting so as to do work.

(*c*) Explain why a falling weight, on striking an object, will produce pressure in excess of its own weight. (Regents)

42. Using a meter stick (weight = 140 gm.) a boy finds the weight of a trout to be 1260 gm. by suspending the fish from one end of the stick and balancing it on the edge of a knife-blade. Find at what point the blade must be placed. (Montclair)

43. Distinguish between kinetic and potential energy.

What is the horse power of an engine which will raise 2376 lb. 1000 ft. in 2 min.? (Lakewood, O.)

44. Show by diagram the three classes of levers. Label parts.

State the law of the lever. Should the weight of the lever be considered? How provided for if it must be considered?

Define the terms velocity, acceleration, gram of force, inertia, equilibrium, erg, and horse power. (San Antonio, Tex.)

45. How many gallons of water (8 lb. each) could a 10-horse-power engine raise in one hour to a height of 60 ft. (Central High, St. Louis)

46. A pile driver moves in a run 16 ft. high.

(*a*) What time will be required for its fall?

(*b*) With what velocity will it strike?

(*c*) If its weight is 200 lb., what momentum will it have?

(*d*) If lifted with a single movable pulley, what force is needed?

(*e*) What power must the motor have to raise it 30 times a minute?

(John Marshall High, Chicago)

47. A workman applies 75 kgm. force at one end of a crowbar 200 cm. long. What weight may be lifted at the other end, distant 25 cm. from the fulcrum? What is the force that must be exerted by the fulcrum?

(Central High, Cleveland)

48. A baseball weighs $5\frac{1}{2}$ oz. It is thrown with a velocity of 120 ft. per second. What is its momentum? Its kinetic energy?

(North, Columbus)

49. A pendulum bob weighs 20 kgm. Find the magnitude of the unbalanced force acting upon it while it is displaced 30° to the right.

(Central, Pittsburg)

50. If a meter pendulum beats seconds with 3 cm. amplitude, what is the period when the amplitude is doubled? If the amplitude remains 3 cm., what is the period if the length is doubled? (Central, St. Louis)

51. How many vibrations will a pendulum 289 cm. long make in one minute if the seconds pendulum is 100 cm. long? (Lincoln, Neb.)

52. If gravity is equal to 32.173 at Duluth, what is the length of a pendulum that will beat seconds? (Duluth, Minn.)

53. A pendulum is 156.4 in. long. (*a*) What is its period? (*b*) What is its number of vibrations per minute? (*c*) How would you regulate a pendulum clock that gains time? (Erasmus Hall, Brooklyn)

54. (*a*) For what purpose is a pendulum used in the mechanism of a clock? (*b*) Why does a pendulum tend to stop vibrating? (*c*) What causes it to continue to vibrate? (State of Pa.)

55. The force of gravity at Newark, Ohio, is 980.1 cm.-per-second per second. What is the length of a pendulum that beats seconds here? (Newark, O.)

56. How can two double pulleys be arranged to give an advantage of 5? (South Chicago High, Chicago)

57. How and for what purpose are rifle balls given a rotary motion? What is the cause of the parallelism of the earth's axis? Explain the cause of the continued axial rotation of the earth. (International Y. M. C. A.)

58. If a barrel weighs 260 lb. and a wagon is 5 ft. high, find:
(*a*) the force required to lift the barrel directly into the wagon.
(*b*) the force to roll it up a plank 15 ft. long.
(*c*) the work done in each case.
(*d*) the mechanical advantage in using the plank. (East, Cleveland)

59. State clearly the difference between mass and weight.
A body weighs 100 lb. on the surface of the earth. What will it weigh 2000 mi. above the surface? What will it weigh 1000 mi. under the surface? (Wheeling)

60. A grocer has a platform balance, the ratio of whose arms is 9 to 10. If he sells 20 lb. of merchandise to one man, weighing it on the right-hand pan, and 20 lb. to another man, weighing it on the left-hand pan, what per cent will he lose or gain by the two transactions? State the law by means of which you solved the problem. (Y. M. C. A.)

61. A bullet, the mass of which is 30 gm., and which is moving with a speed of 3×10^6 cm. per second, strikes a piece of wood and penetrates 10 cm. What is the wood's resistance to penetration? Give answer in joules. (Lake View High, Chicago)

62. What is meant by the mechanical advantage of a machine? Draw a simple diagram of a machine, other than the lever, whereby a man exerting 50 kgm. may raise a load of 1500 kgm. (West High, Cleveland)

63. The handles of a wheelbarrow, measured from the axis of the wheel, are 5 ft. long. A load of 380 lb. is placed 16 in. from the axis of the wheel. How much must a man lift to wheel the load?

To what class of levers does the wheelbarrow belong?

(Barringer, Newark)

64. If the capstan of a ship is 12 in. in diameter and the levers are 5 ft. long, what force must be exerted by each of 4 men in order to raise an anchor weighing 2000 lb.? (Central, Scranton)

65. At what point in its length must a 40-inch evener (double-tree) be attached to the wagon, so that one force may draw 1200 and the other 800 lb. of every ton? (Boys', Brooklyn)

66. A weight of $\frac{1}{2}$ ton is raised by a jackscrew. What force must be applied if the lever is 1 ft. long and the screw threads are six to the inch?

(Spokane)

67. In an hydraulic press the small piston has an area of 5 sq. in. and a pressure of 70 lb. is applied to it. If a pressure of 7000 lb. is desired at the large piston, what must be its area?

State the law which governs the action of the press. Who discovered this law? (Erasmus Hall, Brooklyn)

68. If an 800-lb. weight can be lifted through 10 ft. of space, with a system of two fixed and two movable pulleys, by exerting an effort of 250 lb., what is the efficiency of the combination?

(South Division, Milwaukee)

69. In the course of a stream is a waterfall 22 ft. high. It is shown by measurement that 450 cu. ft. of water per second pour over it.

How many foot-pounds of energy could be obtained from it? What horse power? What becomes of this energy if not used in driving machinery? (Morris, New York)

70. If the roof of this schoolhouse is 80 ft. from the ground, and an engine lifts, in 5 hr., steel girders which weigh 20 tons to the top of this building, what is the horse power of the engine? (Newark, O.)

71. With a system of 3 movable pulleys, what force must be exerted to balance a weight of 2 tons? (Central, Scranton)

72. A gun weighing 100 lb. discharges a projectile weighing 0.1 lb.

(*a*) Compare (1) as to both magnitude and direction, the force acting on the gun with that acting on the projectile, (2) the duration of force action on the projectile with that on the gun.

(*b*) Explain why the velocity and the kinetic energy of the projectile are different from those of the gun, at the instant the projectile leaves the gun. (Regents)

73. A 200-lb. barrel is rolled up a 12-ft. plank into a wagon 4 ft. high. Compute the force required, and show that the same amount of work is done as would be required to lift it in directly. (Boys', Brooklyn)

74. It is desired to raise ore from a mine 550 ft. deep at the rate of 3 tons per minute. What horse power must the hoisting engine be able to develop? (Oil City, Pa.)

75. Explain in detail what becomes of the potential energy of the powder used to fire a ball from a rifle, and state what principle of physics is illustrated by your answer. (Regents)

76. Under what conditions does force do work?

What is the difference between work and energy?

A body weighing 500 grams has a velocity of 1000 cm. per sec. How much energy does it possess? Name the unit in which it is measured. (South High, Cleveland)

77. What is the horse power of an engine which is able to lift 180 T. of ore per hour from the bottom of a mine 1100 ft. deep? (Barringer, Newark, N.J.)

78. I have 3 fixed pulleys and 2 movable ones by which I intend to raise a weight of 600 lb. How shall I arrange these pulleys so that I shall use the least effort? Find the effort required. (Newark, O.)

79. A train weighing 2000 tons going up a grade was lifted 20 ft. a minute. What was the horse power of the engine?

State and explain the law of gravitation. (Englewood, Chicago)

80. Why is the metric system of weights and measures more convenient for scientific purposes than the English system?

A gram weight in the left-hand pan of a balance with unequal arms supports 1.01 gm. on the other side. Which is the longer? If it is 12 cm. long, what is the length of the other arm? (University of Oxford)

81. (*a*) How would the energy of a given amount of water vary with the height of the receptacle containing it? Explain your answer fully.

(*b*) What portion of the force used in hauling a sled is useful and what portion does no work? Draw a diagram and assume such numerical values as are necessary to a solution of the problem. (State Board, N. Dak.)

82. At 2000 mi. above the earth's surface a body would weigh 100 lb. What is its weight at the earth's surface? (Normal, Charleston)

83. A lever is 10 ft. long and weighs 10 lb. per running foot. What resistance can be overcome by a force of 100 lb. applied at one end, 7 ft. from the fulcrum? (Ft. Wayne, Ind.)

84. The radius of the handwheel of a copying press is 7 in., each hand exerts upon the wheel a force of 25 lb., the screw has 5 threads to the inch. Find the pressure exerted by the screw. (Central, Pittsburg)

85. The radius of a wheel is 10 ft., the radius of the axle attached is 1 ft. The load to be lifted is 1000 lb. It is found that 125 lb. are necessary to move it up, while 75 lb. will allow it to go down without acceleration. Find the per cent efficiency of the machine. (Joliet)

86. A barrel is being rolled up a plank, which is 12 ft. long, into a doorway which is 4 ft. high. If the barrel weighs 300 lb., what force must be applied to prevent the barrel rolling back? (Central, Scranton)

87. The pitch of a jackscrew is $\frac{1}{2}$ in. It is turned with a lever 2 ft. long. What is the mechanical advantage?

If an effort of 50 lb. is used, what weight can be lifted if the efficiency of the machine is 1? What weight will be lifted if its efficiency is 80%? (Normal, Charleston)

88. Why do revolving fly wheels sometimes break, and how much will this tendency be increased if the number of revolutions per minute is doubled? (South Division, Milwaukee)

89. A toy engine weighs 1 kgm. and runs on a circular track whose diameter is 10 ft. If this engine makes 5 complete turns around the track in one minute, what is the centrifugal force in dynes? What is the centripetal force? (Newark, O.)

90. Define specific gravity.

The weight of a stone in air is 146 gm. Its weight in water is 94 gm. Find its specific gravity. (Ft. Wayne)

91. The density of a stone is 2.5. If a boy can lift 120 lb., how heavy a stone can he lift to the surface of a pond? (Shaw, East Cleveland)

92. (*a*) Why does the human body float and why is it so easy for it to sink?

(*b*) A man's weight is 145 lb. and his volume is $2\frac{1}{2}$ cu. ft. What is his specific gravity? (Y. M. C. A.)

93. A cube of stone, 3 cm. on each edge, weighs 81 gm. in water. What will it weigh in air? In alcohol whose density is .8? (Man. Tr., Denver)

94. Compute the pressure per square inch at a depth of 100 ft. in sea water of density 1.02. (Paterson)

95. A bottle weighs 29 gm. empty, 68 gm. filled with water, and 89 gm. filled with nitric acid.

(*a*) What is the capacity of the bottle? The specific gravity of the acid?

(*b*) What would the bottle weigh filled with saturated salt solution whose specific gravity is 1.20? (East, Cleveland)

96. Into a U-tube of uniform bore mercury (sp. gr. 13.6) is poured until the surfaces in the two arms reach a certain level. Alcohol (sp. gr. 0.8) is poured into one side, and glycerine (sp. gr. 1.26) is poured into the other side until the two surfaces are at the same level, which is 30 cm. above the former level of the mercury. Find the lengths of the columns of alcohol and glycerine. (Ed. Dept., Ontario)

97. What is the specific gravity of cork if it will float with one-third of its mass out of water? (Joliet)

98. State Archimedes' Principle.

A solid which weighs 100 gm. in air weighs 90 gm. in water and 92 gm. in oil. What is the density of the oil? (Chattle, Long Branch)

99. (*a*) State the principle concerning the transmission of pressure through a liquid.

(*b*) How much pressure would the water in a cistern exert against one of its retaining walls if the surface against which the pressure is exerted is 8 ft. long and 5 ft. high? (Grand Rapids, Mich.)

100. (*a*) Explain the meaning of the expression, "The specific gravity of copper is 8.8."

(*b*) What will 10 c.c. of copper weigh in air? What will the same volume of copper weigh in water? (Regents)

101. A rectangular vessel whose base is 30 cm. square and whose height is 60 cm. is $\frac{2}{3}$ filled with alcohol, specific density .8. Find the total pressure upon the bottom. (Central, Pittsburg)

102. How would you find the density of a solid?

Give reasons for each step. (South, Cleveland)

103. The diameters of the cylinders of an hydraulic press are 6 in. and 1 in. respectively. What is the pressure on the larger piston when a force of 100 lb. is applied to the smaller piston? (North, Minneapolis)

104. (*a*) A body loses 25 gm. when immersed in water, 23 gm. in oil, and 20 gm. in alcohol. Find the density of oil and alcohol.

(*b*) What is the volume of a copper ball whose mass is 130 gm. and whose density is 8.87 $\frac{\text{gm.}}{\text{c.c.}}$? (Central, Columbus)

105. Find the lateral pressure upon a vertical water gate which is 4 ft. square and is set in the bottom of a dam in 20 ft. of water.

(Norfolk, Va.)

106. A closed cubical box, each inside edge of which is 2 ft., is filled with water. A vertical pipe 20 ft. high, also filled with water, is inserted in the top of the box. Calculate the pressure upon one side of the box. (1 cu. ft. of water weighs 62.5 lb.). (East, Cleveland)

107. A wrought-iron rivet weighs 96 gm. in air and 84 gm. in water. Find the weight of 1 cu. ft. of iron.

What would be the apparent weight of a wrought-iron anchor having a volume of 3 cu. ft. when suspended in fresh water, which weighs 62.5 lb. per cubic foot? (Morris, New York)

108. If a pair of Magdeburg hemispheres are 6 in. in diameter, what force will it take to separate them when the atmospheric pressure is 14.8 lb. per square inch? (Yeatman, St. Louis)

109. A cone-shaped vessel filled with water has a base of 200 sq. cm. and a height of 100 cm. Find the force acting on the bottom.

(Central, Scranton)

110. A vessel 12 ft. long, 8 ft. wide, and 7 ft. deep is full of water. What is the pressure on the bottom? What is the pressure on the end?

(Birmingham)

111. Describe a barometer.

What is there between the mercury and the top of the tube? (Malden)

112. How large a balloon is needed to raise a weight of 300 lb.? Explain. (Englewood, Chicago)

113. A tank is 10 cm. by 20 cm. by 25 cm. It is filled with sulphuric acid whose density is 1.8.

(*a*) How many liters does the tank contain?
(*b*) How many grams does it weigh?
(*c*) How many kilograms does it weigh?
(*d*) About how many quarts does it contain?
(*e*) What does the acid weigh in pounds? (Central, Scranton)

114. A rectangular block of cut stone having a specific gravity of 2.6 is 100 cm. long and 75 cm. wide and weighs 975 kgm.; what is the height of the block? (Regents)

115. What is the pressure on a dam which holds back the water in a lake 1½ mi. long, if the lake is 100 ft. wide and the water is 20 ft. deep at the dam? (Barringer, Newark)

116. Distinguish between the weight of the air and the pressure of the atmosphere. (West, Cleveland)

117. A rectangular block, weighing 900 gm., floats in water; the base of the block is 20 cm. long and 15 cm. wide. How far is the base below the surface of the water? If the density of the block is 0.5, what is the volume? (Regents)

118. (*a*) If the volume of a body were 10 cu. ft., how much would it lose in weight if it were wholly submerged in water?

(*b*) If a floating body weighs 100 tons, how much water is it displacing?

(*c*) State the principles connected with the above. (Grand Rapids)

119. State why hydraulic pressure, instead of air at the same pressure, is used in determining the safety of boilers or of gas cylinders. (Regents)

120. A boat 30 × 15 ft. sank 4 in. when loaded with coal. Find the weight of the coal in tons. (Central, Scranton)

121. Find the specific gravity and the volume of a block of wood from these data:—

Weight of wood in air, 450 gm.

Weight of sinker in water, 300 gm.

Weight of wood and sinker in water, 250 gm. (Morris, New York)

122. What is Boyle's Law, and how far is it to be relied on?

A mass of air 3 c.c. in volume is introduced into the space above a barometer's column which originally stands at 760 mm. The column sinks until it is only 570 mm. high. Find the volume now occupied by the air. (University of Oxford)

123. The diameters of the piston and cylinder of a hydrostatic press are, respectively, 3 in. and 30 in. The piston rod is attached 2 ft. from the fulcrum of a lever 12 ft. long. What force must be applied at the end of the lever to make the press exert a force of 5000 lb.? (East, Cleveland)

124. Make a diagram of a common lift pump, showing the position of the valves on the first down stroke.

How high may water be lifted by such a pump? Why? (Malden)

125. How high will a lift pump raise water if it is located upon the side of a mountain where the barometer reading is 71 cm.? (Norfolk)

126. If the cylinder of an air pump is $\frac{1}{3}$ the size of the receiver, what fractional part of the original air will be left after 5 strokes? (Shaw, East Cleveland)

127. A diver is working at a depth of 80 ft. in water. What pressure *per square foot* does the water exert on his body? (Morris, New York)

128. A piece of gold weighs 9.7 gm. A flask full of water weighs 95 gm. The gold is dropped into the flask, displacing some of the water. The flask and its contents now weigh 104.2 gm. Find the specific gravity of gold. (West, Cleveland)

129. If the reading of the closed end of a manometer is 32 in., the reading of the mercury in the closed tube 27 in., and in the opposite tube 17 in., what is the pressure in atmospheres? (Ft. Wayne)

130. A tank is 3 ft. in diameter at the top and 4 ft. in diameter at the bottom. If the tank is 8 ft. deep and is full of water, what will be the downward pressure? (Norfolk)

131. An air bubble has a volume of 1 cu. in. at a depth of 80 ft. What will be the size of the bubble on reaching the surface? The barometer reads 29.4 in. The density of mercury is 13.6. (Montclair)

132. A piece of glass weighs 250 gm. in air. Show why it is partly lifted when submerged in a fluid. If the glass weighs 150 gm. in water and 70 gm. in acid, find the weight of 1 c.c. of the acid.
(Morris, New York)

133. A cubical block of iron, sp. gr. 7.8, floats on mercury, sp. gr. 13.6. What fractional part of the iron is immersed? What force would be required to completely immerse a 10-cm. cube of iron in mercury?
(Girls', Brooklyn)

134. A little water lies on the top of the mercury column in a barometer. What error would be made if this barometer were used to measure the pressure of the atmosphere?

Would the error be the same on a cold day as on a warm day?
(University of Cambridge)

135. A block of wood 20 by 20 by 20 cm. floats in water with its top 6 cm. above the surface. What is the mass of the block? What is the specific gravity of the wood? (Central, Pittsburg)

136. What is the amount of pressure exerted against a vertical mill-dam whose length is 250 ft., the water being 8 ft. deep?
(Rayen, Youngstown)

137. If the diameter of an iron cylinder is 20 ft. and if the cylinder is closed when the barometer stands at 30 in., what will be the approximate unbalanced pressure on one end of the cylinder if the barometer falls to 28 in.? [One atmosphere = 15 lb. per square inch approximately.]
(Regents)

138. The Niagara turbine pits are 136 ft. deep and their average horsepower is 5000. Their efficiency is 85%. How much water does each turbine discharge per minute? (Shaw, East Cleveland)

139. What is the atmospheric pressure per square centimeter when the barometer reads 74 cm.? What would be the reading of a water barometer at the same time? (North Division, Milwaukee)

140. (*a*) A submarine boat displaces 1600 cu. ft. of water and, with ballast tanks half full, weighs 48 tons. Will it sink or float? Why?

(*b*) The total downward pressure on the upper surface of the boat, when submerged to a depth of 8 ft., is 80,000 lb. What will this pressure become at a depth of 32 ft.? Why? (East, Cleveland)

141. (*a*) Why is a wagon going along an uneven road more likely to upset if loaded with hay than if loaded with an equal weight of iron?

(*b*) Why is it better to make the depths of floor timbers greater than their widths?

(*c*) When a ship passes from fresh water into salt water will it sink deeper or rise higher? Why? (Portsmouth, N. H.)

142. A boy who could lift 100 lb. undertook to carry a gallon (231 cu. in.) of mercury. Did he succeed? (South Division, Milwaukee)

143. Where will a pendulum beat fastest, on a mountain or at sea level? Why? Where will a barometer read higher, on a mountain or at sea level or in a mine below sea level? Why? (Joliet)

144. Make a diagram showing a tumbler of water and a tumbler of mercury (in vertical cross section) with one fine and one coarse capillary tube standing in each. Represent the relative levels and the general shapes of the liquid surfaces in the tumblers and tubes. State *one* law of capillary action illustrated by your diagram. (Regents)

145. How do you account for the spherical form of a tear-drop, a raindrop, shot? Is the earth spherical for the same reason? What experiments have you performed or seen performed illustrating the subject? (Y. M. C. A.)

146. What phenomena indicate molecular forces in solids, liquids, gases? (Englewood, Chicago)

147. Why did not the man meet with success who attempted to run a water wheel placed in a pit near a lake by water drawn from the lake by means of a pipe, and then to return the water from the pit by means of a siphon, the end of the siphon in the lake being carried down lower than the end in the pit? (Y. M. C. A.)

148. A lump of copper sulphate, dropped into a tumbler of water, dissolves, forming a blue layer at the bottom, while the upper layers of water remain uncolored; after a few days the blue liquid is found to have extended upward. Explain. (Regents)

149. Mention one contribution to science made by each: Galileo, Pascal, Archimedes, Torricelli, Boyle. (Lima)

150. Discuss in ten lines the essential difference between solids, liquids, and gases. (West, Cleveland)

151. Upon what does the velocity of sound depend?

An echo is heard by a speaker 3 sec. after the sound is produced. How far away is the reflecting surface if the temperature is 24° C.? (Joliet)

152. Steam was seen to escape from the whistle of a ferryboat, and the sound was heard 4 sec. later. The temperature was 20° C. How far was the boat from the observer? (Barringer, Newark)

153. Define fundamental, overtones, and octave.

At a temperature of 20° C., how long would it take sound to travel 1892 m.? (Lakewood)

154. Give the laws of vibrating strings.

What is pitch and upon what does it depend? (Bradford)

155. The sound from a tuning fork making 250 vibrations per second passes from a room at a temperature of 25° C. to a place where the temperature is only 5° C. What is the difference in wave length between the two places? (Hyde Park, Chicago)

156. Describe water waves and sound waves as follows:

(1) Make a rough sketch of each, naming parts.

(2) Describe the action of the particles of the medium.

(3) Indicate on the drawings one wave-length and define what it is.

(4) Describe briefly how to get the velocity of sound waves by the air resonator. (East Side, Bay City)

157. A sound is heard by two listeners, one of whom is distant $\frac{1}{4}$ mi. and the other $\frac{1}{2}$ mi. from the sound. How much louder does it sound to one than to the other? Why? (Y. M. C. A.)

158. A stone falls in a mine 1033.6 ft. deep. How soon will the sound which it makes on striking the bottom reach the mouth of the mine? (Normal, Charleston)

159. A glass jar containing water responds most loudly to a tuning fork when the length of the column of air is 17.5 cm. If the temperature is 24° C., what is the frequency of the fork? (Joliet)

160. Represent graphically two coincident trains of sound waves, the length of the waves of one being three times that of the other. (Norfolk)

161. A bullet, fired from a rifle with a speed of 1200 ft. per second, is heard to strike the target 6 sec. afterward. What is the distance to the target if the speed of sound is 1126 ft. per second? (Polytechnic Inst., Baltimore)

162. Determine the vibration frequencies of each of the first three overtones of C (= 240 vibrations), and designate the pitch of each. (Joliet)

163. (*a*) Describe and illustrate resonance.

(*b*) Find the number of vibrations per second of a fork which produces resonance in a pipe 1 ft. long. (Take speed of sound as 1120 ft. per second.) (Shaw, East Cleveland)

164. If the tension of a string at one time was 80 kgm., and at another 20 kgm., what would be the relative frequencies? (Norfolk)

165. Build up a diatonic scale on C = 264. (North, Minneapolis)

166. If a string 120 cm. long gives C, what lengths of string will give each note of the major chord? (Joliet)

167. Explain the effect of filing away a portion of the prongs of a tuning fork.

In what respects may sound sensations differ? (Central, Pittsburg)

168. Explain why there is a rise in the pitch of a locomotive whistle when the train is rapidly approaching an observer, and a corresponding fall when it is moving away. (Regents)

169. What are the source, speed, medium, and wave character of sound? What phenomena indicate this? How? (Englewood, Chicago)

170. (*a*) Why will the sense of touch not always tell us whether two bodies are of equal temperature?

(*b*) In everyday life, how are bodies protected against heat from the outside and loss of heat from the inside? Explain. (Y. M. C. A.)

171. Change 40° C. to Fahrenheit. Change − 60° F. to Centigrade.

Explain the processes of conduction, convection, radiation.

(Englewood, Chicago)

172. (*a*) Convert 20° C. to degrees Fahrenheit.

(*b*) Convert 77° F. to degrees Centigrade. (Allegheny)

173. Describe a maximum and a minimum thermometer.

How would you determine the correctness of the marking on a thermometer?

Will sea water and river water boil at the same temperature? Give reasons for your answer. (University of Oxford)

174. State the difference in calories between the amount of heat in 100 gm. of water at 25° C. and the same amount of water at 35° C.

State the specific heat of water. (Spokane)

175. State and explain briefly the method of heat transference that is most effective in *each* of the following processes: (*a*) heating a flatiron, (*b*) heating the earth by the sun, (*c*) heating a room by a hot-air furnace. (Regents)

176. The original length of a metallic rod is 60 cm. By heating it from 20° C. to 97° C., its length is increased 0.98 cm. Find the coefficient of expansion of the metal. (Guthrie, Okla.)

177. If the length of an iron rod is 4864 mm. at 15° C., what will it be at 65° C. if the coefficient of expansion of iron is 0.000012? (Hyde Park, Chicago)

178. 5 gm. of mercury at 20° C. are mixed with 5 gm. of water at 0° C. and the resulting temperature is 0.634° C. What is the specific heat of mercury? (Joliet)

179. 200 g. of iron (sp. heat .1138) at 300° C. are plunged into 1 kgm. of water at 0° C. What will be the resulting temperature? (Central, Scranton)

180. Define (*a*) thermal unit; (*b*) thermal capacity; (*c*) specific heat. 80 gm. of silver at 100° C. is placed in 220 gm. of water at 24.7° C. The final temperature is 26.2° C. What is the specific heat of the silver? (Normal, Charleston)

181. How long will it take a gas flame supplying 5000 lesser calories per minute to heat a 1 kgm. flatiron from 20° C. to 250° C.? The specific heat of iron is 0.11. (Central, Pittsburg)

182. A piece of ice weighing 65 gm. is dropped into 250 gm. of water at 40° C. and after all the ice is melted the temperature of the mixture is 15° C. From these data compute the latent heat of fusion of ice. (Boys', Brooklyn)

183. The steel rails of a railroad are 30 ft. long. If these rails are put down in winter, how much space must be left between two rails to allow for a change of temperature of 50° C.? Coefficient of expansion of steel is 0.000012. (Joliet)

184. 100 gm. of iron, specific heat .1125, at 200° C. can melt how much ice at 0° C.? Explain.

Explain the draft of a chimney. (Englewood, Chicago)

185. How much heat is required to change 5 gm. of ice at − 10° C. to steam at 100° C.? (Central, Pittsburg)

186. Determine the specific heat of lead, if 200 gm. of lead at 100° C. raised the temperature of a calorimeter and contained water from 20° C. to 22.4° C. The weight of the calorimeter (specific heat 0.1) was 50 gm. and of the water 200 gm. (Polytechnic Inst., Baltimore)

187. Describe the phenomena when ice at − 10° C. is heated until it becomes steam at 115° C. Compute the number of heat units required per gram. (West, Cleveland)

188. Account for the enormous amount of heat given out by the condensation of steam in steam pipes. (Ft. Wayne)

189. How many thermal units would be given out by 10 lb. of water at 45° C. in cooling to zero and then freezing? (Lima)

190. Apply the theory of convection to explain (*a*) trade winds; (*b*) land and sea breezes; (*c*) the heating of buildings by hot water.

What is meant by the hygrometric state of the air?

(University of Oxford)

191. A 6-horse-power machine will take how long to lift 550 tons of coal 300 ft.?

State and solve a problem illustrating the Law of Charles.

(Englewood, Chicago)

192. Why do we apply a flame *under* a vessel of water or air we wish to heat?

Why does wrapping a piece of ice in flannel keep it cool while wrapping the body in flannel keeps it warm? (Y. M. C. A.)

193. Determine the coefficient of linear expansion of a metal rod from the following data:

Initial length of rod, 60 cm.; expanded length, 60.079 cm.; initial temperature, 0° C.; final temperature 76°.6 C. (East, Cleveland)

194. A quantity of gas measures 360 c.c. at 30° C. What will be its volume at 0° C.? (Joliet)

195. Describe the thermal properties of (1) glass, (2) water, (3) rock salt.

Give a list of the principal sources of heat, with an example of each.

(University of Cambridge)

196. If an iron pipe 20 ft. long is heated from 20° C. to 100° C. by steam, what will be the increase in length? [Coefficient of expansion of iron = 0.000012.] (Regents)

197. A waterfall is 112 ft. (34 m.) high. How much warmer is the water at the bottom of the falls than at the top? [One calorie is equivalent to 42,000,000 ergs of heat energy.] (Polytechnic Inst., Baltimore)

198. (*a*) What are the two general results of heating a substance?

(*b*) How many calories of heat are required to change a gram of ice at −20° C. to steam at 100° C.? What is the weight of the steam?

(*c*) Name at least four factors which may affect the boiling point of any substance. (Louisville)

199. A certain quantity of air has a volume of 800 c.c. at 50° C. and 76 cm. pressure. What would be its volume at 150° C. and 152 cm. pressure? (Rayen, Youngstown)

200. 1000 gm. of ammonia liquid at 0° C. enter the pipes of an ice machine and leave it as gas at 0° C. How much water at 10° C. will it change to ice at 0° C.? (Heat of vaporization of ammonia is 450.)
(Joliet)

201. An iron ball, mass 100 gm., at 304° C., is placed on 40 gm. of ice at 0° C., which is melted and its temperature raised to 4° C. Find the specific heat of iron. (Man. Tr. High, Louisville)

202. What will be the result if 60 gm. of ice at −20° C. are put into 40 gm. of water at 90° C.? (Girls', Brooklyn)

203. Name, explain, and give examples of the mode of heat transmission. (South Bend)

204. What is the temperature at the bottom of a pond that is frozen over? Why? (Huntington, W. Va.)

205. In determining the linear coefficient of brass, the following data were obtained: length of brass rod at 18.5° C., 59.8 cm.; length at 98° C., 59.88 cm. Calculate the coefficient. (Paterson)

206. How much heat is equivalent to the energy of a 500 kgm. stone which has fallen 800 m.?
State Charles' Law. What does it mean? (Englewood, Chicago)

207. A metallic ball, of specific heat 0.032, weighing 1000 gm., at a temperature of 100° C., is plunged into ice. How much ice is melted? [Latent heat of fusion of ice = 80.]
Sketch and describe an arrangement of apparatus for preparing distilled water. (University of Cambridge)

208. At 76 cm. of mercury and 0° C., 1.293 gm. of air has a volume of 1000 c.c. What pressure must be exerted upon the air in order to keep its volume unchanged when the temperature is 546° C.? (Central, Pittsburg)

209. A volume of air measured at 15° C. is 170 c.c.; if the pressure on the air remains constant while the temperature is raised to 21° C., what volume will the gas occupy? (Regents)

210. How many calories of heat are generated by the impact of a 200-kgm. bowlder when it falls through 100 m.? (Central, Scranton)

211. Trace the changes in volume that occur when a mass of ice at −6° C. is heated up to 100° C.
A block of solid iron floats on molten iron. Hence show whether iron contracts or expands when it solidifies; and state the effect that an increase of pressure would have on its melting point.
(University of Oxford)

212. What is a saturated vapor? A saturated solution? What is the effect of cooling each?

40 c.c. of gas 80 cm. pressure will have what volume at 66 cm. pressure? (Englewood, Chicago)

213. Find resulting temperature of the following mixture: 100 lb. of ice at 0° C., 10 lb. steam at 100° C., 100 lb. of ice water, 100 lb. of water at 6.3° C., and 200 lb. of water at 5° C. (South Bend)

214. 10 gm. of steam at 100° C. condensed in 1 kgm. of water at 0° C. raised the temperature to 6.3° C. Calculate the heat of vaporization of water. (Crane, Chicago)

215. If the volume of a gas be doubled, what change in temperature is required in order that its pressure remain constant? (Joliet)

216. A 1000-lb. mass, whose specific heat was 1 fell 389 ft. If all the heat produced when it struck went into the body, how many degrees was it warmed? (Central, Detroit)

217. A brass meter scale is correct at 0° C. It is used at 30° C. to measure a distance of 20,000 cm. What is the error? (Coefficient of linear expansion of brass is 0.000019.) (Girls', Brooklyn)

218. (*a*) Explain why snow and ice do not melt all at once on a warm winter's day.

(*b*) Why does a wet sponge tied to a horse's head keep him cool? (Morris, New York)

219. It requires 1.77 calories to raise the temperature of 10 gm. of glass from 74° to 75° C. What is the specific heat of glass? (Joliet)

220. (*a*) Give law of Boyle and Charles combined.

(*b*) 100 c.c. of ice water is poured into a glass tumbler whose mass is 500 gm. and temperature 12° C. and the resulting temperature is found to be 6° C. Find the specific heat of the glass. (Ft. Worth, Tex.)

221. Make a diagram to illustrate the heating of a building by a hot-water system. Upon what principles does it depend? (Bradford)

222. What weight of water at 85° C. will just melt 15 gm. of ice at 0° C.? (Joliet)

223. If 586 c.c. of hydrogen at 20° C. have the temperature changed to −20° C. while the barometer falls from 76 to 72 cm., what will be the new volume? (Central, Columbus)

224. Explain any *two* of the following phenomena:

(*a*) That a mine may be ventilated by means of two shafts with a fire at the bottom of one of them.

(*b*) That dew forms on blades of grass but not on roads or metals.

(*c*) That the polished fire irons in front of a fire are sometimes found to be only slightly warm while the blackened fender is too hot to be touched.

(*d*) That equal weights of pounded ice at 0° C. and salt at 0° C. on being mixed give a temperature below zero. (University of Cambridge)

225. A cloth is wet with water at room temperature and hung up in the room to dry; it is observed that the temperature of the cloth while drying is continually below that of the room. Explain. (Regents)

226. Show that to raise the temperature of a pound of water from 32° F. to 52° F. requires more energy than to raise a 7-ton mass through 1 ft. (Joliet)

227. What is the reason the temperature known as "absolute zero" is so called? (Austin, Chicago)

228. Why is the current of steam which issues from the spout of a kettle invisible in the immediate neighborhood of the spout?

Explain the cause of the deposition of dew. (University of Oxford)

229. Why do clothes dry more quickly on a windy day than on a quiet day?

Why does sprinkling the street on a hot day make the air cooler?

Why will blotting paper soak up ink so much more readily than glazed paper? (Sioux Falls, S. Dak.)

230. A silver cup containing boiling water is placed on a silver tray in a room. Describe the various ways in which the water loses heat.

Give some account of the application of the principle of expansion of solids to industrial purposes. (University of Cambridge)

231. What is the effect of dissolved salt upon the freezing point of water? Name some practical applications of this effect. (Y. M. C. A.)

232. On what grounds, and by what experiments, are we led to regard heat as a form of energy?

Why is glass used in the construction of greenhouses, in spite of the fact that a glass screen will protect us from the heat of a fire? (University of Oxford)

233. Explain the method of producing artificial cold for the manufacture of ice or ice cream. (Lake View, Chicago)

234. Why does ice often form at night in desert regions in tropical countries? (Joliet)

235. How does the temperature of the steam coming out of a steam engine compare with that of the steam entering? Account for this difference. (Central, Pittsburg)

236. (*a*) How could you show that the specific heat of a metal ball was less than that of water?

(*b*) How would you account for the apparent loss of heat which accompanies the fusion of a solid?

(*c*) Explain why, so long as any water remains in a vessel that is being heated, the solder at the bottom does not melt. (University of Oxford)

237. Explain why the coal consumption of a steamship increases so much more rapidly than the speed. (Man. Tr., Denver)

238. Make drawing to show (*a*) the (magnetic) field about a bar magnet, (*b*) the field between two like poles, (*c*) the field between two unlike poles. (Yeatman, St. Louis)

239. In what direction must a current flow through a magnetizing coil in order to produce a north magnetic pole in that end of its soft iron core which faces the observer? Show, by means of a drawing, why this must be the case. (Y. M. C. A.)

240. A dozen steel needles, strung on a thread, are suspended so that the points touch the N. pole of a bar magnet.

What position will the needles assume, and why? Make sketch. (Man. Tr., Louisville)

241. In using a compass why is it not necessary to remove the glass cover? (Central, St. Louis)

242. Explain and illustrate what is meant by the expression: "Magnetic lines of force always form closed circuits and take a path of least resistance." Illustrate your answer by a bar magnet and by a horseshoe magnet. (Y. M. C. A.)

243. (*a*) What is the evidence that the earth is a magnet with fairly well-defined poles?

(*b*) State how a body may be given a positive electrostatic charge.

(*c*) What are conductors and insulators? (Glenville, Cleveland)

244. What is meant by inclination or dip of the needle? By declination? (Oshkosh, Wis.)

245. Why are the iron posts of a fence generally found to be magnetized? What is the polarity (N. or S.) of their upper ends? (Morris, New York)

246. What is the advantage of laminating magnets? What probably occurs in a steel bar when it is magnetized? (Polytechnic Inst., Baltimore)

247. (*a*) Define magnetic pole; permeability; retentivity; line of force.

(*b*) Explain the cause, effect, and cure of local action.

(Man. Tr., Denver)

248. Suppose the needle of a compass has become demagnetized. State in detail how you may magnetize it so that the blue end will again become a N.-seeking pole. (Morris, New York)

249. If an electrified rod is brought near to a pith ball suspended by a silk thread, the ball is first attracted to the rod and then repelled from it. Explain this. (University of Cambridge)

250. What is detected by a gold-leaf electroscope? What by an astatic galvanoscope? Would a sensitive galvanometer be affected by a charged Leyden jar brought near it? Why? (Worcester Academy)

251. Explain, by aid of diagrams, how an electroscope can be charged with either kind of a charge from the same source.

(South Division, Milwaukee)

252. Explain the charging of a Leyden jar, using a diagram.

(East, Cleveland)

253. What is a Leyden jar? electroscope? magnet? Explain the theory of each. (Rockford)

254. (*a*) Diagram a simple battery cell and name all the parts.

(*b*) Name two defects.

(*c*) How can each defect be shown?

(*d*) How can each defect be lessened? (Erasmus Hall, Brooklyn)

255. Describe with diagrams the magnetic field produced by an electric current flowing through (i) a straight wire, (ii) a solenoid.

(Ed. Dept., Ontario)

256. What are polarization and local action? Tell what harm each does and how it is prevented. (Paterson)

257. Explain the general principle of electroplating, employing the following terms; viz. cathode, anode, electrolyte, anions, cathions, and ions. (Y. M. C. A.)

258. State Ohm's Law.

If a conductor carries a current of 4.5 amperes with an electrical pressure of 63 volts, find the resistance of the conductor. (Lakewood)

259. State three effects of an electric current.

Name and define the unit of current strength. Upon which of the three effects is this definition based? What is the strength of a current that will deposit by electrolysis 0.3354 gm. of silver in 10 min.?

(Worcester Academy)

260. Given the electrochemical equivalent of copper equals 0.0003286. How many amperes are necessary to deposit 30 gm. of copper in 8 hr.?
(Man. Tr., Louisville)

261. A dynamo generating 110 volts has 15 ohms resistance. What current strength will the ammeter show when the resistance of the line wire is 25 ohms? (Austin, Chicago)

262. Explain by aid of a diagram how a single loop of wire, in connection with an electro-magnet, may be used to generate an alternating electric current. What modification of the apparatus would be necessary to produce current flowing from the loop in one direction only?
(Regents)

263. There are ten 1500-candle-power arc lamps in series, each having a resistance of 3 ohms. The resistance of the line is 5 ohms. The E.M.F. is 280 volts. What is the current? How many watts are developed? What is the horse power? What is the wattage per candle power?
(Hyde Park, Chicago)

264. (*a*) State the law of divided circuit or joint resistance.
(*b*) What would be the combined or joint resistance of three branches of 5, 10, and 15 ohms each? (Central, Columbus)

265. Find the current through an external resistance of 10 ohms, if 6 cells of an E.M.F. of 1.2 volts, and an internal resistance of 3 ohms, are joined in series; are joined in parallel. (Joliet)

266. Give three laws of resistance.
If 39 ft. of copper wire, 0.020 in. in diameter, have a resistance of 1 ohm, what will be the resistance of 156 ft. of the same kind of wire but of half the diameter? (Central, Columbus)

267. A battery is composed of 8 cells, each giving 2.5 volts electromotive force, and having an internal resistance of 0.3 ohm. What current will the battery supply through an external resistance of 4 ohms? If 2 cells are added to the battery, what would then be the strength of the current? (Polytechnic Inst., Baltimore)

268. Three cells, each having an E.M.F. of 2 volts, and an internal resistance of 5 ohms, are coupled in series with an external resistance of 4.5 ohms. What current will they give? (Ft. Wayne)

269. (*a*) Explain what is meant by a *shunt*.
(*b*) A galvanometer, whose resistance is 80 ohms, is supplied with three shunts which permit only $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{1000}$ of the entire current in the circuit to pass through it. Find the resistance of the shunts.
(Ed. Dept., Ontario)

270. What is the resistance of a divided circuit of two wires, one having a resistance of 10 ohms and the other of 5 ohms. (Joliet)

271. (*a*) If an arc lamp requires a current of 10 amperes and a difference of potential at its terminals of 50 volts, how great is its resistance?

(*b*) How many watts does it require?

(*c*) What part of a horse power does it use? (Central, Scranton)

272. Two telegraph sounders, each having a resistance of 20 ohms, are connected by a line wire whose resistance is 6 ohms. What current will be sent through the circuit by a battery whose E.M.F. is 13.2 volts, and whose resistance is 12 ohms? (East, Cleveland)

273. An inclosed arc lamp requires 80 volts and 4.5 amperes to run it properly. What is its resistance? (Lima)

274. Represent by a drawing the essentials of an outfit for silver plating. Describe the work accomplished by each part of the outfit.

If a coulomb of electricity deposits 0.01725 grain of silver, how much will be deposited by 2 amperes in 3 hours? (Y. M. C. A.)

275. Two cells, each 1.4 volts and .33 ohm, connected in series, supply current through wires of 1.2 ohms resistance to 2 electric bells in parallel of 2 and 3 ohms resistance respectively. Diagram the circuit. Calculate the main current and the current through each cell. (Central, Pittsburg)

276. What is the result of approaching a coil of wire with another coil in which current is flowing in a clockwise direction, the axes of the coils being in the same line? What would be the effect of breaking the circuit in the coil carrying the current? How is the break in the circuit of a Ruhmkorff coil made to occupy the least possible time?

(Polytechnic Inst., Baltimore)

277. Two incandescent lamps are in series on a 120-volt circuit. A $\frac{1}{4}$-ampere current is flowing through the line.

(*a*) What is the resistance of each lamp?

(*b*) If the lamps were in parallel, what would be the current.

(*c*) Define an ohm. (Wadleigh, New York)

278. A motor has a resistance of 7 ohms, and is to be run with 3 Daniell cells each having an E.M.F. of 1.08 volts, and an internal resistance of 3 ohms. How should the cells be connected, and what current will be obtained? (Show all work.) Write law involved. (Man. Tr., Denver)

279. Explain the action of a shunt.

A galvanometer has a resistance of 171 ohms. With what resistance must it be shunted in order that only one tenth of the current in the main circuit may pass through it? (University of Cambridge)

280. The E.M.F. of a dynamo is 839.02 volts. The current from it passes through 16 arc lamps, each having a resistance of 4.51 ohms. The resistance of the line wire is 0.8 ohm. The current is 10.04 amperes. Find the resistance of the dynamo. (Lakewood)

281. If a copper wire 200 m. long and 1 mm. in diameter has a resistance of 1 ohm, what resistance will a wire 500 cm. long and 5 mm. in diameter have? (Central, Pittsburg)

282. The receiver of a telephone consists of a thin iron disk that is free to vibrate in the field of a permanent magnet whose nearest pole is surrounded by a coil of fine wire. Show how this instrument can act either as a transmitter or receiver. (State Normal, Ypsilanti)

283. A transformer receives 1000 volts and 10 amperes at its primary. If the secondary pressure is 100 volts, how great will the secondary current be? What is the ratio of the turns of wire. (Rayen, Youngstown)

284. (*a*) How many watts will it take to maintain a current of 3.5 amperes through a resistance of 23 ohms?

(*b*) How many horse power will be generated by a dynamo giving 500 volts and 50 amperes? (Omaha, Neb.)

285. Four wires, of 4, 6, 8, and 12 ohms respectively, are joined in parallel to the terminals of a battery whose E.M.F. is 16 volts, and whose internal resistance is 2 ohms. Draw a diagram of the circuit and compute the current that flows through the battery. (Worcester Academy)

286. Five incandescent lamps, each having a resistance of 220 ohms, are joined in parallel (multiple) to the mains of an electric circuit, between which there is maintained a constant difference of potential of 110 volts.

(*a*) What current would flow through the lamps?

(*b*) What current would flow if the lamps were connected in series?

(*c*) Which of these connections, the parallel or the series grouping, is better adapted for lighting purposes? Why? (Regents)

287. How many calories of heat are developed per second by a 110-volt incandescent lamp carrying a current of 0.6 ampere?

(Central, Scranton)

288. Distinguish between a permanent, a temporary, and an electromagnet.

Five cells of E.M.F. 2 volts each, and an internal resistance of 1.2 ohms each, are coupled in series through an external resistance of 10 ohms. Find the current. (Chester, Pa.)

289. How many amperes will be required for a 10-horse-power motor, the efficiency being 80 per cent, and the electromotive force 250 volts?

(Polytechnic Inst., Baltimore)

290. Why is an electromotive force generated when a coil of wire revolves in a magnetic field? How is an E.M.F. produced in the secondary of an induction coil? Answer these questions carefully in detail.

(Morris, New York)

291. (*a*) A galvanoscope shows the same deflection with 30 ft. of copper wire, or 3.7 ft. of iron wire, or 1.7 ft. of German silver wire in the external circuit. The diameter of the wires being the same, find the specific resistances in terms of the copper.

(*b*) What is the general result of connecting cells in series? in parallel? (Memphis, Tenn.)

292. Ten glow lamps connected in parallel have together a resistance of 22 ohms, and require a current of 5 amperes.

(*a*) Allowing for 3 ohms resistance of line wire, what voltage must the dynamo maintain?

(*b*) What is the output of power in watts?

(*c*) If the dynamo has an efficiency of 80 per cent, what horse power must the engine furnish to the dynamo? (Central, Cleveland)

293. What is meant by polarization in an electric cell? How would you demonstrate its effect on the current? (University of Cambridge)

294. If an arc lamp requires a current of 10 amperes and a potential difference of 50 volts, how great is its resistance?

How many watts does it require?

What part of a horse power? (Marshall, Chicago)

295. Two branches of a divided circuit have resistances of 7 and 5 ohms, respectively, the total current being 24 amperes; state

(*a*) What current flows in each branch.

(*b*) What voltage is required at the terminal to maintain this current.

(Regents)

296. Four Daniell cells, each having an E.M.F. of 1.1 volts, and an internal resistance of 2.5 ohms, are joined in parallel. What will be the current through an external resistance of 1 ohm? Compute the current through the same external resistance when same cells are in series.

(Crane, Chicago)

297. What E.M.F. will force 20 amperes through 500 ohms resistance? Express the power in watts, in horse power.

How much heat would be given in 10 sec.? (Englewood, Chicago)

298. Describe some of the physical phenomena of the carbon voltaic arc. (Y.M.C.A.)

299. How many calories of heat will be generated by a current of 4 amperes passing through a resistance of 2 ohms for one half hour. How much will the temperature of $2\frac{1}{2}$ kgm. of water be increased by this heat, no allowance being made for radiation or conduction?

(Polytechnic Inst., Baltimore)

300. Explain carefully the structure and principle of some form of dynamo, naming each part and stating the function of each. Use a diagram. (Barringer, Newark)

301. Estimate the horse power it would take to run 200 arc lights. Explain. (Roswell, N.M.)

302. (*a*) Make a labeled diagram of an incandescent lamp bulb.

(*b*) Explain why air is removed from the bulb.

(*c*) Explain why the wires conveying current through the glass are of different material from the light-giving filament.

(*d*) Explain why the nature and the dimensions of the filament determine the proper voltage of the circuit on which the lamp may be used.

(Regents)

303. An electric current is sent through a solution of copper sulphate, then through a conductor of high resistance, then through a coil of wire wound upon an iron core, and then back to the generator. State clearly what would be the main effect produced in each of the three parts of this circuit. What quantitative change would be produced in each part by doubling the current? (Morris, New York)

304. Give

(*a*) Two ways of increasing the strength of an electro-magnet.

(*b*) Two ways of decreasing the deflection of the galvanometer with a given current.

(*c*) Three ways of increasing the E.M.F., or current generated by a dynamo. (East Side, Bay City)

305. State briefly the part that each essential piece of apparatus plays in the utilization of the energy of a waterfall for electric lighting, giving clearly the energy changes involved. (Regents)

306. Define an *ampere*.

How can two voltmeters be used to measure a current, and at the same time check each other's result? (Ed. Dept., Ontario)

307. Looking at the end of a 2-pole dynamo, the N. pole is at the left, and the S. pole at the right. A coil of wire of the armature rotates clockwise. Indicate the direction of induced current in the coil when it is in a horizontal position. (Man. Tr., Louisville)

308. State the rules for direction of induced currents.
What is self-induction? (South Bend)

309. Draw a diagram of a Wheatstone bridge, and name each part. What relation exists between the resistances involved when the bridge is "balanced"? (Regents)

310. Distinguish between electromotive force and counter electromotive force. Describe briefly any two processes in which the development of counter electromotive force in an electric device is necessary to its successful operation. (Y. M. C. A.)

311. What is the principal difference between the structure of the lamp producing the arc light and that producing the incandescent light? (State of Pa.)

312. A lineman wishes to know the direction of the current in a telegraph wire. How may he determine it? (Hyde Park, Chicago)

313. Figure this light bill:
4 16-candle-power lamps on 110-volt circuit, drawing $\frac{1}{2}$ ampere each, 3 hr. daily for 30 da., at 10¢ per K.W. hour.
50 street arcs on 220-volt circuit, drawing 4 amperes each, 10 hr. daily for 30 da., at the same rate. (Dallas, Tex.)

314. Describe any two voltaic cells and explain carefully the means used to prevent polarization. (Allegheny)

315. Draw a diagram of a two-station telegraph line showing receiving and sending instruments at each station and a relay at one station. (Portsmouth)

316. (*a*) Make a drawing of the induction coil and explain its operation.
(*b*) What is the general effect of self-induction on currents flowing through a helix with an iron core? (Central, St. Louis)

317. Diagram and explain the use of the Wheatstone bridge. (East, Minneapolis)

318. Make a diagram showing the essential parts of some piece of practical electric apparatus whose action depends on electro-magnetic induction. State the purpose of each part. (Regents)

319. Explain differences in motor, dynamo, and magneto. (Girls' High, Mobile, Ala.)

320. Give the meaning of the terms: inertia, center of gravity, moment of force.
State the following laws, or principles:
Pascal's Law, Principle of Archimedes, Boyle's Law, Ohm's Law. (West, Cleveland)

321. A tree stands upon the bank of a pond of still water. Show how to construct the image, and show the actual path of light which causes the observer on the opposite bank to see the image.

State the laws of reflection. (Morris, New York)

322. If sunlight travels in straight lines and its rays are parallel, how is daylight disseminated through a room so that it reaches every nook and corner? How does a room with only a north window get any light at all? (Y. M. C. A.)

323. A ray of light, coming from the northeast, strikes a plane mirror at such an angle that the reflected ray travels due west. How much must the mirror be turned, clockwise, in order that the reflected ray may travel due north? Show by drawing. (Hyde Park, Chicago)

324. (*a*) Locate and define the elements of a concave mirror (figure).

(*b*) Construct the image of an object placed beyond the center of curvature. (Central, Columbus)

325. Construct the image of an object in a concave mirror when it is placed

(1) at the focal distance,

(2) between the focal point and the mirror,

(3) between the focal point and the center of curvature,

(4) at the center of curvature. (Shaw, East Cleveland)

326. What is the position of the image of a man who stands on the floor before a plane mirror which is tilted so as to make an angle of 45° with the floor?

(Make a drawing to illustrate your answer.) (Joliet)

327. (*a*) Explain briefly by a diagram how an image of a luminous object outside may be formed by means of a hole in a window shutter.

(*b*) Explain also how a lens acts so as to improve the definition of this image. (Central, Cleveland)

328. The index of refraction of water is 1.33. What is the apparent depth of a pool whose actual depth is 16 ft.? (Worcester Academy)

329. What is meant by principal focus, vertex, optical center, secondary axis, conjugate foci? Answer for both mirrors and lenses. (Joliet)

330. Define critical angle and give and explain construction for same from diamond to air. Index of refraction for diamond is $\frac{5}{2}$. (Man. Tr., Louisville)

331. A ray of light, passing from the air, enters water at an angle of 30° with its surface; determine the direction of the ray of light after it enters the water. Use diagrams. [Index of refraction $= \frac{4}{3}$.] (Regents)

332. The image of a candle placed 10 in. in front of a concave mirror is seen on a screen 30 in. from the mirror. What is its focal length?

(Joliet)

333. An object is placed 10 cm. from a convex lens whose focal length is 15 cm. Describe the image formed.

The object is 4 in. long. How long is the image?

(Worcester Academy)

334. Consider an object at a great distance from a convex lens. Where is the image formed? What is the change in location and size of image as the object approaches the lens? Draw diagrams.

(Glenville, Cleveland)

335. Show by diagram the path of a ray of light passing horizontally from left to right through a rectangular plate of glass lying in the horizontal plane. The incident ray should make an angle of about 45° with the edge of the plate. Account for each change of direction in the ray.

(Regents)

336. A body with an area of 1 sq. mm. is placed before a stereopticon lantern at a distance of 1 ft. Find size of shadow cast on a screen at a distance of 20 ft. (Joliet)

337. What *two* effects may be noted when light is passed through a triangular glass prism, and what explanation may be given of them? Explain. (Morris, New York)

338. An object is 15 cm. in front of a convex lens of 12 cm. focal length. What is the nature of the image, its size, and its distance from the lens? (Central, Scranton)

339. If the focal length of a lens is 35.05 cm. and an image of a distant object is focused 35.051 cm. from the lens, how far distant is the object? If the image is 1 cm. high, how high is the object? (Joliet)

340. A pin, $\frac{1}{4}$ in. long, is placed at a distance of 3 in. from a convex lens of focal length 2 in. Draw to scale a diagram to show the position and length of the image. Explain your construction.

(University of Oxford)

341. Why are images formed through small apertures inverted? On what does the size and distinctness of the image depend? Why?

(Birmingham)

342. In spearing a fish, if looking obliquely at the water, would you strike above or below the apparent position of the fish? Why? (Joliet)

343. I can see to read a book at a distance of 5 ft. from a gas flame. I can also read the book equally well at a distance of 12 ft. from a 16-candle-power electric lamp. What is the candle power of the gas flame?

(Hyde Park, Chicago)

344. Show by diagram how an image can be found in a plane mirror. Is the image real or virtual? Distinguish between a real and a virtual image.

State the theory of color. On what does the color of objects depend?

(San Antonio)

345. An object is 20 cm. from a lens and its image is 72 cm. from the lens on the same side. Find the focal length. Is it concave or convex?

(Joliet)

346. Where does a convex lens form:

(*a*) A real image,
(*b*) a vertical image,
(*c*) an enlarged image,
(*d*) a diminished image,
(*e*) an erect image,
(*f*) an inverted image?

(Central, Scranton)

347. With a diagram show how a simple microscope magnifies an object. Explain. (Middletown, Conn.)

348. (*a*) A room to be used for reading or writing may be illuminated either by a single powerful source centrally placed, or by several distributed weaker sources. If the two systems give out the same amount of light, which would be the better? Why?

(*b*) How is the illumination of a room modified by the character and the color of the walls? (Regents)

349. Explain how and why an image is formed in an opera glass.

(Roswell)

350. Draw and explain figures to illustrate the formation of a virtual image of an object (*a*) with a convex lens, (*b*) with a concave lens.

The real image of an object 3 cm. long and 6 cm. from a lens is at a distance of 12 cm. from it. Draw a diagram to scale to find (i) the length of the image, (ii) the focal length of the lens, and state the values so found. (University of Cambridge)

351. At what distance from a screen will a 100-candle-power lamp give as much light to it as a 25-candle-power lamp at a distance of 6 ft. from the screen? (Erasmus Hall, Brooklyn)

352. A telescope has an objective of 30 ft. focal length and an eyepiece of 1 in. focal length. What is its magnifying power?

(Polytechnic Inst., Baltimore)

353. When white light falls upon piece of glass *A*, red and yellow pass through it; through *B* pass violet and blue; through *C* pass violet, green, and yellow; through *D*, violet and red. What will each transmit when yellow light (from sodium vapor) falls upon it?

(Central, Pittsburg)

354. Name three ways in which light differs from sound.

If a church steeple throws a shadow 40 ft. and a pole 10 ft. high throws a shadow 2 ft., how high is the steeple?

Explain the principle for measuring the candle power of light.

(Wheeling)

355. By constantly increasing the heat of a metal until it is white hot, a continuous spectrum is formed. Explain. When it is heated to a vaporous state a bright line spectrum is formed. Explain. (Roswell)

356. Name the color of the solar spectrum having (*a*) the most heat; (*b*) the most chemical power; (*c*) the greatest wave length. (State of Pa.)

357. How much heat will a 16-candle-power light give out in one hour?

(East, Minneapolis)

358. What are the Fraunhofer lines? Show their use and importance in scientific research. (Barringer, Newark)

359. (*a*) Compare the images seen in plane mirrors with those seen in convex mirrors.

(*b*) What kind of lenses should be used for far-sighted eyes? Why?

(Malden)

360. What are the guiding principles to be followed in the construction of images formed by mirrors and lenses? (Birmingham)

361. If the sun were suddenly to explode and disappear, how long would it take astronomers on the earth to know of the trouble?

(Central, Scranton)

362. Explain briefly any five of the following, stating the physical principle upon which the experiment depends:

(*a*) Why does blowing across a page freshly written with ink cause it to dry quickly?

(*b*) Why do persons not fall out of the car when "looping the loop"?

(*c*) Why will a needle float on the surface of water?

(*d*) Why does ice beneath a dark-colored object exposed to the sunlight melt more quickly than beneath a light one?

(*e*) Why do delicate tints appear to change color when viewed by sunlight and by gaslight?

(*f*) Why does a clock usually gain time in winter?

(*g*) Why does the heated air over a stove appear to dance or shimmer?

(*h*) Why does blowing across the surface of hot tea or coffee cool it?

(East, Cleveland)

363. What are the physical principles involved in the camera, pumps, electric bell, telegraph, dynamo, steam engine, piano?

(West, Cleveland)

364. Contrast light waves with sound waves, telling (*a*) how they are produced; (*b*) how they originate; (*c*) in what medium they travel.
(Girls', Mobile)

365. State the laws of refraction of light.
Trace a ray of light from glass into air.
When do we have total internal reflection?
(Fogg High, Nashville, Tenn.)

366. By diagram show how an image is formed by a convex lens.
Show by diagram how a convex lens is used as a simple microscope.
(South, Cleveland)

367. (*a*) What is chromatic aberration? How is it corrected in lenses? (*b*) If a lamp is 65 cm. and a standard candle 35 cm. from a translucent screen when it is equally illuminated by the two lights, what is the intensity of the light from the lamp? (Richmond, Va.)

368. When a certain piece of paper is illuminated by white light, the eye receives violet and red light from it. What color or colors will it send to the eye when placed in the red of the spectrum? In the yellow? In the violet? (Central, Pittsburg)

369. Compare sound waves and light waves as to (*a*) method of propagation, (*b*) transmission in vacuum, (*c*) speed in air, (*d*) change of velocity on passing from air to water, (*e*) wave length. (Atlantic City, N. J.)

370. State the theory of light.
How do we know white light is made up of different colors?
What is the fundamental difference between two colors?
What are complementary colors? (Fogg, Nashville)

THE SOLUTION OF NUMERICAL PROBLEMS

The solution of any numerical problem in physics depends simply upon the application to a concrete case of those physical facts and relations which have been expressed in definitions and laws. A definition or a law may frequently be expressed more briefly and concisely in the *language of mathematics* than in words. Accordingly in a perfectly natural way many so-called *formulas* have found their way into physics *for the purpose of simplifying the physics.* Unfortunately the student does not learn to express himself in the language — shorthand if you like — of mathematics any more readily than he learns to speak German or French. As a result the meaning of the symbols in the formulas is likely to be lost sight of and the student attempts to *select a formula* and *substitute* with altogether too little regard for the *idea* for which the formula stands.

The following solutions are not given so that the student may *find a solution* like that of the problem he wishes to solve and imitate it, but to illustrate how easy it is, *in most cases*, to go back to the fundamental ideas and apply them *numerically*. These solutions have to do chiefly with the *form* of solving numerical problems.

1. Distance traversed when the Average Speed and Time are known.

Problem. A train travels 15 hr. at an average rate of 30 mi. an hour. How far has it gone?

Solution. In 1 hr. the train travels 30 mi. Hence, in 15 hours, it will travel $30 \times 15 = 450$ mi. *Ans.*

The same result might be obtained by recalling the generalized statement **distance = average speed × time**, hence distance $= 30 \times 15 = 450$ mi.

Evidently the simplest of analyses would give the value for either *time* or *average speed* if the *distance* and *either* time or speed were known.

2. Distance traversed when the Motion is uniformly accelerated.

Problem. A falling body is accelerated at the rate of 980 cm. per second in every second. How far will it fall in 10 sec.?

Solution. If the body starts from rest and if it gains velocity at the rate of 980 cm. per second in a second, its velocity at the end of 10 sec. is $980 \times 10 = 9800$ cm. per second. Since we suppose that the acceleration is uniform, the average velocity during this time has been $\frac{0 + 9800}{2} = 4900$ cm. per second. Hence, in 10 sec. the body will pass over $4900 \times 10 = 49{,}000$ cm. *Ans.*

It will be readily seen that it is first necessary to calculate the average velocity, then to find distance as in 1. The problem cannot be solved when the average velocity cannot be calculated.

3. Average Acceleration when Distance and Time are known.

Problem. A body starts from rest and rolls down an inclined plane 72 ft. long in 3 sec. What is the average acceleration?

Solution. The average velocity is $\frac{72}{3} = 24$ ft. per second. Since the body starts from rest, the final velocity is twice the average velocity, or $24 \times 2 = 48$ ft. per second. Since this was produced in 3 sec., the average acceleration is $\frac{48}{3} = 16$ ft. per second in every second. *Ans.*

4. Distance traversed in any Particular Unit of Time.

Problem. A body fell $\left(g = 32 \frac{\text{ft.}}{\text{sec.}^2}\right)$ for 10 sec. How far did it travel during the *third* second?

Solution. The velocity at the *beginning* of the third second was $32 \times 2 = 64$ ft. per second. The velocity at the *end* of the third second was $32 \times 3 = 96$ ft. per second. The average velocity during this second was $\frac{64 + 96}{2} = 80$ ft. per second. In other words the body fell 80 ft. in the third second. *Ans.*

5. Velocity after passing over a Distance with a Certain Acceleration.

Problem. A ball fell from a height of 100 ft. with an acceleration of 32 ft. per second in every second. With what velocity was it then traveling?

Solution. Expressing the physical facts algebraically, $d = \frac{1}{2} gt^2$ and $v = at$. By a purely algebraic elimination of t between these equations, $v^2 = 2\,ad$, which means that the velocity at any instant is equal to the square root of the product, 2 × acceleration × distance, or $v = \sqrt{2\,ad}$. Hence, *velocity* $= \sqrt{2 \times 32 \times 100} = \sqrt{6400} = 80$ ft. per second. *Ans.*

6. Parallelogram of Forces, or Velocities, or Directed Quantities in General.

Problem. What is the resultant of a force of 100 grams-weight directed north and a force of 200 grams-weight directed east?

Solution. By the *principle of the parallelogram of forces*, two concurrent forces at right angles and their resultant are related as are the concurrent legs and diagonal of a rectangle. Hence, by the Pythagorean Theorem, the numerical value of the resultant is the square root of the sum of the squares of the components, or $\sqrt{100^2 + 200^2} = \sqrt{5000} = 223.6$ gm. The exact direction of this resultant cannot be found without the use of trigonometry. Approximately it lies between northeast and east. Its position may be more definitely determined by constructing a rectangle to scale and measuring the angle between either leg and the diagonal which extends in a northeasterly direction.

7. Force required to produce a Known Acceleration in a Known Mass.

Problem. What is the force which produces in 20 sec. a change of velocity of 10,000 cm. per second in a mass of 10 gm.?

Solution. By definition 1 dyne produces an acceleration of 1 $\frac{\text{cm.}}{\text{sec.}^2}$ in 1 gm. Then 10 dynes produce an acceleration of 1 $\frac{\text{cm.}}{\text{sec.}^2}$ in 10 gm. and $10 \times 10{,}000$ dynes produce an acceleration of 10,000 $\frac{\text{cm.}}{\text{sec.}^2}$ in 10 gm. If this

force will produce the acceleration required in 1 sec., the average force to produce the same effect in 20 sec. is $\frac{100000}{20} = 5000$ dynes. *Ans.*

Analogous reasoning will analyze any relation between *force*, *mass*, and *acceleration*.

8. Moments of Force.

Problem. A bar 4 ft. long and weighing 20 lb. whose center of gravity is 1 ft. from the heavy end, is balanced about a point $1\frac{1}{2}$ ft. from that end when a weight of 10 lb. is hung at the heavy end and an unknown weight at the light end. What is the weight at the light end?

Solution. Since the bar is in equilibrium, the moments about any point in the bar are equal. The whole mass of the bar may be considered as concentrated at its center of gravity. Take the point about which the loaded bar balances, namely, $1\frac{1}{2}$ ft. from the heavy end, as the center of moments. Then, moment of 10 with arm $1\frac{1}{2}$ + moment of 20 with arm $\frac{1}{2}$ = moment of unknown weight with arm $(4 - 1\frac{1}{2} =) 2\frac{1}{2}$, or $15 + 10 = 2\frac{1}{2} \times$ unknown weight. Therefore the unknown weight is 10 lb. *Ans.*

Note. — For any problem in moments of force it is always advisable to construct a figure drawn, at least roughly, to scale.

9. Kinetic Energy.

Problem. A falling mass of 100 gm. strikes the ground with a velocity of 2940 cm. per second. What is its kinetic energy?

Solution 1. To acquire a velocity of 2940 cm. per second the body must fall for $\frac{2940}{980} = 3$ sec. if it starts from rest. Its average velocity is $\frac{0 + 2940}{2} = 1470 \frac{\text{cm.}}{\text{sec.}}$ and the distance from which it fell $1470 \times 3 = 4410$ cm. Its potential energy before falling was $100 \times 4410 = 441{,}000$ gm.-cm. or $441{,}000 \times 980 = 432{,}180{,}000$ ergs. By the principle of the conservation of energy, the potential energy at the beginning of the fall is equal to the kinetic energy at the close of the fall. Hence, the kinetic energy is 432,180,000 ergs. *Ans.*

Solution 2. It has been found that kinetic energy, mass, and velocity are thus related, *kinetic energy* $= \frac{1}{2}$ *mass* $\times$ (*velocity*)^2. Hence, kinetic energy of this mass is $\frac{1}{2} \times 200 \times (2940)^2 = 432{,}180{,}000$ ergs. *Ans.*

10. Pendulum.

Problem. What is the period of a pendulum whose length is 3920 cm. at a place where the acceleration due to gravity is 980 cm. per second in every second?

Solution. The relation between period and length of pendulum, and the

acceleration due to gravity is — period is equal to *2 π times the square root of the quotient* $\frac{\textit{length}}{\textit{gravity}}$. Hence in this case, period $= 2\pi\sqrt{\frac{1920}{960}} = 4\pi$ $= 4 \times \frac{22}{7} = \frac{88}{7} = 12\frac{4}{7}$ sec. *Ans.*

N. B. The *laws of the pendulum* (p. 67) may frequently be used to construct simple proportions from which the numerical values desired may be obtained.

11. Simple Machines.

The principle of the conservation of energy affords such a simple solution to problems on this subject that specific examples will not be given. Difficulties arise chiefly from failure to separate the solution of a problem into *successive* steps each of which is easy.

12. Specific Gravity.

Problems are solved in the text, p. 119.

13. Pressure in Liquids, Boyle's Law, and the Law of Charles.

The Theorems as stated in the text are very easily applied.

14. Conversion of Centigrade Readings to Fahrenheit and vice versa.

Problem. What reading on the Fahrenheit thermometer corresponds to 20° C.?

Solution. Since 0 to 100 on the centigrade scale corresponds to 32 to 212 on the Fahrenheit scale, 100 centigrade degrees are equal to 180 Fahrenheit degrees. Then 20 centigrade degrees are equal to $180 \times \frac{20}{100}$ $= 36$ Fahrenheit degrees. Since the centigrade zero begins at 32° F., 20° C $= 36 + 32 = 68°$ F. *Ans.*

N. B. The only difficulty in such problems arises from failure to refer both scales to the same zero at the moment of conversion.

15. Specific Heat. Heat of Fusion. Heat of Vaporization. Final Temperature of a Mixture.

Problem. 20 gm. of mercury at 40° C. are mixed with 20 gm. of water at 10° C., and the resulting temperature is 10.96° C. What is the specific heat of mercury?

Solution. The water equivalent of 20 gm. of mercury is 20 times the specific heat of mercury, or 20 H. In cooling from 40° C. to 10.96° C. the mercury lost $20\,H(40 - 10.96) = 580.8\,H$ calories. In warming to 10.96° C. from 10° C. the 20 gm. of water gained $20(10.96 - 10) = 19.2$ calories. By the principle of the conservation of energy these amounts of

heat are equal, or $580.8\ H = 19.2$. Hence H, the specific heat of mercury, is .033. *Ans.*

N. B. The principles involved in calculating heat of fusion, etc., are not essentially different from those applied in this problem on specific heat.

16. Ohm's Law. Resistances in Series and Parallel. Heating Effect of an Electric Current.

Refer to pp. 321–323 of the text.

17. Real Images in Mirrors and Lenses. Sizes of Image and Object.

Problem. An object 2 in. high stands 8 in. from a converging lens whose focal length is 5 in. Where is the image located and how large is it?

Solution. The principal and conjugate focal distances of any lens or mirror are thus related,

$$\frac{1}{\text{focal distance}} = \frac{1}{\text{image distance}} + \frac{1}{\text{object distance}}.$$

Hence, in this problem, $\frac{1}{5} = \frac{1}{\text{image distance}} + \frac{1}{8}$,

or $$\frac{1}{\text{image distance}} = \frac{1}{5} + \frac{1}{8} = \frac{3}{40}.$$

Hence, image distance is $13\frac{1}{3}$ in. *Ans.*

It is known from experiment that $\frac{\text{image distance}}{\text{object distance}} = \frac{\text{image size}}{\text{object size}}$.

Hence, $\frac{13\frac{1}{3}}{8} = \frac{\text{image size}}{2}$, or the size of the image is $3\frac{1}{3}$ in. *Ans.*

18. Virtual Images in Mirrors and Lenses.

Problem. What is the focal length of a concave lens which forms an image 5 cm. distant from the lens when the object is 30 cm. distant?

Solution. Since the focal distance of a concave lens is regarded as negative and since principal and conjugate focal distances are related as follows:

$$\frac{1}{\text{focal distance}} = \frac{1}{\text{image distance}} + \frac{1}{\text{object distance}},$$

our values must be so assigned as to make the focal distance negative,

or $$\frac{1}{\text{focal distance}} = \frac{1}{30} - \frac{1}{5} = -\frac{25}{150} = -\frac{1}{6}.$$

Therefore the focal length of the lens is 6 cm. *Ans.*

Note. — In problems involving virtual images the scientific convention according to which signs are adopted is somewhat complex. Hence the student is cautioned simply to select the negative quantity in such a way that the result may be *reasonable*. He will rarely, if ever, be misled.

LIST OF TABLES

(Numbers refer to pages.)

Absolute value of gravity, 73.
Acceleration due to gravity at various points on the earth's surface, 73.

Boiling points, 223.

Coefficient of linear expansion, 213.
Conductivity for electricity, 261.
Conductivity for heat, 204.
Critical angles, 354.

Declination, magnetic, 246.
Density of water, 217.

Efficiency of some machines, 102.
Electrical conductivities, 261.
Expansion, coefficient of linear, 213.

Heat conductivities, 204.

Magnetic declinations, 246.
Melting points, 223.
Musical nomenclatures, 190.

Pressure of saturated water vapor, 226.

Specific gravities, 118.
Specific heats, 230.
Speed of sound, 174.

INDEX

(Numbers refer to pages.)

Abscissa, 4.
Absorption of light, 342.
Acceleration, 13; earth's, 61; in circular motion, 36, 37; unit of, 14.
Air pump, 140.
Alternating current, 293, 300.
Ammeter, 325.
Ampere, the, 306, 329.
Amplitude, 66.
Angle, critical, 354; of deviation, 362; of incidence, 344, 351; of reflection, 344; of refraction, 351.
Anode, 285.
Antinode, 156.
Archimedes, principle of, 116.
Armature, 299, 314.
Atmosphere, composition of earth's, 131.
Atmospheric pressure, 129; normal, 130.
Averaging, 15.
Axis, of X, 4; of Y, 4; optical, 369; of a lens, 369.

Balance, beam, 80; chemical, 81; principle of, 81.
Barometer, 131; weather bureau, 132.
Battery, 284; gravity, 286; Leclanché, 287; storage, 288.
Beats, in music, 185.
Bell, telephone, 301; transmitter, 301.
Block and tackle, 98.
Boiling, 225; law of, 225.
Boyle's Law, 142.
British Thermal Unit (B. T. U.), 228.
Buoyancy, 116.

Calorie, 228.
Camera, pinhole, 376; photographic, 376.
Capillary phenomena, 122.
Cathode, 285.
Cell, gravity, 286; Leclanché, 287; storage, 288; voltaic, 281.
Center, of gravity, 74–75.
Center of mass, 75; determination of, 76.
Centigrade, or Celsius, thermometric scale, 201.
Centimeter, 7.
Centrifugal force, 35, 37; applications of, 37, 38.
Centripetal force, 36.
Characteristics of a musical tone, 181.
Charge, electric, 258; free and bound, 268.
Charles, law of, 219.
Chemical, effects of electric currents, 302; equivalent, 306.
Circuit, closed, 283; open, 284; primary, 296; secondary, 296; shunt, 323.
Clock, pendulum, 68.
Clockwise, 5.
Closed pipes, 190.
Coefficient of, elasticity, 107; expansion of a liquid, 215; friction, 54; linear expansion, 212.
Cohesion, 36.
Coil, induction, 297.
Color, 362, 363.
Commutator, 290, 300.
Compass, mariner's, 244.
Component forces, 40.
Condenser, 269.
Conduction of heat, 202.
Conductors, 260, 283; of first class, 281; of second class, 282.
Conservation of energy, 90, 235.
Constant quantities, 6.

Convection currents, 206.
Converging lens, 365.
Counterclockwise, 5.
Couple, 251.
Critical angle, 354.
Currents, 163; alternating, 293, 300; direct, 294; convection, 206; electric, 275; induced, 292; rule for direction of electric, 277; summary concerning electric, 329.

D'Arsonval galvanometer, 280, 317.
Declination, magnetic, 246.
Definition of, acceleration, 13; the ampere, 306; amplitude, 66; anode, 285; battery, 284; beats, 186; boiling point, 225; calorie, 228; cathode, 285; center of gravity, 75; closed circuit, 284; coefficient of expansion, 212, 215; cohesion, 36; complete vibration, 66; conductor, 283; convection currents, 206; density, 27; dew-point, 226; dyne, 35; elasticity, 105; electrical resistance, 309; electrodes, 284; electrolyte, 283; energy, 85; equilibrium, 42; force, 34; foot-pound, 86; fundamental, 185; gram, 25; gram-centimeter, 86; horse-power, 103; inertia, 22; interference, 160; interval, musical, 188; kilogram, 25; kilogram-meter, 86; linear momentum, 28; mass, 25; meter, 7; moment of force, 33, 45; motion, 9; ohm, the, 310; open circuit, 284; overtones, or harmonics, 185; parallel, or multiple, 284; period, 66; phase, 66; physics, V, 21; polarization, 288; poles, 284; refraction, 151; rotational inertia, 29; series, 284; solid, liquid, gas, fluid, 104; specific gravity, 117; specific heat, 229; surface tension, 126; temperature, 197; unit of heat, 228; volt, the, 322; voltaic cell, 282; water equivalent, 229; watt, the, 328; work, 84.
Density, 27.
Deviation of light, 362.
Dew-point, 226.
Dielectric, 264.
Diffraction of light, 339.
Dioptric, 367.
Dip, magnetic, 247.
Direct current, 294; generator, 300.
Direction of current, 277.
Dispersion of light, 362.
Diverging lens, 365.
Dynamo, 299; experiment, 298.
Dyne, 34, 35.

Efficiency, 102.
Elasticity, 21, 104, 105; coefficient of, 107; limit of, 106.
Electric, charge, 258; door bell, 315; motor, 318.
Electric currents, effects of, chemical, 302; heating, 307; magnetic, 312; summary concerning, 329.
Electrical pressure, 285; in series, 321; unit of, 322.
Electrical resistance, 309; in parallel, 324.
Electrification, 258; law of, 264; positive and negative, 261.
Electrodes, 284.
Electrolysis, 302; Faraday's laws of, 305.
Electrolyte, 283.
Electro-magnet, 256, 313.
Electro-motive force, 322; back E. M. F., 319.
Electrophorus, 267.
Electroscope, 259.
Electrostatics, 258; summary of, 273.
Energy, 21, 87; applications to machines, 94; conservation of, 90; dissipation of, 88; kinetic, 88; matter a vehicle of, 83; potential, 88; summary concerning, 93.
Engine, steam, 237; gas, 241.
Equilibrium, 42; heavy bodies, 77; non-parallel forces, 42; parallel forces, 47; stable, unstable, and neutral, 78.
Erg, 85.
Expansion, coefficient of linear, 212; of gases, 217; of liquids, 215; of solids, 211; of water, 216.
Eye, the human, 378.

Fahrenheit thermometric scale, 202.
Falling bodies, summary concerning laws of, 63.

Faraday's, discovery, 292; laws of electrolysis, 305.
Field, electric, 265; magnetic, 299.
Flame, manometric, 176.
Fluid, 104.
Focal length, 366.
Foci, conjugate, 373.
Focus, principal, 366.
Foot, 7.
Foot-found, 86.
Force, 33, 34; back electro-motive, 319; centrifugal, 35; centripetal, 36; component, 40; electro-motive, 322; of friction, 53; line of, 249; parallelogram of forces, 41; resolution of, 41; resultant, 40; resultant of non-parallel, 41; resultant of parallel, 47, 48.
Force-pump, 135.
Frequency, 183.
Friction, 53; coefficient of, 54; laws of sliding, 54; rolling, 55.
Fulcrum, 94.
Fundamental, 185.
Fusion, heat of, 231.

Galileo, 57; his experiment on falling bodies, 58; his inclined plane, 58, 59; his method of averaging, 15; his thermometer, 219.
Galvani, 279.
Galvanometer, 278; D'Arsonval, 280, 317.
Galvanoscope, 279.
Gas engine, 241.
Gases, 104; expansion of, 217; properties of, 127.
Generator, direct current, 300.
Gram, 25.
Gram-centimeter, 86.
Gravitation, 21; universal, 70; Newton's law of, 71; summary concerning, 82.
Gravity, cell, 286; center of, 75; value of, 72, 73.

Harmonics, 185.
Heart, human, as pump, 137–139.
Heat, 195; and temperature, 196; conduction of, 202: equation, 230; of fusion, 231; latent, 231; mechanical equivalent of, 233, 234; radiation of, 209; of vaporization, 231.
Helix, 278.
Homogeneous, 335.
Hooke's Law, 105.
Horror vacui, 128.
Horse power, 103.
Humidity, 227.
Hydraulic press, 111.
Hydrostatics, 108.

Ideal gas thermometric scale, 221.
Illumination, 336.
Image, 370; real and virtual, 371, 372.
Incandescent lamp, 311, 324.
Incidence, angle of, 344.
Inclined plane, 99.
Index of refraction, 351.
Induction, coil, 297; of electric currents, 291; electro-magnetic, 292; electrostatic, 266.
Inertia, 21, 22, 23; moment of, 30; rotational, 29.
Insulator, 260.
Intensity of light, 336.
Interference, 159; of light waves, 359.
Interval, musical, 188.
Irradiation, 338.
Isochronous, 69.

Joule, the, 85.
Joule's laws of heating, 308.

Key, or switch, 290.
Kilogram, 25.
Kilogram-meter, 86.
Kinetic energy, 88; measure of, 91.
Kinetic theory of matter, 235.
Kirchhoff's law of radiation, 211.
Krakatoa, 173.
Kundt's, method, 169; tube, 170.

Lamp, incandescent, 311, 324.
Lantern, projection, 388.
Latent heat, 231.
Laws, boiling, 225; Boyle's, 142; Boyle and Charles's, 221; Charles's, 220; falling bodies, 17, 63; Faraday's — of electrolysis, 305; friction, 54; Hooke's, 105; Joule's — of heating, 308; Kirchhoff's, 211; Lenz's, 295; lever, 95; machines, 102; melting, 223; Newton's laws of motion, 31, 33, 50; uniformly accelerated motion, 16;

Ohm's, 321; parallelogram law, 41; Pascal's, 111; pendulum, 67; principle of Archimedes, 117; plane reflection, 345; refraction, 352; universal gravitation, 71.
Leclanché cell, 287.
Length, unit of, 7; standard, 6.
Lenses, 364, 369; achromatic, 375.
Lenz's law, 295.
Lever, 94; mechanical advantage of, 95.
Leyden jar, 269.
Light, 331; rectilinear propagation of, 333; a wave motion, 358.
Lightning protectors, 273.
Limit of elasticity, 106.
Line of force, 249.
Liquids, 104; properties of, 108; expansion of, 215.
Lodestone, 244.
Longitudinal waves, 154.
Loop, 156.
Loudness, 181.

Machines, efficiency of, 101; law of, 102.
Magnet, 244; field, 299.
Magnetic, declination, 246; dip, 247; field, 249; induction, 254; line of force, 249; poles, 244, 251.
Magnetism, 244; summary, 257.
Magnetoscope, 246.
Magnification, 381.
Manometric flame, 176.
Mariner's compass, 244.
Mass, 25; comparison of masses, 28, 29, 82; unit of, 25; and weight, 72.
Matter, 21; general properties of, 21; kinetic theory of, 235; special properties of, 21, 108; states of, 104.
Measurement of, areas, 8; length, 7; quantities, 6; volumes, 8.
Mechanical advantage of, inclined plane, 100; lever, 95; pulley, 99; screw, 100; wheel and axle, 96.
Mechanical equivalent of heat, 233, 234.
Mechanics, 94.
Melting, 223; points, table of, 223.
Meter, 7.
Microphone, 302.
Microscope, 380.
Mirrors, concave, 368; convex, 368; spherical, 367.
Moment, of force, 33, 45; of inertia, 30; principle of moments, 46, 47.
Momentum, linear, 27, 28.
Motion, 9; angular periodic, 65; linear periodic, 64; Newton's laws of, 31, 33, 50; perpetual, 90; rectilinear, 9; rotary, 9; laws of uniformly accelerated, 16.
Motor, electric, 318.
Multiple, connection in, 284.
Musical interval, 188.

Negative, 4; electrification, 261; pole, 284.
Newton's laws of motion, 31, 33, 50.
Node, 155.
Noise, 179.
Non-conductors, 260.
Normal atmospheric pressure, 130.

Octave, 188.
Oersted, 279.
Ohm, the, 310, 330.
Ohm's law, 321, 322, 323.
Opaque, 332.
Opera glass, 383.
Optical instruments, 376.
Optics, summary, 386.
Ordinate, 4.
Organ pipes, 191; open and closed, 190.
Origin, 4.
Overtones, 185.

Parallel, connection in, 284; forces, 47, 48.
Parallelogram of forces, 41.
Particle, 3.
Pascal's law, 111.
Pendulum, clock, 68; compound, 67; laws of, 67; simple, 66.
Penumbra, 338.
Period, 66; pendulum, 68.
Periodic motion, 64.
Perpetual motion, 90.
Phase, 66.
Photography, 377.
Photometer, 336.
Physics, V, 21; purpose of study of, 1; subjects studied under, 20.

Pipes, organ, open and closed, 190.
Pitch, 182.
Polarization, 288.
Pole, magnetic, 244; of cell, 284.
Position, 4, 5.
Positive, 4; electrification, 261; pole, 284.
Potential energy, 88, 92.
Pound, weight of a, 80.
Power, 102; of a lens, 367; units of, 103, 328.
Press, hydraulic, 111.
Pressure, 109; effect on melting point, 223; electrical, 285, 321, 322; normal atmospheric, 129.
Primary circuit, 296.
Problems, solution of, 17, 422.
Proof plane, 260.
Properties of matter, 20, 21; general, 21; special, 21, 108.
Pulley, 96; mechanical advantage of, 99.
Pumps, 135; air, or vacuum, 140; force, 135; lift, 137.

Quality of a musical tone, 183.
Quantities, constant and variable, 6; directed, 43; measurement of, 6.

Radiation of heat, 209.
Rate, 11.
Ray of light, 335, 361.
Real images, 371–373.
Rectilinear, motion, 9; propagation of light, 333.
Reference, point of, 3; line of, 5.
Reflection, angle of, 344; of light, 342; laws of plane, 345; total, 352; of water waves, 152.
Refraction, angle of, 351; index of, 351; laws of, 352; of water waves, 150.
Resistance, electrical, 309; in parallel, 324; unit of, 310.
Resolution of forces, 41, 42.
Resonance, 160.
Resultant, forces, 40; of parallel forces, 47, 48.
Review, questions for, 389.
Roemer, Ole, 347.
Rotation, pure, 9.
Rule, right hand, for direction of current, 277.
Scale, major, 189.
Screw, 100.
Secondary circuit, 296.
Series, connection in, 284; -wound, 300.
Shadow, 337.
Shunt, circuits, 323; -wound, 300.
Siphon, 134.
Solids, 104; expansion of, 212.
Solution of problems, 17, 422.
Sound, 164; character of sound waves, 171; production of, 166; representation of sound waves, 175; speed of, 174.
Sounder, telegraphic, 314.
Specific gravity, 117; of liquids, 120; methods of finding, 118; table of, 118.
Specific heat, 229.
Spectacles, 379.
Spectrograph, 386.
Spectroscope, 385.
Spectrum, prismatic, 363.
Speed, 10, 11; of light, 348; of sound, 174.
Starting box, 319.
Steam, engine, 237; turbine, 240.
Stereopticon, 388.
Storage cell, 288.
Strain, 105.
Strength, 106.
Stress, 107.
Summary, electric currents, 329; electrostatics, 273; energy, 93; falling bodies, 63; friction, 56; gravitation, 82; light, a wave motion, 361; magnetism, 257; Newton's laws of motion, 51, 52; on optics, 386; water waves, 153; waves in strings, 160.
Surface tension, 108, 122, 126.

Table of, absolute value of gravity, 73; coefficient of linear expansion, 213; critical angles, 354; density of water, 217; efficiency of some machines, 102; electrical conductivities, 261; heat conductivities, 204; magnetic declinations, 246; melting and boiling points, 223; musical nomenclatures, 190; pressure of saturated water vapor, 226; specific gravities, 118; specific heats, 230; speed of sound, 174.

Telegraph, 314.
Telephone, 301.
Telescope, 381.
Temperature, 197; heat and, 196; measurement of, 198.
Thermodynamics, 234.
Thermometer, 200; fixed points of a, 201.
Thermometric scales, centigrade, 201; Fahrenheit, 202; Galileo's, 219; ideal gas, 221.
Time, 11, 12.
Torricelli's experiment, 129.
Total reflection, 352.
Transformer, 296.
Translation, pure, 9.
Translucent, 332.
Transmitter, Bell, 301.
Transparent, 332.
Transverse waves, 154.
Turbine, steam, 240.

Umbra, 338.
Units, of acceleration, 14; C. G. S. and F. P. S. units, 26; of current, 306; electrical, 329; of electrical pressure, 322; fundamental and derived units, 26; of heat, 228; of length, 7; of mass, 25; of power, 103; of resistance, 310; of velocity and speed, 12; of volume, 8; of work, 85.

Vaporization, heat of, 231.
Variable quantities, 6.
Velocity, 11; of light, 346; of sound, 174.
Vibration, complete, 66; damped, 149.
Virtual images, 371–373.
Voice, human, 193.
Volt, the, 322, 330.
Volta, 281.
Voltaic cell, 281, 282; typical, 283.
Voltameter, 305.
Voltmeter, 325.

Water, equivalent, 229.
Water waves, 148.
Watt, the, 328.
Wave front, 361.
Wave length, 150.
Wave motion, 147.
Waves, 146; longitudinal, 154; stationary, 152, 155; in strings, 153; transverse, 154; water, 148.
Weight, and mass, 72.
Wheel and axle, 96.
Work, 84; units of, 85, 86.

Yard, 7.
Young's experiment, 359.

The Macmillan Company desires to call attention to the following courses in the elementary sciences, for which special text-books have been prepared:

Agriculture

WARREN, G. F. **The Elements of Agriculture**

Cloth, 12mo, $1.10 net

This book is designed for use in high schools, academies, and normal schools. Its purpose is to make the subject of agriculture in the schools comparable in extent and thoroughness with physics, mathematics, history, and literature.

Astronomy

BALL, Sir ROBERT E. **The Elements of Astronomy**

Cloth, 16mo, $.80 net

A clear and simple presentation of the basal truths of astronomy by an eminent scientist. There are eleven full-page plates and an adequate index.

Biology

BAILEY, L. H., and COLEMAN, W. M. **First Course in Biology.**

Cloth, 12mo, $1.25 net

In three parts: PART I, PLANT BIOLOGY; PART II, ANIMAL BIOLOGY; PART III, HUMAN BIOLOGY. The book is profusely illustrated, so that much of the difficulty of obtaining specimens of all the species is obviated.

Botany

BAILEY, L. H. **Beginner's Botany**

Cloth, 12mo, $.60 net

This is Part I of Bailey and Coleman's First Course in Biology.

BAILEY, L. H. **Botany, an Elementary Text-book for Schools**

Cloth, 12mo, $1.10 net

The subjects treated here are: The nature of the plant, the relation of the plant to its surroundings, the minute structures of plants, and the determination of the kinds of plants.

Botany

OSTERHOUT, W. J. V. **Experiments with Plants**

Ill., Cloth, 8vo, $1.25 net

The central idea of the book has been to select those significant experiments which illustrate fundamental laws and far-reaching principles, and so to simplify them that they may be in every way available to school work.

SETCHELL, W. A. **Laboratory Practice for Beginners in Botany**

Cloth, 12mo, $.90 net

In order to avoid the necessity for much apparatus, the larger plants have been chosen to work with, and elaborate laboratory outfits are therefore not required.

Chemistry

RICHARDSON, G. M. **Laboratory Manual and Principles of Chemistry**

Cloth, 12mo, $1.10 net

A course of experiments arranged for elementary students of chemistry, containing also a clear and concise statement of some of the fundamental theories and principles of chemistry.

ROSCOE, Sir H. E., and HUNT, J. **Inorganic Chemistry for Beginners**

Cloth, 16mo, $.75 net

This book is intended for pupils beginning the study of chemistry. The elementary principles of the science are fully treated, while the description of the elements and their compounds is restricted to a few well-chosen examples.

SNYDER, HARRY. **Chemistry of Plant and Animal Life**

Cloth, 12mo, $1.25 net

Although the book is highly technical, it is put in popular form and made comprehensible to the intelligent reader and student, even though he lack the advantage of a course in chemistry. It treats chemistry from the standpoint of the farmer, deals largely with those questions which arise in his experience, and will prove an invaluable aid in countless directions.

Chemistry

WADDELL, JOHN. **A School Chemistry**

Cloth, 12mo, $.90 net

The aim of this book is to aid the pupil in the discovery of new facts, to enable him to see connections, to show how facts lead to theory, and how theory aids in investigation and in the discovery of further facts.

Geography

GONNER, E. C. K. **Commercial Geography**

Cloth, 12mo, $.75 net

Designed as a text-book to be used in schools and classes, and to furnish an outline sketch which may be filled in and elaborated by the teacher or lecturer.

REDWAY, JACQUES W. **The New Basis of Geography**

Cloth, 12mo, $1.00 net

This book sets forth in an elementary manner the relations between human activities and geographic environment. It is intended, not for a class-room manual, but for the preparation of the teacher in the educational side of geography.

Physical Geography

TARR, RALPH S. **New Physical Geography**

Cloth, 12mo, $1.00 net

Every topic generally considered as properly belonging to physical geography receives concise, accurate, and interesting treatment. The illustrations constitute a very strong feature of the book, being 568 in number. This text will be found to be the most teachable of all the physical geographies now on the market.

TARR, RALPH S. **First Book of Physical Geography**

Cloth, 12mo, $1.10 net

This is a thoroughly scientific work, and yet its treatment is so elementary that it may be used profitably in the higher grades of the high school. The vast amount of information with which the book is packed is given to the student through observation, careful reading, and discriminating thinking. The work is an admirable presentation of practical pedagogy.

TARR, RALPH S. **Elementary Physical Geography**

Cloth, 12mo, $1.40 net

A more advanced book than the others on the same subject by Professor Tarr, designed especially for students in the latter part of the high school course.

Physiology

COLEMAN, WALTER MOORE. **Series of Physiologies**

Each volume, 12mo, Cloth

A Health Primer	$.35 *net*
Physiology for Beginners	$.40 *net*
Lessons in Hygienic Physiology	$.50 *net*
Elements of Physiology	$.90 *net*

The books of this series are distinguished by an interesting and attractive style and by the thoroughness and variety of the methods by which the application of the study is made to the pupil's own life and surroundings. Throughout the course the pupil is led to regard healthful living not as an irksome duty, but as the simplest, happiest, and most reasonable living.

FOSTER, Sir M., and SHORE, E. **Physiology for Beginners**

Cloth, 12mo, $.75 net

This little work is intended for those who, without any previous knowledge of the subject, desire to begin the serious study of physiology. A sound knowledge of physiology cannot be gained without some acquaintance with chemistry and physics, and consequently the main body of the text is preceded by a chapter on chemical and physical facts.

PUBLISHED BY

THE MACMILLAN COMPANY

64–66 Fifth Avenue, New York

Cloth

5 net

) net

' net

net

and

ods

ife

rd

it,

1 milli-gram (mgm.)	= 0.001 gram (gm.)	10 mgm.	= 1 cgm.
1 centi-gram (cgm.)	= 0.01 gram	10 cgm.	= 1 dgm.
1 deci-gram (dgm.)	= 0.1 gram	10 dgm.	= 1 gm.
1 gram (gm.)	= 1 gram		
1 deka-gram (Dgm.)	= 10 gram	10 gm.	= 1 Dgm.
1 hecto-gram (hgm.)	= 100 gram	10 Dgm.	= 1 hgm.
1 kilo-gram (kgm.)	= 1000 gram	10 hgm.	= 1 kgm.

1 kilogram = 2.204622 avoirdupois pounds.

For foreign postage 30 grams is the equivalent of the avoirdupois ounce.

Customary, or British Measures

Length

12 inches (in.)	= 1 foot (ft.)
3 ft.	= 1 yard (yd.)
5½ yd.	= 1 rod (rd.)
40 rd.	= 1 furlong
8 furlongs	= 1 mile (mi.)

Area

144 sq. in.	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
30¼ sq. yd.	= 1 sq. rd.
160 sq. rd.	= 1 acre (A.)
640 A.	= 1 sq. mi.

Volume

1728 cu. in.	= 1 cu. ft.
27 cu. ft.	= 1 cu. yd.

Dry Measure

4 gills (gi.)	= 1 pint (pt.)
2 pt.	= 1 quart (qt.)
8 qt.	= 1 peck (pk.)
4 pk.	= 1 bushel (bu.)

Liquid Measure

16 fluid ounces (fl. oz.)	= 1 pt.
2 pt.	= 1 qt.
4 qt.	= 1 gallon (gal.) = 231 cu. in.
31½ gal.	= 1 barrel (bbl.)

Weight (or Mass)

Avoirdupois

16 drachms	= 1 ounce (oz.)
16 oz.	= 1 pound (lb.)
2000 lb.	= 1 ton (T.)

Troy

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pwt.	= 1 ounce (oz.)
12 oz.	= 1 pound (℔.)

Apothecaries

20 grains (gr.)	= 1 scruple
3 scruples	= 1 dram
8 drams	= 1 ounce
12 ounces	= 1 pound

5760 grains = 1 ℔. apothecaries or troy.
7000 grains = 1 lb. avoirdupois.

Customary and Metric Equivalents

1 inch	= 2.54001 cm.	1 cm.	= 0.3937 in.
1 foot	= 30.4801 cm.	1 m.	= 39.37 in.
1 yard	= 91.4402 cm.	1 m.	= 1.0936 yd.
[illegible]	[illegible]	[illegible] km.	= 0.62137 mi.
[illegible]	[illegible]	[illegible] q. cm.	= 0.1550 sq. in.
[illegible]	[illegible]	[illegible] . m.	= 1.196 sq. yd.
[illegible]	[illegible]	[illegible]	= 0.0610 cu. in.
[illegible]	[illegible]	[illegible] dm.	= 61.023 cu. in.
[illegible]	[illegible]	[illegible] m.	= 1.308 cu. yd.
[illegible]	[illegible]	[illegible]	= 0.9081 qt. dry
[illegible]	[illegible]	[illegible]	= 1.0567 qt. liquid
[illegible]	[illegible]	[illegible]	= 0.03215 ounce troy
[illegible]	[illegible]	[illegible]	= 0.03527 ounce avoirdupois
[illegible]	[illegible]	[illegible]	= 2.2046 lb. avoirdupois

[illegible] $\pi^2 = 9.8696$.

2. 1 [illegible] second = 33,000 ft.-lb per minute = [illegible]

3. 1 ca[illegible] ram-meter.

4. 7.8 [illegible] of water 1° Fahrenheit.

5. 1 [illegible] .001118 gm. per second.

6. [illegible] 332 meters per second or about [illegible] neter per second or about 2 feet [illegible]

7. [illegible] ers per second or 186,000 miles per [illegible]

8. Th[illegible] alories per gram.

9. The b[illegible] C. is 536 calories per gram.

10. The spe[illegible]

11. Circumf[illegible] ter multiplied by 3.14159 (about $\frac{22}{7}$), or [illegible]

12. Area of [illegible] multiplied by 3.14159, or *area* $= \pi r^2$.

13. Volume [illegible] cube of the diameter multiplied [illegible]

14. [illegible] gled triangle, extract the [illegible] base and altitude (Pythago-

15. One liter of air, N.T.P., weighs 1.293 gm.

16. $\sqrt{2} = 1.414$, $\sqrt{3} = 1.732$, $\sqrt{5} = 2.236$, $\sqrt{10} = 3.162$.